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Externality control and endogenous market structure under uncertainty: The price vs. quantity dilemma☆

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ABSTRACT

In a competitive industry where production entails a negative externality, a welfare-maximizing regulator considers, as control instruments, setting a cap on the industry output or levying an output tax. We embed this scenario within a dynamic setup where market demand is stochastic and market entry is irreversible. We firstly determine the industry equilibrium under each policy and then determine the cap level and the tax rate that maximize welfare in each case. We show that a first-best outcome can be achieved through the tax policy while the cap policy may only qualify as a second-best alternative.

1. Introduction

Weitzman (1974) has studied the dilemma concerning which policy tool to use – taxation or a cap on quantity – when maximizing welfare in a context where production entails a negative externality. His seminal work has led to decades of intense study of this dilemma within the fields of public economics and environmental economics. Koenig (1985), in which the adverse externality is due to the entry of foreign firms that negatively affects the domestic industry, Spulber (1985), who assumes that market entries increase pollution, and Anderson (1993), where the development of new properties harms residents by reducing open space, are a few examples for studies dealing with this dilemma and the wide range of topics to which it is relevant.

Weitzman's model was static, and his main result was that a cap on quantity performs better than a tax if the marginal benefit curve is steeper than the marginal cost curve, otherwise a tax does better. As per the survey by Tang et al. (2019), much of the subsequent research has remained within a static framework, and in particular, very few studies have explored

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this issue under the rather realistic assumptions of stochastic profitability, irreversibility of the investment, and flexibility in choosing the investment timing. The literature on the industry equilibrium under these conditions is vast, and Dixit and Pindyck (1994) presents much of it. So far, the only study analyzing Weitzman's "tax vs. cap" dilemma within the typical framework of this literature is Baldursson and von der Fehr (2004) who study the efficacy of price and quantity controls in a setup where the investment in abatement is irreversible for some firms in the industry and reversible for others. Their main finding repeats Weitzman's result, as they conclude that the relative slope of the cost curve with respect to the slope of the benefit curve determines whether tax or cap is the better policy. Yet, their modeling does not follow the standard lines of the literature on investment under uncertainty and, in particular, the uncertainty in their model springs from the unorthodox assumption that the number of the firms adopting a reversible abatement technology is stochastic.

Other studies which add adverse externalities to the typical framework of the investment under uncertainty literature are few and they focus either on taxes or on a cap policy, or possibly analyze the equilibrium under both policies but not comparing the two.[1] Thus, for example, Jou and Lee (2008) who consider a real estate market assuming that newly developed properties, by reducing open space, have an external cost, have focused on tackling this externality with a tax policy. Similarly, Lee and Jou (2007) show how the regulator can correct the negative externality by imposing a density ceiling control. Di Corato and Maoz (2019) search for the optimal cap on private firms' entries in markets where production has adverse externalities and the output price is stochastic, but do not consider the possibility of a tax policy.[2]

To fill this void, in this paper, we set up a model analyzing the industry equilibrium under perfect competition in a dynamic setup where market demand is stochastic and entry is irreversible. Production generates an external cost for Society, which is assumed increasing and convex in the industry output. We then consider the following two polar policy instruments for regulating the negative externality: (i) a quantity control exerted by introducing a cap on the industry output and then rationing market entries, (ii) a price control exerted by imposing an output tax. We characterize the industry equilibrium under each policy and try to find which of the two policy tools leads to greater welfare.

Our main findings are as follows. In the case of a cap policy, we find that optimality leads to on an internal welfare-maximizing cap level. Rather intuitively, this level is the quantity at which the marginal market surplus is equal to the marginal social cost, i.e., the sum of private and external costs. If the current market quantity is still below this level, then it is optimal to set this level as a cap on market quantity and allow the market to expand towards it over time, based on the strategic entry considerations of the firms. If, on the other hand, the current market quantity is already above this welfare-maximizing cap level, then, due to irreversibility, the market cannot revert to this welfare-maximizing cap level, and it is optimal to set the cap at the current level, i.e., to immediately ban any further entry. We also find that the welfare-maximizing cap level is increasing in the level of market uncertainty, which implies that greater profit uncertainty makes the policy maker allow more entries. This result is based on the same effect by which the uncertainty premium counterbalances the external cost as described in Di Corato and Maoz (2019).

The result that the optimal policy in this case is based on an internal welfare-maximizing cap level is novel because the only other study searching for an optimal cap within a competitive environment with profit uncertainty and investment irreversibility is Di Corato and Maoz (2019) which reaches a different result. Specifically, they assume that the external cost is a linear function of the quantity produced, and therefore reach the result that it optimal to have either no cap at all, if the uncertainty is high enough, and otherwise to set the cap at the current market quantity. In contrast, in this study we assume that the external cost is a convex function of market quantity, as the empirical literature about pollution damages often suggests, and therefore reach the result of an internal welfare-maximizing cap level.

In the case of a tax policy, we show that the output tax can be viewed as an additional cost of production for the private firm whose impact can be studied using the model by Leahy (1993). In his model, the price threshold triggering market entries is increasing in the cost of production, therefore, the introduction of an output tax, by raising the entry threshold, delays market entries with respect to the scenario where the industry is not regulated. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold. We then determine the tax rate maximizing welfare and find that it must be set equal to the marginal external cost associated with the industry output supplied at each time point. This implies that further market entries become less and less likely as the industry output increases since the higher the tax burden, the higher the entry threshold.

Finally, when comparing the cap and the tax policies, a relevant trade-off emerges. With the cap, the industry output is bounded but the cap does not affect its temporal evolution with respect to the scenario where the industry is not regulated. In contrast, with the output tax, there is no limit to market entries but the tax affects the temporal evolution of the industry output by delaying market entries.

We then show that a first-best outcome can be achieved only by adopting a tax policy and that, in this respect, the ability to affect the entry timing is crucial. In fact, by setting the tax rate equal to the marginal external cost associated with the

[1] Other studies incorporating externality control within the typical framework of investment under uncertainty, but not addressing the Weitzman's dilemma are Chao and Wilson (1993), Xepapadeas (2001) and Zhao (2003). Chao and Wilson (1993) show that the option value affects the investment in abatement under uncertain permit prices. Xepapadeas (2001) studies how a firm respond to environmental policy when deciding their investment in abatement and location under uncertainty about the output price, the policy context and the technology. Zhao (2003) shows that when considering uncertainty about the abatement costs the magnitude of the option value is larger when introducing taxes rather than permits.

[2] Their model is based on Bartolini (1995) where, differently, no external damages associated with firms' investment and production are explicitly considered and the cap is taken as exogenous.

industry output supplied at each time point, the externality is fully internalized by the firms and consequently entries occur only when the exogenous stochastic shifts in market demand yield an associated gain in terms of market surplus covering the marginal social cost of an additional unit of the good. In contrast, a cap policy may only be considered as a second-best alternative since, in the presence of a cap, market entries occur at a socially suboptimal time. In fact, when the cap is not binding, firms keep entering the market using the same strategy that would be followed in the absence of regulation while, when the cap is binding, market entries do not occur at all, even when they would be beneficial from a welfare perspective.

The main reason for the difference between our result of tax superiority and Weitzman's result springs from the dynamic setting that we portray, in contrast to the static analysis provided by Weitzman (1974). Due to that, while the uncertainty that policy makers face in Weitzman's model is about the current situation, in our model they have a perfect view of current situation and the uncertainty they face is about its future development. Thus, at each point in time, the policy makers in our model can fit the best tax rate for the current situation. With a cap policy, this is not possible because, by the very nature of this policy tool, the cap level is assumed to be fixed credibly over a sufficiently long time period. In that sense, the dynamic setting gives an advantage to the tax policy.

To shed more light on the role of the dynamic modelling in making the tax policy better than the cap policy, we study, in the final section of this article, a case in which the tax rate is constant as in Weitzman (1974) and much of the related literature. We have taken the resemblance to the static Weitzman's model to the extreme by assuming also that the tax is levied immediately at time 0. This Weitzman-like modeling of the tax policy has indeed led to a result that resembles Weitzman result that the cap policy may be the better one if, as a function of the industry output, the external cost curve is sufficiently steeper than the benefit (market surplus) function. Yet, the analysis of this case within a dynamic framework allows us to show the time inconsistency of such a policy. Moreover, it takes very little deviating from this extreme case to make the tax policy doing, once again, better than the cap policy in terms of welfare maximization. We demonstrate it with allowing the government the freedom to choose the time at which the tax is imposed. We show that, once again, this makes the tax policy dominate the cap policy regardless of the specifics of the benefit and cost functions.

As a side product of that analysis, we also develop a dynamic version of the measure for the relative steepness of the external cost function with respect to that of the benefit curve. The need for that arises because in a static model this measure is based on the slopes at the single equilibrium point while in our dynamic model the equilibrium moves from one point to another as firms endogenously enter the market over time.

The paper remainder is as follows. In Section 2, we present our model set-up. In Section 3, we determine the industry equilibrium under no policy intervention. In Section 4, we introduce the two policy instruments for externality control and determine the optimal entry strategy under each policy. We determine the optimal cap and the optimal tax rate, compare the two policies and discuss our findings. In Section 5, we provide some final remarks and conclude.

2. The basic model

Within a continuous time setting, we consider a competitive industry comprised of a large number of identical firms that producing a certain good. Their individual size, dn, is infinitesimally small with respect to the market and they are all price takers.[3]

At each time point $t \geq 0$, the demand for this good is given by:

$$P_t = X_t \cdot \varphi(Q_t), \tag{1}$$

where P_t and Q_t are the market price and quantity of the good, respectively, $\varphi(Q_t)$ is a deterministic component of the market demand with $\varphi(Q_t) > 0$ and $\varphi'(Q_t) < 0$ for any $Q_t > 0$, and $\lim_{Q_t \to \infty} \varphi(Q_t) = 0$. The term X_t, is a demand shift factor that evolves stochastically over time according to the following Geometric Brownian Motion:

$$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t, \tag{2}$$

where $\mu > 0$ is the drift parameter, σ is the instantaneous volatility, and dZ_t is the increment of a standard Wiener process satisfying $E(dZ_t) = 0, \quad E(dZ_t)^2 = dt$ at each t.

Each firm rationally forecasts the future evolution of the whole market. Market entry is free and an idle firm can enter the market at any time. By entering the market, the firm commits to offer permanently one unit of the good at each t. This implies that the industry output, Q_t, equals the number of active firms in the industry. Producing one unit of the good has a cost equal to $M > 0$.

Production entails a negative externality that firms do not incur. Its cost for Society, $D(Q_t)$, is a function of the industry output Q_t. We take the standard assumptions that $D'(Q_t) > 0$ and $D''(Q_t) > 0$ for any $Q_t > 0$ and $D(0) = 0$, implying that the external cost is positive, increasing and convex in the industry output.

Last, firms are risk-neutral profit maximizers and discount future payoffs using the interest rate r.[4] As standard in the literature, we assume that $r > \mu$ to secure that the firm's value is finite.

[3] Assuming that firms are of infinitesimally small size is standard in models investigating the competitive equilibrium in a dynamic setting. See for instance Jovanovic (1982), Hopenhayn (1992), Lambson (1992), Leahy (1993), Dixit and Pindyck (1994, Ch. 8), Bartolini (1993, 1995) and Moretto (2008).

[4] Note that introducing risk aversion would not affect our results, but merely require the development of the analysis under a risk-neutral probability measure. See Cox and Ross (1976) for further details.

3. Industry equilibrium under no policy intervention

Let start by considering a scenario where no control policies are present. Under our model setup, a firm contemplating market entry is facing the same situation as the investors in Leahy (1993). Therefore, in the following, we use Leahy's analysis in order to determine the optimal entry strategy.[5]

At each time t, an idle firm has to decide whether to enter the market or not. By assumption, a firm entering the market commits to produce permanently one unit of the good at a cost equal to M. The present value of the associated flow of production costs, i.e. M/r, can be viewed as the irreversible investment that a firm must undertake in order to enter the market. As future revenues are uncertain, market entry will occur when the expected profitability of such investment is sufficiently high.

Let $V(X,Q)$ be the value of an active firm given the current levels of X and Q. The standard no-arbitrage analysis in Appendix A shows that

$$V(X,Q) = Y(Q)\cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r}, \tag{3}$$

where $\beta > 1$ is the positive root of the quadratic equation

$$\tfrac{1}{2}\cdot\sigma^2\cdot x^2 + \left(\mu - \tfrac{1}{2}\cdot\sigma^2\right)\cdot x - r = 0. \tag{3.1}$$

In (3), the term $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$ represents the expected present value of the flow of the firm's future profits conditional on Q remaining forever at its current level. Therefore, the first term, $Y(Q)\cdot X^{\beta}$, accounts for how future market entries reduce the value of the firm, as the firm's profit falls when the industry output Q increases.

Two boundary conditions are required for finding the threshold function $X^*(Q)$ triggering market entry. The first one is the *Value Matching Condition*:

$$V[X^*(Q),\ Q] = 0, \tag{4}$$

and the second one is the *Smooth Pasting Condition*:

$$V_X[X^*(Q),\ Q] = 0. \tag{5}$$

Condition (4) is a standard zero-profit condition at the entry requiring that the value of an idle firm, which is null under free entry,[6] must equal the value of an active one. Condition ((5), in contrast, is an optimality condition that concerns the evolution of the demand shift, X_t, over time. Each time the process $\{X_t,\ t\geq 0\}$ hits the threshold $X^*(Q)$ a new firm enters the market and the price of the good, $P(Q)$, lowers since the supplied market quantity output has increased (see Dixit and Pindyck, 1994, Ch. 8, pp. 252–260). Thus, $X^*(Q)$ is an upper reflecting barrier regulating the process $\{X_t, t\geq 0\}$ by keeping its level over time below $X^*(Q)$.

Solving the system [4–5] yields the following result:

Proposition 1. *Entry in a perfectly competitive market occurs every time the process* $\{X_t,\ t\geq 0\}$ *hits the threshold:*

$$X^*(Q) = \frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}}{\varphi(Q)}, \tag{6}$$

where $\hat{\beta} \equiv 1 + \frac{1}{\beta-1} > 1$.

Proof. Follows from applying (3) in (4) and (5).

From $\varphi'(Q_t) < 0$ it follows that the threshold $X^*(Q)$ is an increasing function of Q, implying that the larger the market quantity supplied, the stronger the competition and then, ceteris paribus, the higher the profitability required for entering the market. Fig. 1 schematically shows the entry dynamics based on the threshold $X^*(Q)$.

In time intervals where Q is not changed, the changes in X are translated, via (1), to changes in P. Based on standard properties of Brownian Motions, in such time intervals the proportional connection between X and P, as captured by (1), implies that P is also a Geometric Brownian Motion, and with the same parameters as X. On the other hand, at time instants when X hits the threshold function $X^*(Q)$, then a rise in X is not translated into a rise in P but leads to an increase in Q which keeps P unchanged. This occurs at the following level of P:

$$P^* = X^*(Q)\cdot\varphi(Q) = \hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}, \tag{7}$$

Which makes P^* an upper reflecting barrier regulating the process $\{P_t, t\geq 0\}$ and preventing the price from going above the level P^*. Fig. 2 provides an illustration of these dynamics.

[5] In the following, we will drop the time subscript for notational convenience.

[6] The option to wait is valueless under free-entry since, as entry is attractive for other firms, the firm, by postponing its entry, may lose the investment opportunity Dixit and Pindyck, 1994, Ch. 8, pp. 256-258).

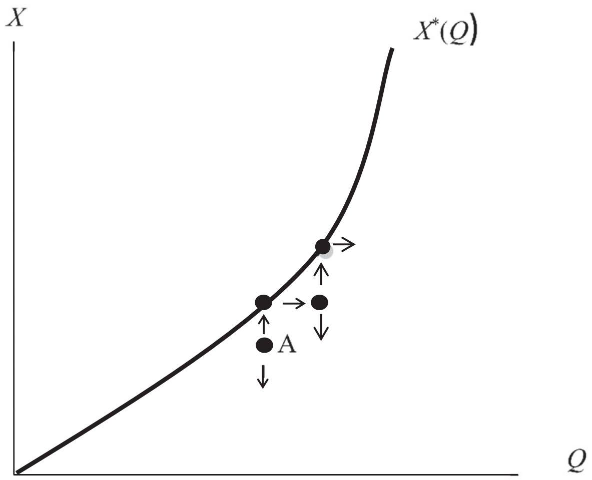

Fig. 1. Demand swings and entry dynamics in a competitive industry. When the market is at a point like A, below the entry threshold, the swings in the demand shit factor, X, do not affect the market quantity. When X hits the threshold function $X^*(Q)$, firm entry leads to an incremental increase in Q making X once again below the threshold line.

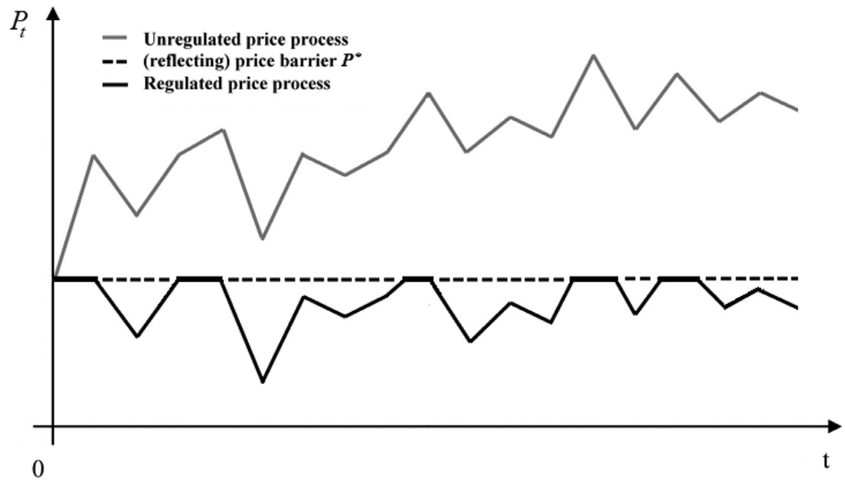

Fig. 2. Price dynamics in a competitive industry.

Note that, by the Marshallian rule, a firm should enter the market as long as $P = X \cdot \varphi(Q) \geq (r - \mu) \cdot \frac{M}{r}$. Hence, by (6), the term $\frac{1}{\beta - 1} > 0$ in $\hat{\beta}$ is the wedge by which the entry threshold should be adjusted in order to take the uncertainty and irreversibility into account (see Dixit and Pindyck, 1994, Ch. 5, Section 2). Last, note that $\frac{dP^*}{d\sigma^2} > 0$ which follows from (7) taken together with the definition of $\hat{\beta}$ and $d\beta/d\sigma < 0$ which is established in appendix A. This means that the higher the demand volatility, the higher the price threshold triggering firm's entry, which implies that market entry is delayed. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold.

4. Industry equilibrium under policy intervention

The optimal entry strategy based on Eq. (6) does not account for the external cost associated with the negative externality that production entails once the firm has entered the market. In this section, we consider two policies for the reduction of the external cost: i) a cap on the industry output and ii) a tax on each unit of output. We first determine the industry equilibrium under each policy and then the level of the cap and the tax rate, respectively, maximizing welfare.

4.1. Industry equilibrium and welfare under a cap on the industry output

Assume that the regulator sets a cap on the industry output. Further, assume that entry licenses are distributed when the cap is announced. Each license allows producing one unit of output and their number is equal to difference between the cap, $\bar{Q}$, and the current level of the industry output, Q. We abstract from how the licenses are distributed since for our purposes their distribution has no other implications than providing to each firm owning a license the right to enter the market.[7]

4.1.1. The optimal entry strategy

The analysis of the firm's optimal entry under rationing is technically similar to the analysis in Section 3. The relevant difference between the two cases is that in this case the option to enter is an asset having a positive value that the firm gives up by entering the market. Thus, alongside the function $V(X,\ Q)$ that represents the value of an active firm, we define the function $F(X,\ Q)$ that stands for the value of the option to enter the market. A standard no-arbitrage analysis, similar to the one conducted in Appendix A for determining the value of an active firm, yields:

$$F(X,\ Q) = H(Q) \cdot X^{\beta}, \tag{8}$$

$$V(X,\ Q) = Y(Q) \cdot X^{\beta} + \frac{P(X,\ Q)}{r-\mu} - \frac{M}{r}, \tag{9}$$

where $H(Q)$ is to be found alongside the threshold $X^*(Q)$ by imposing the following *Value Matching Condition*:

$$V[X^*(Q), Q] = F[X^*(Q), Q], \tag{10}$$

and *Smooth Pasting condition*:

$$V_X[X^*(Q),\ Q] = F_X[X^*(Q), Q]. \tag{11}$$

Condition (10) asserts that the value of the option to enter, that is, the implicit cost of market entry, equals the value of an active firm, that is, the implicit return associated with market entry. Condition (11) secures optimality by imposing that the marginal cost of market entry equals its marginal return. As shown by Dixit (1993), Condition (10) holds for any entry threshold and merely reflects a no arbitrage assumption, while Condition (11) is an optimality condition that holds only at the optimal threshold.

Proposition 2. *In a perfectly competitive market with a cap on the industry output, as long as the quantity in the market, Q, is below the cap, new entries to the market occur every time the process$\{X_t,\ t \geq 0\}$ hits the threshold:*

$$X^*(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \frac{M}{r}}{\varphi(Q)}, \tag{12}$$

or, equivalently, when the process $\{P_t, t \geq 0\}$ hits the barrier P^*, as captured by (7).

Proof. Follows from applying (8) and (9) in (10) and (11).

Notably, the threshold function (12) does not depend on $\bar{Q}$ and is equal to the threshold function (6) determined under no policy intervention. The relevant difference here is that $X^*(Q)$ applies only until the cap $\bar{Q}$ is reached.

By *Proposition 2* and (7), a new firm enters the market every time the process $\{P_t, t \geq 0\}$ hits the upper reflecting barrier P^*. As explained above, this prevents the price from going above the level P^*. However, under a cap policy, the regulation of the price through the barrier control applies only until the cap $\bar{Q}$ is reached and, once there, the output price starts moving freely over time following only the evolution dictated by (2). Fig. 3 provides an illustration of these dynamics.

4.1.2. Welfare and the optimal cap

Once determined the industry equilibrium, in this section we determine the cap level maximizing welfare. This optimal level will trade off the welfare gains associated with lower negative externalities and the losses, in terms of market surplus, due to a lower quantity of the good available on the market once the cap has been reached.

Following a procedure similar to the one conducted in Appendix A for determining the value of an active firm, the expected discounted social welfare, given the current levels of X, and Q and the cap set at $\bar{Q}$, is:

$$W\left(X, Q,\ \bar{Q}\right) = C\left(Q,\ \bar{Q}\right) \cdot X^{\beta} + \int_0^Q \left[\frac{P(X,\ q)}{r-\mu} - \frac{M + D'(q)}{r}\right] \cdot dq, \tag{13}$$

The integral in (13) represents the expected present value of welfare if the current industry output level, Q, will never change. For each unit supplied, the term $\frac{P(X,q)}{r-\mu}$ is the expected present value of the flow of market surplus associated with

[7] Note that, as shown by Bartolini (1995), the government may fully extract the producer's surplus by allocating licenses through a competitive auction.

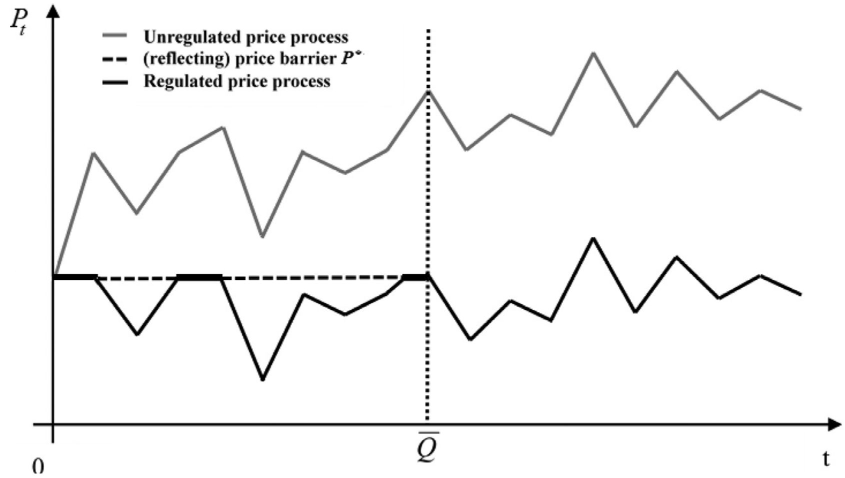

Fig. 3. Price dynamics under a cap on the industry output.

the supply of each unit of the good, whereas the term $\frac{M+D'(q)}{r}$ is the present value of the flow of social costs associated with its production, i.e. private production costs plus external costs. The first term, $C(Q,\ \bar{Q}) \cdot X^{\beta}$, captures instead the contribution of future market entries to welfare.

At $X^*(Q)$ the following *Value Matching Condition* must hold:

$$W_Q\left[X^*(Q),\ Q,\ \bar{Q}\right] = 0 \tag{14}$$

Condition (14) is a standard boundary condition stating that each market entry raises welfare by $\frac{P[X^*(Q),\ Q]}{r-\mu} - \frac{M+D'(Q)}{r}$ via the supply of an additional unit of the good, but at the same time, it also lowers welfare by $C_Q(Q,\ \bar{Q}) \cdot X^*(Q)^{\beta}$ in that the forgone market entry lowers the value of the contribution to welfare by future market entries.

Further, at $Q = \bar{Q}$ we must impose that:

$$C\left(Q,\ \bar{Q}\right) = 0 \tag{15}$$

The intuition behind Condition (15) is that the term $C(Q,\ \bar{Q}) \cdot X^{\beta}$ in (12) captures the welfare associated with future entries to the market. No such changes are possible once Q has reached the cap $\bar{Q}$ and thus $C(Q,\ \bar{Q})$ must be null at $Q = \bar{Q}$.

Based on (13), (14) and (15) we show in Appendix B that:

$$C\left(Q,\ \bar{Q}\right) = \int_{Q}^{\bar{Q}} \left[\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot \frac{1}{X^*(q)^{\beta}} \cdot dq. \tag{16}$$

Differentiating $C(Q,\ \bar{Q})$ with respect to $\bar{Q}$ yields:

$$C_{\bar{Q}}\left(Q,\ \bar{Q}\right) = \left[\frac{P^*}{r-\mu} - \frac{M+D'\left(\bar{Q}\right)}{r}\right] \cdot \frac{1}{X^*\left(\bar{Q}\right)^{\beta}}. \tag{17}$$

Eq. (17) leads to the following Proposition:

Proposition 3. *(a) If the current industry output level, Q, is sufficiently large so that* $\frac{P^*}{r-\mu} \le \frac{M+D'(Q)}{r}$ *then it is optimal to set the cap at the current Q, i.e., to immediately ban any further market entry; (b) otherwise, if the current industry output level, Q, is sufficiently small so that* $\frac{P^*}{r-\mu} > \frac{M+D'(Q)}{r}$ *then the optimal level of the cap, denoted by* $\bar{Q}^*$*, is the root of the following equation:*

$$\frac{P^*}{r-\mu} = \frac{M+D'\left(\bar{Q}^*\right)}{r}, \tag{18}$$

Proof. Follows from (17) and the convexity of $D(Q)$.

By *Proposition 3*, if $\frac{P^*}{r-\mu} \le \frac{M+D'(Q)}{r}$, a ban deterring any further market entry is optimal. This is because the expected present value of the flow of market surplus added by the firm entering the market, i.e. $\frac{P^*}{r-\mu}$, does not cover the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. Otherwise, if

$\frac{P^*}{r-\mu} > \frac{M+D'(Q)}{r}$, it is optimal setting a cap at a level higher than the current industry output level Q. Firms will then be allowed to enter the market until the industry output level $\bar{Q}^*$ is reached and where $\frac{P^*}{r-\mu} = \frac{M+D'(\bar{Q}^*)}{r}$.

Implicit differentiation of (18) yields that:

$$\frac{d\bar{Q}^*}{d\sigma^2} = -\frac{1}{D''\left(\bar{Q}^*\right)} \cdot \frac{M}{(\beta-1)^2} \cdot \frac{d\beta}{d\sigma^2} > 0, \tag{19}$$

where the inequality follows from $D''(Q) > 0$, $\beta > 1$ and $\frac{d\beta}{d\sigma^2} < 0$. Thus, the higher the demand uncertainty the larger the optimal cap and the larger the industry output that the regulator is going to allow for. The reason for that is that a higher σ^2 leads, via its effect on the option wedge $\hat{\beta}$, to a higher P^* and, consequently, to a slower entry process in expected terms. This implies that while, on the one hand, the external cost increases at a slower speed, on the other hand, we incur into losses of market surplus since, having a higher entry barrier, P^*, market prices may reach relatively higher levels before a new firm enters the market. Further, one must account for the fact that, even tough, once reached the cap, the external cost stops increasing, there is a loss of market surplus in that the output price evolves freely being absent, as no firms may enter the market, the barrier control preventing it from going above the level P^*. The loss of market surplus may be relevant and consistently,

$$\lim_{\sigma^2\to\infty} \bar{Q}^* = \infty, \tag{20}$$

which means that setting an internal cap, $\bar{Q}^*$, is not optimal since restricting firms' entry is too costly in the presence of high levels of market uncertainty.

Implicit differentiation of (18) also yields:

$$\frac{d\bar{Q}^*}{dM} = \frac{1}{D''\left(\bar{Q}^*\right)} \cdot \frac{1}{\beta-1} > 0, \tag{21}$$

where the inequality follows from $D''(Q) > 0$ and $\beta > 1$. Thus, the higher the production cost the larger the optimal cap and therefore the larger the market size that the regulator is going to allow for. The reason for that is that the larger M, the higher the price that triggers entry, i.e. P^*, and, consequently, the slower the entry process in expected terms. This has, as above, implications for the speed at which the external cost increases and the magnitude of the flow of market surplus.

Last, based on *Proposition 3* and (13), in the case where the optimal cap is at the current Q, the expected discounted social welfare is equal to:

$$W^{cap}(X,Q) = \int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot dq \tag{22}$$

otherwise, when the optimal cap is $\bar{Q}^*$, the expected discounted social welfare is:

$$\begin{aligned} W^{cap}(X,Q) = &\int_Q^{\bar{Q}^*} \left[\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot \left[\frac{X}{X^*(q)}\right]^\beta \cdot dq + \\ &+ \int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot dq \end{aligned} \tag{23}$$

As Dixit and Pindyck (1994, pp. 315–316) show, the term $[\frac{X}{X^*(q)}]^\beta$ is equal to the discount factor $E[e^{-r\cdot T(q)}]$, where $T(q)$ is the time when process $\{X_t,\ t > 0\}$, starting from its current level X, hits the threshold level $X^*(q)$ for the first time. This insight enables the following, rather intuitive, view of the resulting formula for the welfare function, as captured by (23):

- The last term $\int_0^Q [\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}] \cdot dq$ is the integral, over the already supplied Q units of the good, of the expected present value of the flow of social welfare associated with each of those units, i.e. the flow of market surplus $\frac{P(X,q)}{r-\mu}$ minus the flow of social costs $\frac{M+D'(q)}{r}$.
- The first term, therefore, represents the expected present value of the flow of social welfare associated with future entries, those that will add units from the current quantity Q, to the maximum allowed by the cap, that is $\bar{Q}^*$. The value of each future entry comprises two parts:
  - $\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$ which is the expected present value of the flow of social welfare that the added unit of the good would yield, from the moment in which the firm producing it enters the industry, i.e. when the market price is equal to P^*.
  - $[\frac{X}{X^*(q)}]^\beta$ the factor by which the payoff $\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$ is discounted back to current time.

4.2. Industry equilibrium and welfare under an output tax

Assume that the regulator levies a tax $\tau > 0$ per unit of output.

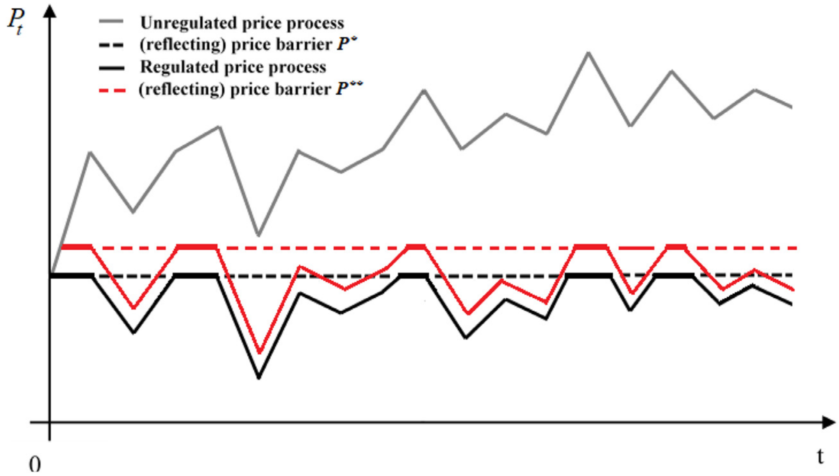

Fig. 4. Price dynamics under an output tax.

4.2.1. Optimal entry strategy

The analysis of the industry equilibrium under an output tax is technically identical to the one conducted in Section 3. The only difference is that here the cost for producing one unit of output is equal to $M + \tau$. Hence:

Proposition 4. *Entry in a perfectly competitive market under an output tax occurs every time the process* $\{X_t, t \geq 0\}$ *hits the threshold:*

$$X^{**}(Q) = \frac{\hat{\beta} \cdot (r - \mu) \cdot \frac{M+\tau}{r}}{\varphi(Q)} > X^*(Q), \tag{24}$$

or, equivalently, the process $\{P_t, \ t \geq 0\}$ hits the barrier:

$$P^{**} = \hat{\beta} \cdot (r - \mu) \cdot \frac{M + \tau}{r} > P^* \tag{25}$$

Proof. Follows from repeating the proof of *Proposition 1*, this time with a private production cost equal to $M + \tau$.

Fig. 4 provides an illustration of the dynamics of the price process in the presence of an output tax policy.

4.2.2. Welfare and the optimal tax rate

The expected discounted social welfare, given the current levels of X and Q, is:

$$W(X, Q, \ \tau) = C(Q, \tau) \cdot X^{\beta} + \int_0^Q \left[\frac{P(X, q)}{r - \mu} - \frac{M + D'(q)}{r} \right] \cdot dq \tag{26}$$

The tax payments from the firms to the government lower their profits but raise the government revenues by the same amount and therefore cancel out of the social welfare. Thus, the only remaining channel by which the output tax affects social welfare is via its effect on the firms' entry thresholds and therefore on entry times. Thus, when setting an optimal tax policy, the regulator is in fact setting an optimal threshold policy. We find that the optimal tax should be set at a level such that both the following *Value Matching Condition*:

$$W_Q[X^{**}(Q), Q] = 0, \tag{27}$$

and *Smooth Pasting Condition*:

$$W_{QX}[X^{**}(Q), Q] = 0 \tag{28}$$

hold.

As above, Condition (27) is a boundary condition stating that at each market entry we have an increase in welfare associated with the supply of an additional unit of the good, i.e. $\frac{P[X^{**}(Q), \ Q]}{r-\mu} - \frac{M+D'(Q)}{r}$, minus the welfare loss associated with the just foregone market entry, i.e. $C_Q(Q, \ \tau) \cdot X^{**}(Q)^{\beta}$. Condition (28), on the other hand, is an optimality condition that leads to the entry pattern that is optimal from the regulator's perspective and to the tax rate that induces it.

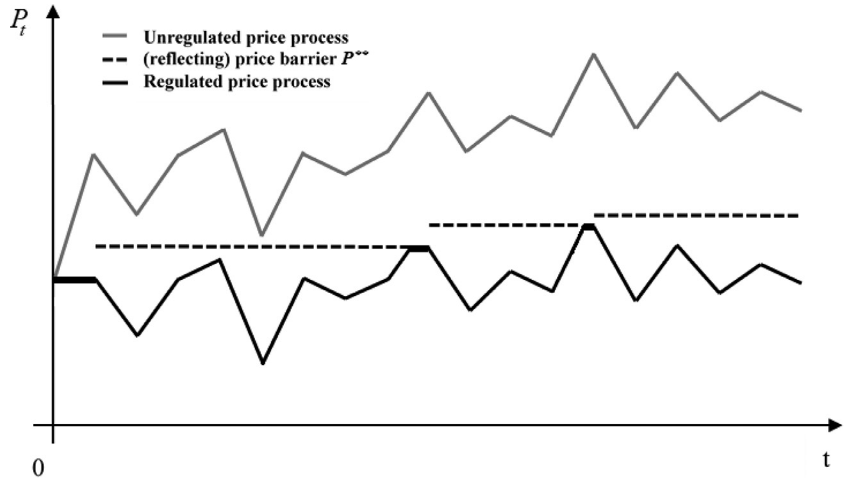

Fig. 5. Price dynamics under optimal output taxation.

Proposition 5. *The government maximizes the expected discounted social welfare by levying the Pigouvian tax:*

$$\tau^*(Q) = D'(Q), \tag{29}$$

and, consequently, each market entry raises welfare.

Proof. (29) follows from applying (26) in (27) and (28). Substituting (29) in (24) yields that the optimal entry threshold satisfies:

$$X^{**}(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \frac{M+D'(Q)}{r}}{\phi(Q)}, \tag{30}$$

which, applying (29) in (25), leads to

$$\frac{P^{**}}{r-\mu} = \hat{\beta} \cdot \frac{M+D'(Q)}{r} > \frac{M+D'(Q)}{r}, \tag{31}$$

which implies that in the equilibrium, market entries are always beneficial since the expected present value of the flow of market surplus added by a new firm entering the market, i.e. $\frac{P^{**}}{r-\mu}$, covers always the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. This is because at each entry the price at entry, P^{**}, is adjusted upward as $\tau^*(Q)$ rises with Q.

Fig. 5 provides an illustration of the dynamics of the price process that follows from the optimal tax policy.

Conditions (27) and (28) also yield:

$$C_Q(Q,\ \tau^*) = -\left[\frac{P^{**}}{r-\mu} - \frac{M+D'(Q)}{r}\right] \cdot \frac{1}{X^{**}(Q,\ \tau^*)^{\beta}} \tag{32}$$

To integrate (32) we use the following boundary condition:

$$\lim_{Q\to\infty} C(Q,\ \tau) = 0 \tag{33}$$

The intuition behind Condition (33) is immediate. In (26), the term $C(Q,\ \tau) \cdot X^{\beta}$ captures the welfare associated with future increases of the industry output. No such changes are expected when $Q \to \infty$ because in that case the entry threshold (24) goes to infinity since, by assumption, $\lim_{Q\to\infty} \varphi(Q) = 0$.

Integrating (32) and applying (33) yields:

$$C(Q,\ \tau^*) = \int_Q^{\infty} \left[\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot \frac{1}{X^{**}(q,\ \tau^*)^{\beta}} \cdot dq. \tag{34}$$

Applying (34) and (1) in (26) yields that the expected discounted social welfare when the tax rate is optimally set is equal to:

$$W^{tax}(X,Q) = \int_Q^{\infty} \left[\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot \left[\frac{X}{X^{**}(q,\ \tau^*)}\right]^{\beta} \cdot dq + \int_0^{Q} \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot dq \tag{35}$$

As in (23), the second integral is the expected present value of the flow of social welfare associated with the already supplied Q units of the good. The first integral represents, therefore, the expected present value of the social welfare associated with future entries that will add units from the current quantity, Q, up to infinity. The expected discounted value of the welfare generated by each future entry is $\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r}$, and it is discounted back to current time through the stochastic discount factor $[\frac{X}{X^{**}(q,\tau^*)}]^{\beta}$.

4.3. Social optimum and industry equilibrium under the two policies

We start by characterizing the social optimum in the considered industry. For the social planner, the relevant problem concerns whether and when to expand the quantity of the good supplied so as to maximize the social welfare (see Dixit and Pindyck, 1994, Chapter 9, Section 1.A).

The expected discounted social welfare, given the current levels of X and Q, is:

$$W(X,Q) = C(Q)\cdot X^{\beta} + \int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right]\cdot dq \tag{36}$$

Denoting the socially optimal threshold for market entry by$X^{SP}(Q)$ and maximizing (36) subject to:

$$W_Q\left[X^{SP}(Q),Q\right] = 0 \; (\textit{Value Matching Condition}), \tag{37}$$

$$W_{QX}\left[X^{SP}(Q),Q\right] = 0 \; (\textit{Smooth Pasting Condition}), \tag{38}$$

$$\lim_{Q\to\infty} C(Q) = 0, \tag{39}$$

yields:

$$X^{SP}(Q) = \frac{\widehat{\beta}\cdot(r-\mu)\cdot\frac{M+D'(Q)}{r}}{\varphi(Q)} = X^{**}(Q,\tau^*) > X^*(Q). \tag{40}$$

By (40), it immediately follows that:

Proposition 6. *A first-best outcome can be achieved through the optimal tax policy while the optimal cap policy may serve only as a second-best alternative.*

The optimal tax policy found in the previous section in a decentralized setting leads to the same supply path $\{Q_t,\ t\geq 0\}$ that the social planner would choose in a centralized setting. This is because in the decentralized case of a regulator choosing an optimal tax policy, the only effect the tax has on welfare is via the timing of market entries. Formally, the correspondence can be easily proven by substituting $\tau^* = D'(Q)$ into Conditions (27) and (28) which would then yield Conditions (37) and (38).

The result above deserves some further comment. For a social planner maximizing welfare, a market entry is desirable as far as the associated gain in terms of market surplus covers its marginal social cost. In our set-up where firms may enter the market at any time point over an infinite time horizon, there is always a time point where this condition is met. Therefore, in a decentralized setting, a first-best policy should be one able to delay market entries so that they occur at the "right" time from the social planner's perspective. Our analysis shows that this is feasible only via price control and, specifically, by equating the tax rate to the marginal external cost associated with the industry output supplied at each time point. This allows a complete internalization of the external cost by the firm when setting the entry strategy and, consequently, the industry equilibrium secures a first-best outcome. In contrast, quantity control exerted through a cap policy may only qualify as a second-best alternative. This is because the resulting industry equilibrium is suboptimal for two reasons. First, the cap has no impact on the timing of market entries since firms keep setting their entry strategy without internalizing the associated external cost and, second, once the cap has been reached, there is a loss of potential welfare gains associated with blocked market entries.

From *Proposition 6*, it follows, as a corollary, that

Proposition 7. *The welfare achieved through the optimal tax policy, as captured by (35) is higher than the welfare achieved through the optimal cap policy, as captured by (23), where the welfare gap between the two policies is positive and equal to:*

$$\begin{aligned} &W^{tax}(X,Q) - W^{cap}(X,Q) \\ &= \int_{\bar{Q}^*}^{\infty} \left[\tfrac{P^{**}}{r-\mu} - \tfrac{M+D'(q)}{r}\right]\cdot\left[\tfrac{X}{X^{**}(q,\tau^*)}\right]^{\beta}\cdot dq \\ &+ \int_{Q}^{\bar{Q}^*} \left\{\left[\tfrac{P^{**}}{r-\mu} - \tfrac{M+D'(q)}{r}\right]\cdot\left[\tfrac{X}{X^{**}(q,\tau^*)}\right]^{\beta} \right. \\ &\left. - \left[\tfrac{P^{*}}{r-\mu} - \tfrac{M+D'(q)}{r}\right]\cdot\left[\tfrac{X}{X^{*}(q)}\right]^{\beta}\right\}\cdot dq > 0 \end{aligned} \tag{41}$$

Proof. See Appendix C.

Eq. (41) illustrates the sub-optimality of the cap policy. By *Proposition 5*, the first term in (41) is positive. This term represents the welfare that is created only under a tax policy as it springs from units of the good added after the cap level, $\bar{Q}^*$, is reached. The second integral refers to market entries until $\bar{Q}^*$ is reached, therefore to units added under both policies. The expression inside the integral shows the welfare trade-off between the two policy tools for units in that range:

- The expected present value of the flow of social welfare is higher under the tax policy because the tax makes firms raise their entry threshold and thus leads, via (7) and (25), to $\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r} > \frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$.
- The expected pace at which these units are added is faster under a cap policy, and therefore under the cap policy the welfare generated by each unit is expected to be less heavily discounted, as reflected by $[\frac{X}{X^{**}(q,\,\tau*)}]^\beta < [\frac{X}{X^*(q)}]^\beta$.

As shown in Appendix C, the expression inside the second integral is positive as well, implying that for these units the surplus effect dominates the discounting effect.

We now wish to explore the excess external cost induced by the tax policy with respect to the external costs arising under a cap policy. We denote this gap by *ECG*, which stands for "external cost gap", and based on the analysis so far it satisfies:

$$
\begin{aligned}
ECG = & \int_{\overline{Q}^*}^{\infty} \frac{D'(q)}{r} \cdot \left[\frac{X}{X^{**}(q,\tau^*(q))}\right]^\beta \cdot dq \\
& - \int_{Q}^{\overline{Q}^*} \frac{D'(q)}{r} \cdot \left\{1 - \left[\frac{X^*(q)}{X^{**}(q,\tau^*(q))}\right]^\beta\right\} \cdot \left[\frac{X}{X^*(q)}\right]^\beta \cdot dq
\end{aligned}
\tag{42}
$$

The first term on the RHS of (42) represents the external cost that the tax policy adds by allowing the production of more units, i.e. from $\overline{Q}^*$ onward. The second term in (42) represents the external cost that the tax policy reduces by delaying market entries in the interval $(Q,\ \overline{Q}^*)$. The higher the tax rate $\tau^*(q)$, the higher $X^{**}[q,\ \tau^*(q)]$, the longer the delay, and the higher the resulting reduction in the external cost.

Similarly, we now wish to explore the excess market surplus induced by the tax policy with respect to the market surplus generated under a cap policy. We denote this excess surplus by *MSG*, which stands for "market surplus gap", and based on the analysis so far it satisfies:

$$
\begin{aligned}
MSG = & \int_{\overline{Q}^*}^{\infty} \left(\frac{P^{**}}{r-\mu} - \frac{M}{r}\right) \cdot \left[\frac{X}{X^{**}(q,\tau^*(q))}\right]^\beta \cdot dq \\
& - \int_{Q}^{\overline{Q}^*} \left\{\left(\frac{P^*}{r-\mu} - \frac{M}{r}\right) - \left(\frac{P^{**}}{r-\mu} - \frac{M}{r}\right) \cdot \left[\frac{X^*(q)}{X^{**}(q,\tau^*(q))}\right]^\beta\right\} \cdot \left[\frac{X}{X^*(q)}\right]^\beta \cdot dq
\end{aligned}
\tag{43}
$$

On the RHS of (43), the first term represents the market surplus associated with units of the good added from $\overline{Q}^*$ onward. The second term represents the loss, in terms of lower expected discounted flow of market surplus, which occurs since market entries in the interval $(Q, \overline{Q}^*)$ are delayed under a tax policy more than they would under a cap policy. In this respect, it is worth highlighting the role played by the term $[\frac{X^*(q)}{X^{**}(q,\tau^*(q))}]^\beta$ that lowers the payoff $\frac{P^{**}}{r-\mu} - \frac{M}{r}$ received from each unit under the tax policy. This term is the stochastic discount factor accounting for the time that the process $\{X_t\}$ needs in order to reach the barrier $X^{**}(q,\ \tau^*(q))$ moving from $X^*(q)$, that is, in other words, the additional discounting due to the entry delay induced by taxation. Note that, consistently, the higher the tax rate $\tau^*(q)$, the higher $X^{**}(q,\ \tau^*(q))$, the longer the delay and the higher the loss.

From *Proposition 5* it follows that the first term of *MSG* exceeds the first term in *ECG*, implying that (with respect to the cap policy) within the range of quantities above $\bar{Q}^*$ the excess market surplus induced by the tax policy exceeds the additional external cost that this policy brings.

Comparing market surplus gain and loss from the tax policy (with respect to the cap policy) in the interval $(Q, \overline{Q}^*)$, we show in Appendix C, that:

$$
\begin{aligned}
& \int_{Q}^{\overline{Q}^*} \frac{D'(q)}{r} \cdot \left\{1 - \left[\frac{X}{X^{**}(q,\tau^*(q))}\right]^\beta\right\} \cdot \left[\frac{X}{X^*(q)}\right]^\beta \cdot dq \\
& > \int_{Q}^{\overline{Q}^*} \left\{\left(\frac{P^*}{r-\mu} - \frac{M}{r}\right) - \left(\frac{P^{**}}{r-\mu} - \frac{M}{r}\right) \cdot \left[\frac{X^*(q)}{X^{**}(q,\tau^*(q))}\right]^\beta\right\} \cdot \left[\frac{X}{X^*(q)}\right]^\beta \cdot dq
\end{aligned}
\tag{44}
$$

which implies that the gain in terms of lower expected flow of external costs dominates the loss in terms of lower expected flow of market surplus. Thus, the tax policy does better than the cap policy both in the range $(Q, \overline{Q}^*)$ and in the range above $\overline{Q}^*$.

Last, it is important to note that the comparative advantage of a tax over a cap policy in terms of welfare does not imply that the tax policy yields a lower expected discounted value of the flow of external costs, i.e., that $ECG < 0$. It only implies that even tough, by allowing further entries, the external cost increases under a tax policy (with respect to the cap policy), these entries increase the market surplus by more than they increase the external cost.

4.3.1. The model in the absence of uncertainty

In this sub-section, we show that the results so far, and in particular the superiority of the tax policy over the cap policy, do not depend on the presence of market uncertainty.

In our model, the price volatility σ captures market uncertainty, so analyzing the case where uncertainty vanishes requires looking at the limit for $\sigma \to 0$. It should be noted though that the uncertainty associated with $\sigma > 0$ is about future levels of market demand and, in that sense, it does not produce only uncertainty but it is also responsible for part of the dynamics in the model. Thus, if $\sigma \to 0$, the dynamics in our model are only due to the drift in the demand process.

Following the analysis undertaken in the previous sections, from 3.1) it follows that $\lim_{\sigma \to 0} \beta = r/\mu$ and then, from ((12), that $\lim_{\sigma \to 0} X^*(Q) = M/\varphi(Q)$. This leads, via (7), to $P^* = M$, which implies that firms enter when the market price is equal to the private marginal cost and that the "uncertainty wedge" between the two, captured by (12), vanishes.

Although new entries take place at $P^* = M$, they are associated with rising market surplus, as they spring from rising demand in expected terms since $\mu > 0$. Yet, with rising quantity, the external cost rises too, and at an increasing pace, hence, introducing a cap may increase welfare. In the current limit case, the analysis in sub-Section 4.1 is valid and leads, via (18), to a finite level of $\overline{Q}^*$ that can be determined solving the equation $D'(\overline{Q}^*) = M \cdot \frac{\mu}{r-\mu}$.

The analysis in sub-Section 4.2 of the tax policy case is not affected when considering the limit case of $\sigma \to 0$, and, in particular, the result that the optimal tax rule given by (39) is still valid and leads to a complete internalization of the externality. This, while repeating the analysis of sub-Section 4.3, i) makes the tax policy based on (39) a first-best in this limiting case too and ii) keeps the welfare gap between the two policies, as captured by (41), strictly positive.[8]

4.4. Industry equilibrium and welfare under a constant output tax

In the previous sub-sections, we have assumed that the tax rate could be changed freely and we found that it is optimal levying a Pigouvian tax with a rate increasing in the industry output. In the current section, we examine the extreme case where the policy maker commits to a tax rate that will remain constant from time 0 onwards.

We find that implementing this static-like tax policy in a dynamic setting and comparing it with a cap policy, complements the findings in Weitzman (1974) by determining in a dynamic setting the circumstances under which a cap (tax) policy may do better than a tax (cap) policy from a welfare maximizing perspective. Specifically, we find that the cap policy does better if, as the market quantity changes, the external damage grows sufficiently more rapidly than the benefit in terms of market surplus. Otherwise, the tax policy does better than the cap policy.

The analysis of this case is similar to the one in Section 4.2. In Appendix D, we show that the optimal tax rate is equal to:

$$\tau^{opt} = \frac{\int_Q^\infty \varphi(q)^\beta \cdot D'(q) \cdot dq}{\int_Q^\infty \varphi(q)^\beta \cdot dq} \tag{45}$$

The optimal tax rate, τ^{opt}, is a function of the initial Q and does not vary as the industry output increases due to further market entries. Differentiating with respect to the initial Q yields:

$$\frac{d\tau^{opt}}{dQ} = \frac{\varphi(Q)^\beta}{\int_Q^\infty \varphi(q)^\beta \cdot dq} \cdot \left[\tau^{opt} - D'(Q)\right] > 0, \tag{46}$$

This means that the larger the initial industry output, the larger the external cost and, therefore, the higher should be the tax rate levied in order to control the externality.

It also follows from (45) that τ^{opt} can be infinitely large. More specifically, as q grows, $\varphi(q)$ falls and $D'(q)$ rises, and, in particular, $\varphi(q) \to 0$ and $D'(q) \to \infty$ as $q \to \infty$. Due to that, whether τ^{opt} is infinite or finite depends on the specific functions chosen for $\varphi(q)$ and $D(q)$.

Rearranging (45) yields:

$$\int_Q^\infty \frac{\tau^{opt}}{r} \cdot \left[\frac{X}{X^{**}(q,\tau^{opt})}\right]^\beta \cdot dq = \int_Q^\infty \frac{D\prime(q)}{r} \cdot \left[\frac{X}{X^{**}(q,\tau^{opt})}\right]^\beta \cdot dq\cdot \tag{47}$$

Recall that the term $[\frac{X}{X^{**}(q,\tau^{opt})}]^\beta$ represents the expected discount factor for payoffs associated with each market entry. Thus, (47) shows that at the initial time point where the industry output is Q, the expected discounted flow of tax revenue equals the expected discounted flow of external costs associated with future market entries. It follows that, as τ^{opt} does not vary, this equality will cease to hold as soon a new firm enters the market, with the gap between the tax collection and the external cost expanding at every new entry. This implies that the tax policy chosen is time inconsistent. As there is

[8] See Appendix C for further details.

no reason for not adjusting the tax rate as Q varies so that (47) holds over time, it becomes then doubtful that a constant output tax policy would ever be implemented (see Kydland and Prescott, 1977).

Comparing welfare under the optimal tax policy to welfare under the optimal cap policy, as captured by (23), yields:

$$\begin{aligned} & W^{tax}(Q,X) - W^{cap}(Q,X) \\ & = \int_{Q}^{\infty} \frac{\left(\hat{\beta}-1\right)\cdot M^{\beta}}{r\cdot X^{*}(q)^{\beta}\cdot (M+\tau^{opt})^{\beta-1}} \cdot dq \cdot X^{\beta} - \int_{Q}^{\overline{Q}^{*}} \frac{\left(\hat{\beta}-1\right)\cdot M - D'(q)}{r\cdot X^{*}(q)^{\beta}} \cdot dq \cdot X^{\beta} \end{aligned} \tag{48}$$

Recall from *Proposition 3* that by the definition of $\bar{Q}^{*}$ the integrand within the second integral is positive throughout the integration range. Let then examine how the gap between the welfare under the two policies changes as the externality function becomes steeper with respect to the benefit function. From (45) it follows that τ^{opt} is a natural measure for this relative steepness within the dynamic setting of our model where the equilibrium moves over time from one quantity to the other as demand rises and firms enter the market. This can be viewed as a generalization of the case of a static model, like, for instance, the one in Weitzman (1974), where the relative steepness is measured at the single market quantity in which the equilibrium rests.

We start with the case where τ^{opt} approaches its lower limit. This happens when the external cost is almost linear, implying that D'(Q) is almost constant and that τ^{opt} converges to D'(Q). In Appendix E, we show that in this limiting case the welfare gap, as captured by (48), is positive, implying that the tax policy is better than the cap policy.

At its opposite extremity, $\tau^{\ opt}$ is infinite, therefore, the first integral in (48) goes to 0 and the welfare gap is negative. This implies that the cap policy is better than the tax policy due to the high relative speed at which the external damage grows with quantity.

To analyze further the comparative advantage of a policy over the other, we rearrange the integral in (48) as follows:

$$\begin{aligned} W^{tax}(Q,\ X) - W^{cap}(Q,X) &= \int_{\bar{Q}^{*}}^{\infty} \frac{(\hat{\beta}-1)\cdot(M+\tau^{opt})}{r} \cdot \left[\frac{X}{X^{**}(q,\tau^{opt})}\right]^{\beta} \cdot dq \\ &+ \int_{Q}^{\bar{Q}^{*}} \frac{(\hat{\beta}-1)\cdot(M+\tau^{opt})}{r} \cdot \left[\frac{X}{X^{**}(q,\tau^{opt})}\right]^{\beta} \cdot dq \\ &- \int_{Q}^{\bar{Q}^{*}} \frac{(\hat{\beta}-1)\cdot M - D'(q)}{r} \cdot \left[\frac{X}{X^{*}(q)}\right]^{\beta} \cdot dq \end{aligned} \tag{49}$$

The first term is positive and shows the welfare that the tax policy yields by allowing entries above the quantity of $\bar{Q}^{*}$. The two other terms refer to the range$(Q,\ \overline{Q}^{*})$. In that range the terms $\frac{(\hat{\beta}-1)\cdot(M+\tau^{opt})}{r}$ and $\frac{(\hat{\beta}-1)\cdot M - D'(q)}{r}$, represent the additional welfare from each additional market entry, under the tax policy and the cap policy, respectively. The terms $[\frac{X}{X^{**}(q,\ \tau^{opt})}]^{\beta}$ and $[\frac{X}{X^{*}(q)}]^{\beta}$ represent, for each policy, the expected discount factor for the welfare generated by each market entry. Note that $(\hat{\beta}-1)\cdot(M+\tau^{opt}) > (\hat{\beta}-1)\cdot M - D'(Q)$ which implies that the welfare that each additional unit supplied yields is greater under the tax policy. However, this welfare is also more heavily discounted under the tax policy, as the tax makes firms delay their entries and raises the entry threshold so that $X^{**}(q,\ \tau^{opt}) > X^{*}(q)$. Thus, depending on the assumed functions and parameter values, either one of the two policies can be the better one in the range $(Q,\ \bar{Q}^{*})$.

4.4.1. *Welfare under a constant output tax imposed at optimal timing*

In the previous sub-section, we have examined a case where the tax policy setting is highly limited, in the sense that the tax rate cannot vary over time in response to changed market conditions. This extreme limitation has revived Weitzman's result that a cap policy can do better than the tax policy if the external cost, as a function of market quantity, is sufficiently steeper than the benefit (market surplus) function. In this sub-section, we mildly relax this constraint and, although we still impose the use of a constant tax rate over time, we let the government choose when the output tax should be levied. As we will now show, this makes the tax policy once again better than the cap policy irrespective of the shape of benefit and cost functions.

To see that, we are not going to search for the optimal timing of action but merely consider the case in which the tax is levied when market quantity reaches $\bar{Q}^{*}$. As we will show, even in this case the welfare under the tax policy is better than under the cap policy. Note that the Government may of course do better by choosing optimally when the tax should be levied.

Since under this policy there is no tax until $\bar{Q}^{*}$ is reached, the welfare associated with both policies is the same. Thus, the comparative advantage of a policy over the other must spring from what happens when the market quantity reaches the level $\bar{Q}^{*}$. Based on (25), the optimal welfare level under a cap policy is equal to:

$$W^{cap}\left[X^{*}\left(\bar{Q}^{*}\right),\ \bar{Q}^{*}\right] = \int_{0}^{\bar{Q}^{*}} \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right] \cdot dq \tag{50}$$

As for the tax policy, the tax rate is determined considering $\bar{Q}^{*}$ as initial market quantity in (45). The rest of the analysis repeats the analysis carried out in Appendix D for the case of a constant output tax with just one difference, that is, considering $\bar{Q}^*$ as initial market quantity. Applying the resulting $C(Q,\ \tau^{opt})$ in (35) yields:

$$W^{tax}\left[X^*\left(\bar{Q}^*\right),\ \bar{Q}^*\right] = \int_{\bar{Q}^*}^{\infty} \frac{(\hat{\beta}-1)\cdot(M+\tau^{opt})}{r\cdot X^{**}(q,\ \tau^{opt})^{\beta}}\cdot dq + \int_{0}^{\bar{Q}^*}\left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right]\cdot dq \tag{51}$$

The first term in (51) is positive and the second term equals $W^{cap}[X^*(\bar{Q}^*),\ \bar{Q}^*]$, implying that $W^{tax}[X^*(\bar{Q}^*),\ \bar{Q}^*] > W^{cap}[X^*(\bar{Q}^*),\ \bar{Q}^*]$.

5. Conclusions

In this paper, we have presented a model of endogenous market structure under uncertainty, with production externalities regulated by a cap on the industry output or via an output tax. The main result is that the tax policy dominates the cap policy when aiming at the maximization of the welfare. In particular, we show that the tax policy allows achieving a first-best outcome since the external cost associated with production is fully internalized.

In the case of a cap policy, we have assumed that entry licenses are distributed when the cap is announced. As Bartolini (1995) shows, in the presence of entry licenses, firms holding a license may optimally exercise their option to invest since the threat of being preempted by others is absent. This creates a dynamic entry pattern in which until the cap is reached firms gradually enter the market at time points in which the entry threshold, based on a sufficiently large profitability, is reached. Otherwise, as Bartolini (1993, 1995) shows, if firms are not licensed, this gradual process last only until a certain quantity is reached, and then a competitive run leading the market quantity instantly to the cap is ignited. We do not consider this case since Di Corato and Maoz (2019) have already shown, in a frame similar to ours, that due to the run it yields lower welfare than under rationed entry. Thus, the superiority of the tax policy over the cap policy in the case of licensing implies that it is also better than the cap policy with no licensing.

It should be noticed that the assumption of perfectly competitive firms is crucial for the complete internalization of the external cost. In that respect, our analysis differs from the strand of the literature that investigates the impact of environmental policy showing, mostly using static models, that the internalization of the external cost depends on the degree of market competition. This is because the regulator must take into account the welfare losses that under imperfect competition may be due to distortions of the industry output and suboptimal market entries (see e.g. Spulber, 1985; Katsoulacos and Xepapadeas, 1995, 1996; Shaffer, 1995; Requate, 1997; Lee, 1999; Lahiri and Ono, 2007; Fujiwara, 2009; Lambertini et al., 2017; and the survey by Millimet et al., 2009).

It would be of interest, as a potential lead for future research, to extend the analysis in order to study how market power impacts, by distorting the industry output, the degree of internalization, and then to examine whether it also alters the result that the tax policy yields more welfare than the cap policy. In this respect, following Grenadier (2002), it would be immediate developing our analysis in a continuous-time Cournot-Nash framework where firms may invest over time in additional capacity to increase their output. However, as shown by Back and Paulsen (2009), the investment equilibrium in Grenadier (2002) is in open-loop strategies, that is, strategies set by a firm regardless of other firms' strategies. This is problematic since, as the investment strategies are not mutually best responses, the equilibrium is not subgame perfect.[9] One should then look for a closed-loop equilibrium but, unfortunately, as far as we know, this question is still open in the literature.

We have shown that the endogenous entry by firms leads implicitly to a barrier capping the market price. In that sense, the introduction of an output tax is equivalent to a price cap regulation and relates the current study to previous research on the impact of a price cap on irreversible investment under uncertainty in the presence of competition (Dixit, 1991), monopoly (Dobbs, 2004) and oligopoly (Roques and Savva, 2009). The main difference is that while in these papers the price cap is a control used for keeping prices low, in the current study, the policy makers adjust this control upwards in order to delay and not foster the market expansion.

Finally, the results of our model are robust to the modification of adding a firm specific production cap alongside the cap on the industry output. More specifically, assume that the regulator announces a cap $\bar{Q}$ on the industry output and impose that each firm may produce not more than $0 < \lambda < 1$ units. As one may immediately see, introducing this variation in our model set-up would have no impact on our results. The only thing that one should keep in mind is that in this case i) the number of active firms in the industry is equal to Q/λ and ii) the maximum number of firms entering the market is equal to $\bar{Q}/\lambda$.

Appendix A – The value of an active firm

In this Appendix, we present the derivation of the value function in (3), i.e. $V(X, Q)$. By a standard no-arbitrage argument (see e.g. Dixit, 1989), $V(X, Q)$ is the solution of the following Bellman equation:

$$r\cdot V(Q,\ X)\cdot dt = [P(X,Q) - M]\cdot dt + E[dV(X,Q)], \tag{A.1}$$

[9] Concerning our model (à la Leahy, 1993), note that the issue does not arise since, as shown by Back and Paulsen (2009), perfectly competitive investment strategies are mutually best response and, consequently, the equilibrium is closed-loop

which states that the instantaneous profit, $[P(X, Q) - M] \cdot dt$, along with the expected instantaneous capital gain, $E[dV(X, Q)]$, from a change in X, must be equal to the instantaneous normal return, $r \cdot V(X, Q) \cdot dt$.

Itô's lemma states that since X is a geometric Brownian motion with parameter μ and σ then $V(X, Q)$, being a twice-differentiable function of X satisfies:

$$dV(X,\ Q) = \left[\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X, Q) + \mu \cdot X \cdot V_X(X, Q)\right] \cdot dt + \sigma \cdot X \cdot dZ. \tag{A.2}$$

Applying (A.2) in (A.1), taking the expectancy recalling that $E(dZ) = 0$, and rearranging, yields:

$$\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X, Q) + \mu \cdot X \cdot V_X(X, Q) - r \cdot V(X, Q) + P(X, Q) - M = 0. \tag{A.3}$$

Trying a solution of the type x^b for the homogenous part of (A.3) and a linear form as a particular solution to the entire equation yields:

$$V(X, Q) = Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta} + \frac{P(X, Q)}{r - \mu} - \frac{M}{r}, \tag{A.4}$$

where $\alpha < 0$ and $\beta > 1$ are the roots of the quadratic equation:

$$\tfrac{1}{2} \cdot \sigma^2 \cdot x \cdot (x - 1) + \mu \cdot x - r = 0. \tag{A.5}$$

Applying $x = 0$ and then $x = 1$, and bearing in mind that $r > \mu$ asserts that (A.5) has two roots, one of them negative and the other exceeds 1.

The first term in (A.4), i.e. $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$, represents the expected present value of the flow of profits conditional on Q remaining forever at its current level. Therefore, the first and second term on the RHS of (A.3) should capture the impact that changes in Q over time have on the value of the firm in expected terms.

By the properties of the Geometric Brownian Motion, when X goes to 0 the probability of ever hitting the barrier triggering a new entry, i.e., $X^*(Q)$, and, consequently, an increase in Q, tends to 0. This leads to the following limit condition:

$$\lim_{X \to 0} \left[Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta}\right] = 0. \tag{A.6}$$

Note that as $\alpha < 0$, A.6) holds only if $Z(Q) = 0$ for any $Q > 0$. Hence, substituting $Z(Q) = 0$ into (A.3) gives ((3).

Finally, applying β for x in (A.5) leads to:

$$\begin{aligned} \frac{d\beta}{d\sigma^2} &= -\frac{\frac{1}{2} \cdot \beta \cdot (\beta - 1)}{\frac{1}{2} \cdot \sigma^2 \cdot (2 \cdot \beta - 1) + \mu} = -\frac{\frac{1}{2} \cdot \beta^2 \cdot (\beta - 1)}{\frac{1}{2} \cdot \sigma^2 \cdot \beta^2 + \frac{1}{2} \cdot \sigma^2 \cdot \beta \cdot (\beta - 1) + \mu \cdot \beta} \\ &= -\frac{\frac{1}{2} \cdot \beta^2 \cdot (\beta - 1)}{\frac{1}{2} \cdot \sigma^2 \cdot \beta^2 + r} < 0 \end{aligned} \tag{A.7}$$

where the third equality follows from (A.4), evaluated at β, and the inequality springs from $\beta > 1$.

Appendix B – Welfare maximization under a cap on the industry output

Substituting the derivative of (13) with respect to Q in (14), applying (12), and rearranging terms, yields:

$$C_Q\left(Q,\ \overline{Q}\right) = -\left[\frac{P^*}{r - \mu} - \frac{M + D'(Q)}{r}\right] \cdot \frac{1}{X^*(Q)^{\beta}}, \tag{B.1}$$

Integrating (B.1) yields:

$$C\left(\overline{Q},\ \overline{Q}\right) - C\left(Q,\ \overline{Q}\right) = -\int_{Q}^{\overline{Q}} \left[\frac{P^*}{r - \mu} - \frac{M + D'(q)}{r}\right] \cdot \frac{1}{X^*(q)^{\beta}} \cdot dq \tag{B.2}$$

The term $C(Q,\ \bar{Q}) \cdot X^{\beta}$ in (13) captures the welfare associated with future increases of the industry output. No such changes are possible if Q has reached the cap $\bar{Q}$. Therefore, the following boundary condition holds at $Q = \bar{Q}$:

$$C\left(\bar{Q},\ \bar{Q}\right) = 0, \tag{B.3}$$

Substituting (B.3) in (B.2) yields:

$$C\left(Q,\ \overline{Q}\right) = \int_{Q}^{\overline{Q}} \left[\frac{P^*}{r - \mu} - \frac{M + D'(q)}{r}\right] \cdot \frac{1}{X^*(q)^{\beta}} \cdot dq. \tag{B.4}$$

Appendix C – Proof of Proposition 6

In this appendix, we show that welfare under an output tax exceeds welfare under a cap on the industry output. We start by noticing from (41) that:

$$\begin{aligned} & W^{tax}(X,Q) - W^{cap}(X,Q) \\ & > \int_{Q}^{\bar{Q}^*} \left\{ \tfrac{M+D'(q)}{(\beta-1)\cdot r} \cdot \left[\tfrac{X^*(q)}{X^{**}(q,\tau^*)} \right]^{\beta} - \tfrac{M+D'(q)-\beta\cdot D'(q)}{(\beta-1)\cdot r} \right\} \cdot \left[\tfrac{X}{X^*(q)} \right]^{\beta} \cdot dq \\ & \int_{Q}^{\overline{Q}^*} \tfrac{M+D'(q)}{(\beta-1)\cdot r} \cdot \left\{ h(q)^{\beta} - 1 + \beta \cdot [1-h(q)] \right\} \cdot \left[\tfrac{X}{X^*(q)} \right]^{\beta} \cdot dq => 0, \end{aligned} \tag{C.1}$$

where the first inequality follows from realizing from *Proposition 5* that the first integral in (41) is positive, as well as from applying (25) for P^{**}. The last inequality follows from defining:

$$h(q) \equiv \frac{X^*(q)}{X^{**}(q,\ \tau^*)} = \frac{M}{M+D'(q)}. \tag{C.2}$$

and noticing from $D'(q) > 0$ that $0 < h(q) < 1$ for any $q > 0$. This leads to the last inequality in (C.1) which holds because any function of the form $g(x) \equiv x^{\beta} - 1 + \beta \cdot (1-x)$ is positive within the range $0 < x < 1$ since:

- $g(0) = \beta - 1 > 0$
- $g(1) = 0$
- $g'(x) = \beta \cdot (x^{\beta-1} - 1) < 0$ for all $0 < x < 1$.

The proof of (44) follows from realizing from C.1) that ((41) is positive even without its first term, and then rearranging the remaining two terms.

Let consider the comparative advantage of a tax over a cap policy in the limit case where $\sigma \to 0$. By Proposition 3, provided that current industry output level, Q, is such that

$$\lim_{\sigma\to 0}\left(\frac{P^*}{r-\mu} - \frac{M+D'(Q)}{r}\right) > 0, \tag{C.3}$$

the optimal level of the cap, $\overline{Q}^*$, solves the following equation

$$D'(\overline{Q}^*) = M \cdot \frac{\mu}{r-\mu}. \tag{C.4}$$

As $\lim_{\sigma\to 0}\beta = r/\mu$, Condition (C.3) can be rearranged as follows:

$$\mu > r \cdot (1 - \frac{M}{M+D'(Q)}) > 0. \tag{C.5}$$

The comparative advantage of a tax, as captured by (41), can be rearranged as follows:

$$\begin{aligned} W^{tax} - W^{cap} = & \int_{\overline{Q}^*}^{\infty} \tfrac{M+D'(q)}{(\beta-1)\cdot r} \cdot \left(\tfrac{X}{X^{**}(q,\tau^*(q))} \right)^{\beta} + \\ & + \int_{\overline{Q}^*}^{\infty} \tfrac{M+D'(q)}{(\beta-1)\cdot r} \cdot \left[h(q)^{\beta} - 1 + \beta \cdot (1-h(q)) \right] \cdot \left(\tfrac{X}{X(q)} \right)^{\beta} \cdot dq \end{aligned} \tag{C.6}$$

Provided that Condition (C.5) holds and $X < \lim_{\beta\to r/\mu} X^*(Q) = \frac{M}{\varphi(Q)} < \lim_{\beta\to r/\mu} X^{**}(Q,\tau(Q)) = \frac{M+D'(Q)}{\varphi(Q)}$, it immediately follows that:

$$\lim_{\beta\to r/\mu}\int_{\overline{Q}^*}^{\infty} \frac{M+D\prime(q)}{(\beta-1)\cdot r} \cdot \left(\frac{X}{X^{**}(q,\tau^*(q))}\right)^{\beta} > 0$$

Moving to the second term in (C.6), it suffices showing that

$$\lim_{\beta\to r/\mu}\int_{Q}^{\overline{Q}^*} \frac{M+D\prime(q)}{(\beta-1)\cdot r} \cdot \left[h(q)^{\beta} - 1 + \beta\cdot(1-h(q))\right] \cdot \left(\frac{X}{X^*(q)}\right)^{\beta} \cdot dq > 0.$$

Note that this is always the case since, as shown above,

$$g(h(q)) = h(q)^{\beta} - 1 + \beta \cdot (1-h(q)) > 0$$

in the interval $0 < h(q) < 1$ for any q and for any $\beta > 1$.

Therefore, summing up, we can conclude that

$$\lim_{\sigma\to 0}\left(W^{tax} - W^{cap}\right) > 0$$

Last, when $0 < \mu \le r \cdot (1 - \frac{M}{M+D'(Q)})$, Condition (C.4) does not hold. In this case, it is optimal to set the cap at the current Q, i.e. $\overline{Q}^* = Q$, which essentially turns the cap policy into a ban. Note that also in this case $\lim_{\sigma\to 0}(W^{tax} - W^{cap}) > 0$ since $\lim_{\beta\to r/\mu}\int_{Q}^{\infty} \frac{M+D'(q)}{(\beta-1)\cdot r} \cdot (\frac{X}{X^{**}(q,\tau^*(q))})^{\beta} > 0$ while the second term in (C.6) disappears.

Appendix D - Industry equilibrium and welfare under a constant output tax

The analysis of this case is similar to the one in Section 4.2 and leads, once again, to the entry threshold described by (24) and (25) and to the general form of the welfare function captured by (26). Boundary condition (27) applies too but only as a mere no-arbitrage condition.

At this point, the analysis parts ways with the one in Section 4.2 as condition (28) is not relevant to the current case. Note that this condition helps find the optimal entry threshold function $X^{**}(Q)$ in the case where the government influences this threshold by changing the tax level as Q changes, and it is therefore not relevant in the current case.

Applying (26) in (27) and then using (24) yields:

$$C_Q(Q,\ \tau) = \frac{D'(Q) - (\hat{\beta} - 1)\cdot M - \hat{\beta}\cdot\tau}{r\cdot X^{**}(Q,\ \tau)^{\beta}}. \tag{D.1}$$

Integrating (D.1) leads to:

$$C(Q,\ \tau) = \int_{Q}^{\infty} \frac{\left(\hat{\beta} - 1\right)\cdot M + \hat{\beta}\cdot\tau - D'(q)}{r\cdot X^{**}(q,\ \tau)^{\beta}}\cdot dq \tag{D.2}$$

Differentiating under the integral sign with respect to τ, while noting from (24) that $X^{**}_{\tau}(q,\ \tau) = \frac{X^{**}(q,\ \tau)}{M+\tau}$, and simplifying leads to:

$$C_{\tau}(Q,\ \tau) = \frac{\beta}{r\cdot(M+\tau)}\cdot\int_{Q}^{\infty}\frac{D'(q) - \tau}{X^{**}(q,\ \tau)^{\beta}}\cdot dq. \tag{D.3}$$

With the term preceding the integral positive, the sign of this derivative takes the sign of the integral. This implies that $C(Q,\ \tau)$ is an inverse u-shape function of τ. Applying (24) and equating (D.3) to 0 shows that $C(Q,\ \tau)$ peaks at:

$$\tau^{opt} = \frac{\int_{Q}^{\infty}\varphi(q)^{\beta}\cdot D'(q)\cdot dq}{\int_{Q}^{\infty}\varphi(q)^{\beta}\cdot dq} \tag{D.4}$$

Further, our assumption that $D''(Q_t) > 0$ for any $Q_t > 0$ implies that:

$$\tau^{opt} > \frac{\int_{Q}^{\infty}\varphi(q)^{\beta}\cdot D'(q)\cdot dq}{\int_{Q}^{\infty}\varphi(q)^{\beta}\cdot dq} = D'(Q) \tag{D.5}$$

and

$$\frac{d\tau^{opt}}{dQ} = \frac{\varphi(Q)^{\beta}}{\int_{Q}^{\infty}\varphi(q)^{\beta}\cdot dq}\cdot\left[\tau^{opt} - D'(Q)\right] > 0, \tag{D.6}$$

Applying (D.4) in (D.2) leads to the following optimized version of $C(Q,\ \tau)$:

$$C(Q,\ \tau^{opt}) = \int_{Q}^{\infty}\frac{(\hat{\beta} - 1)\cdot M^{\beta}}{r\cdot X^{*}(q)^{\beta}\cdot(M+\tau^{opt})^{\beta-1}}\cdot dq \tag{D.7}$$

Applying D.7) in the welfare function ((26) leads to a function that captures welfare given the initial levels of X and Q and the optimal tax rate that we denote by $W^{tax}(Q,\ X)$. Comparing it to welfare under the optimal cap policy, as captured by (23), yields (48).

Appendix E - The welfare gap between the two policies under a linear external cost function

In this appendix, we show that when the external cost function is almost linear, the welfare gap that (48) shows is positive, implying that the tax policy does better than the cap policy. To see that, notice that the linearity, taken together with $D(0) = 0$, implies that the externality is given by $D(Q) = a\cdot Q$ where $a > 0$ is a constant. Thus, $D'(Q) = a$ and, by (45), the optimal tax is $\tau^{opt} = a$.

Two cases should be considered. In the first one, $a \geq (\hat{\beta} - 1)\cdot M$. In that case, the price at entry times, which was denoted by P^{**} and captured by (25), satisfies $P^{*} \leq a + M$. This implies that each entry yields negative (or zero) welfare and the optimal cap should be set at the current quantity, Q. In that case, as the second integral in (48) equals zero, the welfare gap is positive, implying that the tax policy does better than the cap policy.

In the second case, $a < (\hat{\beta} - 1) \cdot M$. In this case $P^{**} > a + M$ implying that each entry yields a positive welfare flow and there should be no cap at all, i.e., $\bar{Q}^*$ should go to infinity. In this case, the welfare gap between the two policies, as captured by (48), satisfies:

$$\begin{aligned} W^{tax}(Q,X) - W^{cap}(Q,X) &= \int_Q^{\infty} \frac{a - \left(\hat{\beta}-1\right) \cdot M \cdot \left[1 - \left(\frac{M}{M+a}\right)^{\beta}\right]}{r \cdot X^*(q)^{\beta}} \cdot dq \cdot X^{\beta} \\ &> \int_Q^{\infty} \frac{\left(\hat{\beta}-1\right) \cdot M - \left(\hat{\beta}-1\right) \cdot M \cdot \left[1 - \left(\frac{M}{M+a}\right)^{\beta}\right]}{r \cdot X^*(q)^{\beta}} \cdot dq \cdot X^{\beta} \\ &= \int_Q^{\infty} \frac{\left(\hat{\beta}-1\right) \cdot M \cdot \left(\frac{M}{M+a}\right)^{\beta}}{r \cdot X^*(q)^{\beta}} \cdot dq \cdot X^{\beta} > 0. \end{aligned} \tag{E.1}$$

where the first inequality follows from $a < (\hat{\beta} - 1) \cdot M$.

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