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Keywords
Monetary policy, bank competition, risk-taking and banking market structure, investment demand elasticity, aggregate risk and social welfare.

JEL Codes
G2, D4, D61

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Monetary Policy with Aggregate Risk:
Real Sector and Welfare

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February 2023

Abstract

This paper shows theoretically the linkages among monetary policy rate, the real sector demand for loans with supply shocks, aggregate risks, and social welfare. We prove that a) when the loans' demand is elastic bank competition and the policy rate decrease risks and increase the amount of lending to firms b) these effects are reinforced as the number of banks in the banking market raises. We provide theoretical support to the empirical findings that a competitive environment, or an elastic demand for investments, renders the monetary policy more effective and increases welfare (Aghion et al 2019), on the contrary, uncompetitive structures obtain opposite effects (Wang et all 2022). The policy implications are that the welfare maximizing policy rate can be lower it could be lower than set by the Central Bank when there is high inflation (Rogoff, 2017). c) As in this economic phase of perfect diversification difficulties because of aggregate risks, the policy rate is more effective in welfare increasing if the banking sector is competitive.

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I. INTRODUCTION

Monetary policy and its real effects are a current concept that recurs for any type of crisis. Central banks use tools, called monetary instruments, such as interest rates to adjust the money supply to keep the economy prosperous in relation to the phases of the business cycle.

Until a few decades ago, economists and experts were quite sure of the effectiveness of monetary policies (Clarida et al 2000).

This work highlights how the effectiveness of monetary policies depends not only on the structure of the aggregate supply and demand curves, Adam and Weber (2019)[1], but also on the structure of the banking system.

Carrillo et al. (2021) highlight how monetary policy can fail due to coordination difficulties and even causes welfare costs. Also, differently to previous studies, Zhu et al (2021) find that the BoE responds relatively less to financial conditions and relatively more to inflation projections when the Global Financial Crisis and Great Recession period is included.

This paper develops a model that analyzes the effects of the policy rate in the presence of a flexible credit demand market, representing the real economy and the demand for investments. Furthermore, monetary policy output is studied considering the level of competitiveness of the banking system.

Wang et al. (2022) find that bank market power explains much of the transmission of monetary policy to borrowers, with an effect comparable to that of bank capital regulation. This aspect is crucial because the structure of the banking system can make monetary policies ineffective. As this has real effects, it is of crucial importance: Huang et al (2021) show that the relationship between inflation and growth is negative or positive depending on the liquidity structure of the firms when investing in R&D. This result is linked with the firm size and the welfare analysis shows that Friedman's rule, in general, is not socially optimal. Aghion et al (2019) use industry-level and firm-level data from the euro area to look at the effects on sectoral growth. This work demonstrates the empirical effect of monetary policy on growth. An important aspect, given that with the runaway epidemiological crisis, monetary policy is one of the tools that can be used to stimulate growth and total welfare.

A considerable amount of paper is analyzing the effects of monetary policy on credit supply and the expansion of the real economy as in Rodnyansky and Olivier Darmouni (2017). However, works that dealt with the reverse demand are scares or absent. That is, what is the effect of monetary policy when the demand for investment is more or less rigid and in relation to the structure of the banking system?

[1] Firm productivity trends deliver radically different predictions for the optimal inflation rate.

Precisely, the structure of the banking channel as transmission of monetary policy has extensively been investigated, dating from Gertler and Gilchrist (1994), Bernanke and Gertler (1995), Clarida et al (2000), Jiménez and Ongena (2012) and Jiménez et al. (2014). While the implication given by the structure of the budget of the enterprises has been studied solely recently and empirically by Jiménez et al (2017). They exploit an administrative dataset of loan applications matched with bank and firm variables covering Spain from 2002 to 2010. Bank balance-sheet strength determines the granting of loan applications only in crisis times, while firm balance-sheet strength – notably leverage – determines strongly this granting in both good and crisis times. Their findings underscore the importance of the strength of corporate balance sheets over credit supply for credit availability.
However, there are some sectors where the demand for loans can be more or less elastic and this may depend on structural factors (eg construction and heavy manufacturing). This aspect has been quite neglected by the financial theory and in the consideration of the effects of monetary policy. This paper aims at filling theoretically this gap and to shed light on the effects of the policy rate in the presence of a more or less flexible credit demand market.
Academic literature and policy makers are very much concerned about different credit frictions and the optimal monetary policy (Cúrdia and Woodford, 2016), about the effects of monetary policy on bank risk-taking (Jiménez, et. al, 2014) and, extensively, on the role of bank capital for monetary policy transmission (Gambacorta and Shin, 2016). Other kind of frictions has also been explored. For instance unemployment and sticky prices (Ravenna and Walsh 2011 and Andrés et al 2013), Andrés et al (2013) analyze optimal monetary policy in a dynamic model with frictions. They find that productivity and financial shocks imply a short-run trade-off between stabilization goals as the traditional macro-literature argues. Such policy trade-offs become amplified as banking competition increases, due to the fall in lending spreads and the resulting increase in financial leveraging. Therefore, the paper shows an important link between policy trade-offs and banking competition without studying, as we do, the direct impact of different banking market structures on the monetary policy. We fill this gap building a simple model that leaves aside any friction, and clearly illustrates how monetary policy is more (or less) effective in competitive markets and with different degree of rigidity in the demand for investments.
As in Tirole (2015), generally, it is believed that competition protects consumers from the political influence of lobbies, and forces producers to deliver products and services at cost. However, since competition is rarely perfect, markets fail, and market power must be kept in check. This aspect is particularly delicate for banks. The aim of our paper is to explain the linkages between different banking market structure and the welfare maximizing policy. We concentrate of the recent different form of monetary policy and analyze them highlighting different credit demand's and supply's

structures. It is important to consider the credit demand and supply structures' because these are basically stable and depend on long-established banking traditions in the various countries.

Monetary policy is the main instrument that Central Banks uses to stabilize prices and to prevent acceleration of prices (Greenspan 2004). The 2007's crisis renews interest by academicians and policy makers on the linkages between monetary policy, risks, and the role of the banking system structure.[2]

On this light, we aim to analyze the kind of effect that a competitive banking system have with this respect. We analyze the welfare implication of easy monetary conditions in relation to the elasticity of the demand for lending and the banking market structure. With different arguments, this paper demonstrates that the structure of the banking market generates different welfare effects regardless of the liquidity aspect. The mechanism, through which economic growth is obtained, passes through monetary policy transmission through the banking channel.

The rest of the paper is organized as follows. Section II introduces the model. The equilibrium is derived in section III. With the equilibrium characterization, we explain results and the properties of the solution. We prove that the policy rate works better as the elasticity of loans demand is high and the banking sector is competitive. These results are robust to banks' diversification choices as proven in section IV, to constant monitoring cost and banks' capital choices (sections V and VI). The welfare is assessed in section VII: the elasticity of loans demand, independently on the number of banks, i.e. the competitive environment in the banking sector, increases total welfare and the policy rate is effective in improving the total output of this simple economy.

These results have important economic implications. If the demand for investments in an economy is elastic, the policy rate can be kept at a lower level and the welfare outcomes are more pronounced. Section VII concludes. The proposition proofs are in the appendix.

II. MODEL

The banking sector is modeled as an imperfect competitive market in which banks compete à la Cournot and choose the risk of their loan portfolios.

[2] In his speech in September 2009, the vice-president of the ECB, Papademos addressed the relation between price stability and financial stability, the policy instruments and the objectives of the Central Bank. He argues that there is not trade-off between price stability and financial stability: monetary policy aimed to preserve price stability promotes financial stability and, conversely, a stable financial system enhances the effectiveness of monetary policy. However, there can be, continues the speech, situations in which there may be a trade-off. These are due to impact of structural or behavioral changes.

A bank picks up a p, probability of success that requires an effort cost represented by the quadratic function $e\frac{p^2}{2}$, where $e > 0$ is the cost of effort. Where e is positive and the bank wishing to increase the loan probability of success has a bigger cost.

The number of banks is indexed by $i = 1,...,N$. Each bank chooses its amount of loans l_i, its amount of deposits d_i, and its level of risk to maximize its profit function.

The demand for loans is represented by the function $R^L(L)$. Where, $R^L(L)$ is a downward sloping inverse demand of loans depending on the whole aggregate amount of loans $L = N \times l_i$. The supply of deposits is $R^D(D)$ which is an increasing inverse function of deposits that depends on the total amount of deposits $D = N \times d_i$. Finally, the bank may invest in a risk-free asset the amount $\Phi_i \geq 0$ defined as

(1) $$\Phi_i = d_i - l_i .$$

We assume that the bond pays the policy rate R^P.

The economy ends in one period only. To summarize, at the beginning of the period each bank chooses the amounts of deposits, loans, and the level of risk in a partial equilibrium framework. The investment in bonds results as consequence of the bank's portfolio choice. The rate on bonds is interpreted as the channel of policy transmission though. Indeed, policy makers affect banks' investment decisions though the level of this rate. Therefore R^P is assumed to be exogenous and *it represents the monetary policy rate*.

III. NO AGGREGATE RISK

In this section, we analyze the equilibrium with no aggregate risk. In this case the bank can fully diversify its portfolio and has the following profit function:

(2) $$\Pi_i = pR^L(L)l_i - R^P\Phi_i - R^D(D)d_i - e\frac{p^2}{2}$$

Substituting (1) in (2) and rearranging terms we obtain

(3) $$\Pi_i = (pR^L(L) - R^P)l_i + (R^P - R^D(D))d_i - e\frac{p^2}{2},$$

where we assume that the revenue satisfies the strict concavity condition $R^L_{LL}(L)\frac{L}{N} + 2R^L_L \leq 0$

.

The equilibrium is for each bank the set of (p,l,d) that solves (2) subject to (1). Therefore, the first order conditions for a symmetric equilibrium are

(4) $$p\left(R^L_L(L)\frac{L}{N} + R^L(L)\right) - R^P = 0$$

(5) $$R^P - R^D_D(D)\frac{D}{N} - R^D(D) = 0$$

(6) $$R^L(L)\frac{L}{N} - ep = 0$$

Now, let's assume the following: $R^L(L) = AL^{-\alpha}$ and $R^D(D) = BD^{\gamma}$ where $\alpha > 0$, $\gamma \geq 1$. We verify the concavity condition of the revenue function, that is $R^L_{LL}(L)\frac{L}{N} + 2R^L_L \leq 0$. Substituting in (4), (5) and (6),

(7) $$AL^{-\alpha} p - R^P - A\alpha L L^{-1-\alpha} p = 0$$

(8) $$R^P - B\gamma D D^{-1+\gamma} - BD^{\gamma} = 0$$

(9) $$A\frac{L}{N}L^{-\alpha} - ep = 0$$

Substituting (9) in (7) and rearranging terms,

(10) $$L^* = \left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{\frac{1}{1-2\alpha}}$$

and

(11) $$p^* = \frac{A\left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{\frac{1-\alpha}{1-2\alpha}}}{eN},$$

then, differentiating p^* with respect to N,

(12) $$p^*_N = \frac{R^P\alpha\left(\frac{eNR^P}{A^2(1-\alpha)}\right)^{-\alpha}}{AN(3\alpha - 1 - 2\alpha^2)},$$

the derivative with respect to R^P is

(13) $$p^*_{R^P} = \frac{\left(\dfrac{eNR^P}{A^2(1-\alpha)}\right)^{\frac{1-\alpha}{1-2\alpha}-1}}{A(1-2\alpha)}.$$

Also differentiating the amount of loans for R^P

(14) $$L^*_{R^P} = \frac{eN\left(\dfrac{eNR^P}{A^2(1-\alpha)}\right)^{\frac{1-\alpha}{1-2\alpha}-1}}{A^2(1-2\alpha)},$$

Observing (13) and (14) the derivatives are positive if $\alpha < \frac{1}{2}$.

While, for completeness, we report the optimal amount of deposits

(15) $$D^* = \left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}},$$

here we observe that deposits increase always in the policy rate, R^P. Although this is a known result, we highlight the ability of the model to capture the deposit dynamics in response to monetary policy changes.

Proposition 1. *In a partial equilibrium environment, for an elastic loans' demand,* $\alpha \leq \frac{1}{2}$*, the bank risk choice is decreasing as* N *increases.*

Proposition 1 states a risk-taking result that holds in partial equilibrium. In general equilibrium, De Nicolo' and Lucchetta (2009) and Lucchetta (2016) show that overall risk in the economy decreases in bank competition. Here, the result is proven in partial equilibrium when the elasticity of demand is high.

Proposition 2. *The monetary policy rate,* R^P*, decreases banks' risk-taking and increases bank lending if* $\alpha < \frac{1}{2}$.

Then, we find that a higher policy rate makes the bank to invest prudently, and it increases the amount of loans.

The intuition behind is the elasticity of the demand for loans. In our example, for the function $AL^{-\alpha}$, a low value of α implies a high elasticity in loans. Note that the demand is vertical as the elasticity decreases (α increases).

It follows that for an elastic demand for loans the banks tend to decrease risk-taking and the policy rate is effective in reducing risks.

As known, the elasticity of demand depends on several factors such as income, substitutes, necessity and durability. The presence of perfect substitutes is a factor that relies on competition or contestability. Consumers can switch easily from one bank to another. Therefore, the more contestable is the market the higher is the efficiency gain in terms of low risk that derives from an increasing number of banks. At the same time the higher is the elasticity of demand and the more is effective in reducing risk-taking the level of the policy rate.

Here we aim to check the sensitivity of the choice of risk and of the amount of loans to the policy rate with respect of different levels of N.

Proposition 3. *The bank level of risk chosen decreases in* R^P *and the amount of loans increases in* R^P *more rapidly as* N *increases.*

This suggests that policy makers should pay attention to the degree of concentration in the banking system as a whole. This renders less effective the monetary policy effect.

IV. NO PERFECT DIVERSIFICATION

This section answers the question whenever the above results continue to hold with no perfect diversification. A financial institution often cannot diversify properly its portfolio, or it can diversify at some cost. Following we analyze the equilibrium with no perfect diversification, i.e. when there is aggregate risk, and with the choice of the level of diversification of the bank.

When banks cannot diversify the profit function to be maximized is

(16) $$\Pi_i = p(R^L(L) - R^P)l_i + (R^P - R^D(D))d_i - e\frac{p^2}{2}$$

the first order condition for the deposits is the same as in the previous diversify case, while the optimality conditions for loans and p are respectively

(17) $$p\left(R_L^L(L)\frac{L}{N} + R^L(L) - R^P \right) = 0$$

and

(18) $$\left(R^L(L)\frac{L}{N} - R^P \right) - ep = 0 \,.$$

Solving with the assumed functional forms

(19) $$L^{**} = \left(\frac{A}{R^P} \right)^{\frac{1}{\alpha}}$$

and

(20) $$p^{**} = \frac{\left(\frac{A^2}{R^P} - R^P \right)\left(\frac{A}{R^P} \right)^{\frac{1}{\alpha}}}{eN} \,.$$

Differently from before, here it is easy to see that the amount of loans do not depend on N and the choice of p decreases in N.

These results hugely contrast with that obtained with perfect diversification. Further, we will assume that perfect diversification inside the bank can be achieved with a technology. This is close to reality. The bank has the ability to diversify its portfolio (although not completely). Therefore, in addition to the monitoring technology, there is a diversification technology that allows reach a level of diversification between $(0,1)$.

There is the possibility to use a costless (by now) new technology that allows a degree of diversification. Offering these free technology called $\theta \in (0,1)$, the bank choose (θ, l, d, p)

(21) $$\Pi_i = \theta(pR^L(L) - R^P)l_i + (1-\theta)p(R^L(L) - R^P)l_i + (R^P - R^D(D))d_i - e\frac{p^2}{2} \,.$$

The first order conditions (θ, l, p) for an optimum are

(22) $$(pR^L(L) - R^P)l_i - p(R^L(L) - R^P)l_i = 0 \,,$$

(23) $$p(1-\theta)\left(R_L^L(L)\frac{L}{N} + R^L(L) - R^P \right) + p\theta\left(R_L^L(L)\frac{L}{N} + R^L(L) \right) - \theta R^P = 0$$

(24) $$(1-\theta)\frac{L}{N}\left(R^L(L) - R^P \right) + \theta\frac{L}{N}R^L(L) - ep = 0$$

Solving for the given assumptions on the functional forms

$$\widehat{p} = 1$$

(25) $$\widehat{\theta} = \frac{eN\left(\frac{A - A\alpha}{R^P}\right)^{-\frac{1}{\alpha}} + R^P - \frac{AR^P}{A - A\alpha}}{R^P},$$

(26) $$\widehat{L} = \left(\frac{A - A\alpha}{R^P}\right)^{\frac{1}{\alpha}}.$$

This means that if the bank may choose a diversification technology, it would be that one that allows to achieve $p = 1$. It is interesting to observe that the level of diversification, (25) is increasing in the number of banks.

Proposition 4. *Whenever the bank diversifies, it would choose a level of diversification such as the number of banks increases the policy rate is more effective and the level of risk decreases.*

Proposition 4 highlights that when the bank diversify, even not completely, the results of the previous section are still verified.

V. CONSTANT MONITORING COST

It is interesting to analyze the case with constant monitoring cost. The bank profit function, for the perfect diversification case, is

(27) $$\Pi_i = (pR^L(L) - R^P)l_i + (R^P - R^D(D))d_i - e\frac{p^2}{2}l_i.$$

Solving with the assumed functional forms

(28) $$L^\circ = \left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{\frac{1}{2\alpha}},$$

(29) $$p^\circ = \frac{A\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{-\frac{\alpha}{2\alpha}}}{e}$$

and finally, the demand of deposits is as before

(30) $$D^\circ = \left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}}.$$

The amount of loans and the level of risk do not depend on the number of banks but on the elasticity of the demand for loans and on the risk-free rate, R^P.

Proposition 5. *With constant monitoring cost, the amount of lending and the level of risk-taking do not depend on the number of banks. However, the lending increase and the level of risk decreases as* $\alpha < \frac{1}{2}$ *decreases.*

Proposition 5 states a result already found in the previous sections. The elasticity of demand for loans plays a key role in the level of lending and the degree of risk taking. The more the demand is elastic and the lower is the risk taken by the bank with constant monitoring cost.

VI. CONSTANT MONITORING COST AND K

It is interesting to analyze the case with constant monitoring cost and capital. The bank profit function, for the non-perfect diversification case, is

(27) $$\Pi_i = p(R^L(L) - R^P)l_i + (R^P - R^D(D))d_i - R^E k - e\frac{p^2}{2}l_i,$$

with $k = l_i - d_i$.
Differentiating we obtain
Solving with the assumed functional forms

(28) $$L^\circ = \left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{\frac{1}{2\alpha}},$$

(29) $$p^\circ = \frac{A\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{-\frac{\alpha}{2\alpha}}}{e}$$

and finally, the amount of deposits is as before

(30) $$D^\circ = \left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}}.$$

The amount of loans and the level of risk do not depend on the number of banks but on the elasticity of the demand for loans and on the risk-free rate, R^P.

Proposition 6. *With constant monitoring cost and capital, the amount of lending and the level of risk-taking do not depend on the number of banks. However, the lending increase and the level of risk decreases as* $\alpha < \frac{1}{2}$ *decreases.*

Proposition 6 states a result already found in the previous sections. The elasticity of demand for loans plays a key role in the level of lending and the degree of risk taking. The more the demand is elastic and the lower is the risk taken by the bank with constant monitoring cost.

VII. WELFARE

In this section, we analyze the optimal policy rate and its welfare effects. The question is how a social planer will choose the rate that maximizes welfare given a perfect diversify bank portfolio, a no perfect diversify bank portfolio and constant monitoring cost.

The set of the optimal allocation is represented by the triplet (L,D,p) that maximizes the utility of a representative bank, π^{B}, and a representative depositor, R^{DC}. These representative agents are the aggregate payoff functions of a bank and a depositor. We can do this because agents are equal. Therefore, a social planer chooses R^{P*} solving

(31) $$Max V \equiv \pi^{B} + R^{DC}(D).$$

Substituting for the competitive allocations:

(32) $$\underset{R^P}{Max} V = \pi_i(L^*, D^*, p^*) + R(D^*).$$

For a perfect diversify economy, the function that must be maximized by regulator is:

(33) $$V = \left(\frac{A\left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{\frac{1-\alpha}{1-2\alpha}}}{eN} A\left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{-\frac{\alpha}{1-2\alpha}} - R^P \right) \frac{\left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{\frac{1}{1-2\alpha}}}{N}$$
$$+\left(R^P - B\left(\frac{R^P}{B(1+\gamma)}\right)\right)\frac{\left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}}}{N} - e\frac{\left(\frac{A\left(\frac{R^P eN}{(1-\alpha)A^2}\right)^{\frac{1-\alpha}{1-2\alpha}}}{eN}\right)^2}{2} + B\left(\frac{R^P}{B(1+\gamma)}\right).$$

The derivative of the above function with respect to R^P simplifies to

$$(34) \qquad V_{R^P} = \frac{N + (1+\gamma)\left(\left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}} - \left(\frac{NR^P e}{(1-\alpha)A^2}\right)^{\frac{1}{1-2\alpha}}\right)}{(1+\gamma)N}.$$

Observing the above derivative, it may be written as

$$(35) \qquad V_{R^P} = \frac{N + (1+\gamma)\left(D^* - L^*\right)}{(1+\gamma)N},$$

then, by the previous results we know that R^P has a different impact according to the value of α. Moreover, the number of banks has an important effect on the optimal policy rate chosen by regulator.

Proposition *7. In the perfect diversify case, as N raises the optimal policy rate, R^{P*} can be set lower and the policy rate increases welfare more rapidly. However, for $\alpha > \frac{1}{2}$, the optimal policy rate, R^{P*}, is smaller than for $\alpha \le \frac{1}{2}$.*

The intuition of proposition 7 is that a competitive banking system renders the policy rate more effective. However, if the demand for loans is relatively elastic the policy rate must be higher than when the demand is elastic.

Here the welfare analysis is applied to no perfect diversify case. The central planer maximizes the following objective function:

$$\tilde{V} = \left(\frac{\left(\frac{A^2}{R^P} - R^P\right)\left(\frac{A}{R^P}\right)^{\frac{1}{\alpha}}}{eN} A\left(\frac{A}{R^P}\right)^{-1} - R^P\right)\frac{\left(\frac{A}{R^P}\right)^{\frac{1}{\alpha}}}{N}$$

$$(36) \qquad + \left(R^P - B\left(\frac{R^P}{B(1+\gamma)}\right)\right)\frac{\left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}}}{N} - e\frac{\left(\frac{\left(\frac{A^2}{R^P} - R^P\right)\left(\frac{A}{R^P}\right)^{\frac{1}{\alpha}}}{eN}\right)^2}{2} + B\left(\frac{R^P}{B(1+\gamma)}\right)$$

The findings are summarized in the following proposition.

Proposition 8*. In the no perfect diversify case, as* N *raises the optimal policy rate,* R^{P*} *can be set lower and the policy rate increases welfare more rapidly. However, for* $\alpha > \frac{1}{2}$*, the optimal policy rate,* R^{P*}*, is lower than for* $\alpha \leq \frac{1}{2}$*.*

Therefore, the no perfect diversify case has the same results and policy implications of the perfect diversify case.

Finally, we analyze welfare maximizing rate for the case of constant monitoring cost.

(37)
$$\hat{V} = \left(\frac{A\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{-\frac{\alpha}{2\alpha}}}{e} A\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{-\frac{1}{2}} - R^P \right) \frac{\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{\frac{1}{2\alpha}}}{N}$$
$$+\left(R^P - B\left(\frac{R^P}{B(1+\gamma)}\right)\right) \frac{\left(\frac{R^P}{B(1+\gamma)}\right)^{\frac{1}{\gamma}}}{N} - \frac{A\left(\frac{A^2(1-2\alpha)}{R^P 2e}\right)^{-1}}{2} + B\left(\frac{R^P}{B(1+\gamma)}\right)$$

Proposition 9*. With constant monitoring cost, welfare increases in* R^P *more rapidly as N increases. The equilibrium policy rate is smaller when the demand is elastic.*

The key result is that a rise in the number of banks increases the monetary policy efficacy for different assumptions on the profit function of the bank. In other terms, when the banking system is less concentrated a smaller policy rate is necessary to optimize welfare.

VII. CONCLUSION

This paper shows, thought a microeconomic framework, the total welfare effect, and the relation between monetary policy and risk-taking with respect to the banking market structure and the demand for loans elasticity. We show that that a) when the demand of loans is elastic the competition and the policy rate decrease risks and increase the amount of lending to firms b) in any case, these effects are reinforced as the number of banks in the banking market raises.

The policy implications are relevant since Central Banks often face the trade-offs between financial stability and price stability. We show that if the banking system is competitive such a trade-off does not exist. Hence, as the number of banks increases and the demand for loans is sufficiently elastic, the monetary policy decreases risks.

In general, we also show that the CVH is not verified even in a partial equilibrium framework. De Nicolo' and Lucchetta (2009) show that whenever a general equilibrium setting is introduced, the CVH's vanish and competition is beneficial in reducing the level of risk. This paper shows that the CVH effect, when the demand for loans is elastic, does not hold at a single bank balance sheet level. We find that the welfare maximizing policy rate is smaller when the demand for loans is elastic. This has the interpretation that a low rate (monetary easing) is not welfare detrimental if the lending demand is sufficiently elastic.

These results are important to support policy decisions regarding market structure and monetary policy rate as stabilizing tool.

APPENDIX

Proposition 1. *In a partial equilibrium environment, for an elasticity of demand* $\alpha \le \frac{1}{2}$*, the bank risk choice is decreasing as* N *increases.*

Proof. Taking the derivative (12), $p^*_N = \dfrac{R^P \alpha \left(\dfrac{eNR^P}{A^2(1-\alpha)} \right)^{-\alpha}}{AN(3\alpha - 1 - 2\alpha^2)}$, it is positive when $3\alpha - 1 - 2\alpha^2 > 0$.

Then, for $\alpha \le \frac{1}{2}$, the relation between p and N is positive. QED

Proposition 2. *The monetary policy rate,* R^P*, decreases banks' risk-taking and increases bank lending if* $\alpha < \frac{1}{2}$.

Proof. Simply observe that (13) and (14) are positive if $1 - 2\alpha > 0$, then when $\alpha < \frac{1}{2}$. QED

Proposition 3. *The bank level of risk chosen decreases in* R^P *and the amount of loans increases in* R^P *more rapidly as* N *increases.*

Proof. Using derivative (13) , $p^*_{R^P} = \frac{\left(\frac{eNR^P}{A^2(1-\alpha)}\right)^{\frac{1-\alpha}{1-2\alpha}-1}}{A(1-2\alpha)}$, we analyze it we respect to N. Formally, we differentiate $p^*_{R^P}$ with respect to N

$$p^*_{R^P N} = \frac{\alpha N\left(\frac{eNR^P}{A^2(1-\alpha)}\right)^{\frac{\alpha}{1-2\alpha}}}{A(N-2\alpha N)^2},$$

this derivative is always positive.

The level of loans in response to monetary policy is gauged by (14). Indeed, taking the derivative with respect to N

$$L^*_{R^P N} = \frac{(\alpha-1)^2\left(\frac{eNR^P}{A^2(1-\alpha)}\right)^{\frac{1-\alpha}{1-2\alpha}}}{NR^P(1-2\alpha)^2}$$

which is also positive. QED

Proposition 4. *Whenever the bank is allow to diversify, it would choose a level of diversification such as the number of banks increases the policy rate is more effective and the level of risk decreases.*

Proof. By (25) the diversification parameter increases in the number of banks. Therefore, as the number of banks increases, the level of diversification raises and the propositions 1, 2 and 3 results apply. QED.

Proposition 5. *With constant monitoring cost, the amount of lending and the level of risk-taking do not depend on the number of banks. However, the lending increase and the level of risk decreases as* $\alpha < \frac{1}{2}$ *decreases.*

Proof. The first order derivative for the equilibrium loans is

$$L^\circ_\alpha = \frac{2^{-1-\frac{1}{2\alpha}}\left(\frac{A^2(1-2\alpha)}{R^P e}\right)^{\frac{1}{2\alpha}}\left(2\alpha+(1-2\alpha)Log\left[\frac{A^2(1-2\alpha)}{2R^P e}\right]\right)}{\alpha^2(2\alpha-1)},$$

which is positive if $\alpha < \frac{1}{2}$. Therefore, lending increases in the elasticity of demand. The same argument applies to p°. QED

Proposition 6. *In the perfect diversify case, as* N *raises the optimal policy rate,* R^{P*} *can be set lower and the policy rate increases welfare more rapidly. However, for* $\alpha > \frac{1}{2}$, *the optimal policy rate,* R^{P*}*, is smaller than for* $\alpha \leq \frac{1}{2}$.

Proof. By proposition 3 the lending increases more rapidly as the number of bank raises and deposits are increasing in the policy rate. By (35), deposits increases welfare, and loans decreases it. Then, the optimal R^{P*} is lower when N is high. By proposition 2, policy rate decreases lending if $\alpha \geq \frac{1}{2}$, therefore in this case the policy rate as to be smaller. QED

***Proposition** 7*. *In the no perfect diversify case, as* N *raises the optimal policy rate,* R^{P*} *can be set lower and the policy rate increases welfare more rapidly. However, for* $\alpha > \frac{1}{2}$, *the optimal policy rate,* R^{P*}, *is lower than for* $\alpha \leq \frac{1}{2}$.

Proof. The first order derivative with respect to the policy rate implies a smaller policy rate as the number of bank increases. Let's set $A = B = \gamma = e = 1$ and $N \to 1$, the first order condition for an optimum is

$$\tilde{V}_{R^P}(N=1) = 2\left(\frac{1}{R^P}\right)^{\frac{1}{\alpha}}\left(R^{P3} + \left(\frac{1}{R^P}\right)^{\frac{1}{\alpha}}(1 - 4R^{P2} + 3R^{P4})\right) + \alpha\left(R^{P3} + R^{P4} - 2\left(\frac{1}{R^P}\right)^{\frac{1}{\alpha}}\left(R^{p3} + \left(\frac{1}{R^P}\right)^{\frac{1}{\alpha}}(3R^{P4} - 1)\right)\right) = 0,$$

while, for $N \to \infty$

$$\tilde{V}_{R^P}(N \to \infty) \to 0.$$

Then, for greater N, the policy rate may be smaller than when the banking system is concentrated. Regarding the value of α, simply observe that when the demand is elastic, the welfare is higher for small value of the policy rate. QED

Proposition 8. *With constant monitoring cost, welfare increases in* R^P *more rapidly as N increases. The equilibrium policy rate is smaller when the demand is elastic.*

Proof. By proposition 5 the lending increase and the level of risk decreases as $\alpha < \frac{1}{2}$ decreases. The welfare increases in R^P more rapidly in the number of banks because $\widehat{V}_{R^P N} > 0$. QED

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