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Solving a Mereological Puzzle

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There is an interesting puzzle about the interaction between mereology, topology, and dependence. It is not only interesting in and on itself, but also reveals subtleties about the aforementioned interaction that have gone unnoticed. The puzzle has it that the following plausible claims are jointly inconsistent: (i) wholes depend on their parts; (ii) boundaries are parts; (iii) boundaries depend on the whole they are part of. In the paper, I first argue that claims (i) – (iii) are not as a matter of fact inconsitent insofar as further assumptions are needed to get the puzzle off the ground. I consider several such assumptions, some more plausible than others. Though I do not take any definite stance as to whether the plusibility of the assumptions considered trump that of claims (i) – (iii), I set forth a suggestion to replace (iii) with something similar yet interestingly different.

Keywords parthood; boundaries; dependence

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1 The puzzle

Here is a mereological puzzle. Well, it's not just a *mereological* puzzle. It's a puzzle about the interaction between mereology, topology, and dependence.[1] It is not only interesting in itself but also reveals subtleties about the aforementioned interaction that have gone unnoticed. For these reasons, I believe, the puzzle should be discussed. One of the aims of the paper is to prompt such a discussion. Consider the following plausible claims:

Part-Whole Dependence. A whole depends on its parts.
Boundaries are Parts. A boundary is a part.[2]
Boundary-Whole Dependence. A boundary depends on the whole it is part of.

It is assumed that dependence is not symmetric. In fact, let's stipulate that it is asymmetric rather than anti-symmetric.[3] The puzzle has it that the three claims above.

> [C]annot be all true. Take a bounded whole. The boundary is part of the whole [by **Boundaries are Parts**]. By **Part-Whole Dependence**, it will follow that the whole depends on its boundary (among other things), while according to **Boundary-Whole Dependence**, the boundary depends on the whole (Smid 2015, 169, slightly modified.)

This violates Asymmetry of dependence. Smid (2015) makes a substantive case for the plausibility of all the claims above. In fact all of them have been thoroughly

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defended in the literature.[4] I shall say some unorthodox things about **Boundary-Whole Dependence**.[5] But as of now, let's accept all the three claims. How should we solve the puzzle? Smid himself suggests different strategies. One might be deflationist about dependence, and claim that there is no relation in the world that is the semantic value of the predicate "depends". One might try to distinguish between *formal* and *material* parts, and claim that wholes depend on their material parts, whereas boundaries are merely formal parts. Finally, one might try to argue that the relation[6] of dependence in **Part-Whole Dependence**, and **Boundary-Whole Dependence** is not the same relation after all. There is indeed a plethora of dependence relations: *conceptual* dependence, *existential* dependence, *identity* dependence, to mention but a few. All of them come in different varieties: *rigid* versus *generic*, *singular* versus *plural*, and so on.[7] I confess I am sympathetic to this solution. I think that a careful scrutiny of what precise relation—if any—is really at stake when we claim, for example, that wholes depend on their parts might prove fruitful. However, as it stands, the puzzle does not even get off the ground. Or so I will argue. Further assumptions are needed to get it going. I will consider several of them throughout the paper, some more controversial than others. Rejecting these additional assumptions will (dis)solve the puzzle. Yet, some of them are plausible enough. I will not take any definitive stance as to whether the plausibility of these assumptions trump the plausibility of the puzzling claims we started with. But I will, as a matter of fact, suggest a solution that endorses at least one such assumption.

2 A solution

Let me make things a little more precise. I will assume, as it is usual in the current literature,[8] plural logic, and standard mereological vocabulary. Double signs such as xx stand for plural terms (both variables and constants), whereas simple signs, such as x, stand for singular ones. In what follows, $x \sqsubseteq y$ abbreviates "x is part of y", $x \prec yy$ abbreviates "x is one of the yy-s", and finally, $x \lhd y$ abbreviates "x depends on y".[9] Consider now a bounded whole w, its parts pp, and its boundary b. In this case, **Part-Whole Dependence**, **Boundaries are Parts**, and **Boundary-Whole Dependence** translate into the following claims respectively:

(1) $$w \lhd pp$$

(2) $$b \sqsubseteq w$$

(3) $$b \lhd w$$

Clearly there is no violation of Asymmetry of $\lhd$ here. Given an instance of the Comprehension Principle for plural logic,[10] that is:

(4) $$\forall y \left(y \prec pp \leftrightarrow y \sqsubseteq w \right)$$

we get:

$$b \prec pp \tag{5}$$

from (2). Yet, even (5) falls short to deliver a fully fledged violation of Asymmetry of $\lhd$, for we are not licensed to infer

$$w \lhd b \tag{6}$$

from (1) and (5).

The overall problem seems obvious. Claim (1) contains a plural term for the dependee, whereas the alleged violation of Asymmetry—delivered by (6)— should contain a singular one, namely *b*, if it is to constitute such a violation. This is what goes wrong with Smid's argument in the quoted passage above. From the claim that a whole *w* depends *plurally*—or *collectively*—on its parts *pp*, it does *not* follow, without any further assumptions, that *w* depends *singularly*—or *individually*—on *each* of them.[11] Let me then suggest one assumption that is enough to get the puzzle off the ground. I shall label it **Strong Distributivity of Dependence**, for obvious reasons. It claims that if *x* depends on the *yy*-s, then it depends on each of the *yy*-s:

$$x \lhd yy \rightarrow \forall z \left(z \prec yy \rightarrow x \lhd z\right) \tag{7}$$

Let $x = w$, and $yy = pp$. **Strong Distributivity of Dependence** and (4) entail that:

$$w \lhd pp \rightarrow \forall x \left(x \prec pp \rightarrow w \lhd x\right) \tag{8}$$

By Modus Ponens and exemplification we get (6).

Clearly (3) and (6) do violate Asymmetry of $\lhd$. (Un)fortunately, **Strong Distributivity of Dependence** is, on the face of it, implausible. Here is why. There are different accounts of dependence on the market: modal-existential (Simons 1987), essential (Fine 1995), explanatory (Correia 2005).[12] According to all these extant account the existence of the dependent entity *necessitates* the existence of the dependee.[13] That is to say that the existence of the dependee is necessary for the existence of the dependent. This—I contend—provides a powerful argument against **Strong Distributivity of Dependence**. If *w* depends on all of its parts *pp* collectively, it is plausible to say that if all of the *pp*-s were to cease to exist, *w* would cease to exist as well. But it is implausible[14] to make the same claim for *each* of the *pp* individually. This would mean that if one single part $p \prec pp$ were to be destroyed, or even removed from *w* so as not to be a part of *w* anymore, *w* would cease to exist. This amounts to claim that mereological change is *impossible*: each proper part of a whole is necessary for that whole to exist. Look at it this way: this is not just the idea that each of the proper parts of a whole somehow *contributes to the identity* of that whole. Rather, it is the more radical idea that it is *indispensable for its very existence*. To put it in a different way: the endorsement of **Strong Distributivity of Dependence** is not simply *mereological extensionalism*, it is *mereological essentialism*.[15]

3 The puzzle strikes Back

In the previous section I argued that **Strong Distributivity of Dependence** is *sufficient* to yield the puzzle. I also argued that it is implausible. A natural question is then facing us: is **Strong Distributivity of Dependence** also *necessary*? Or, to phrase it differently: are there any other less implausible principles about dependence that are strong enough to generate the puzzle?

Here is a suggestion that I shall label **Weak Distributivity of Dependence**.[16] It claims that if x depends on the yy-s, none of the yy-s depends on x:[17]

(9) $$x \lhd yy \rightarrow \neg \exists z \left(z \prec yy \wedge z \lhd x \right)$$

It is not difficult to see that **Weak Distributivity of Dependence** is sufficient to deliver the puzzle. In fact, the reader can easily verify that (6) follows from (3), (4), and (9).

4 Solutions for everyone

What now? We can always opt for one of the tentative solutions I sketched in §1. As I said, I might even be sympathetic. However, I want to provide a few alternatives. First, we need to recognize the dialectical situation we are in. We have three initially plausible claims, namely (1), (2), and (3). Under the assumption that $\lhd$ is asymmetric, it turns out they are incompatible with further principles about dependence, namely **Strong** and **Weak Distributivity of Dependence**. The former is implausible, so we should reject it on independent grounds—or at least, this is what I argued. In absence of any independent, non-question-begging argument in favor of **Weak Distributivity of Dependence** one might reasonably contend that (1), (2) and (3) simply provide a counter-example to it. Let's put it this way. Call *Distributionalists* those who endorse **Weak Distributivity**. Distributionalists and Anti-distributionalists[18] can both play the *shifting the burden of the proof* card. The former will ask to see convincing arguments in favor of (1), (2), and (3). The latter will ask for convincing independent arguments in favor of **Weak Distributivity**. Here is one.[19] **Weak Distributivity of Dependence** is in line with the thought that dependence relations should not "loop back," given that we have endorsed Asymmetry right from the start. As an illustration, suppose the truth of the conjunction $p \wedge q$ depends on p, q. Then the truth of p should not depend on the truth of $p \wedge q$, or so the thought goes. **Weak Distributivity** delivers such a result. Now, as we saw, **Weak Distributivity**, is inconsistent with the conjunction of (1), (2) and (3). Anyone who endorses **Weak Distributivity**, perhaps because she is convinced by the argument above, has to give up one of those claims.[20] In the remainder of the paper I will suggest a few alternatives. Note that these can be endorsed by Distrbutionalists and Mereological Essentialists alike.[21] What about Anti-distributionalists? Well, I am about to suggest to give up their beloved claim (3). But I am suggesting to replace it with something similar, so they might not be completely displeased either. As I anticipated, the suggestion has it that we should reject **Boundary-Whole Dependence**. Rather we should claim something similar, but different. Various options come to mind. According to the first one, we should not endorse **Boundary-Whole Dependence**, but rather the following:

Parts of the Boundary-Whole Dependence. The *parts* of a boundary depend on the whole that the boundary it is part of.

In the case at hand, using once again the Comprehension Principle of plural logic, we can define the plurality *bb* of parts of *b*:

$$x \prec bb \leftrightarrow x \sqsubseteq b \tag{10}$$

Then, according to this suggestion we should replace (3) with:

$$bb \lhd w \tag{11}$$

Note, that, as it stands, this is compatible with **Weak Distributivity**. Yet, the puzzle is a stubborn one. Let $\iota zF(z, xx)$ abbreviate "*z* is the mereological fusion of the *xx*-s".[22] The puzzle would rise again if only the following principle—let me call it **Aggregativity of Dependence**—is endorsed:

$$xx \lhd y \rightarrow \left(\exists z \left(\iota zF\left(z, xx\right) \rightarrow z \lhd y\right)\right) \tag{12}$$

Aggregativity of Dependence informally says that if the *xx*-s depend on *y*, then the mereological fusion of the *xx*-s—provided it exists—depends on *y*. Since, clearly *b* is the mereological fusion of the *bb*-s, this will lead us into the claws and fangs of the puzzle again. The failure of such a proposal helps seeing the limited space of possibilities. Limited, yet existent. Here is a suggestion that deserves further consideration and that, to my knowledge, has not been put forward in the literature. That suggestion still insists that we should replace **Boundary-Whole Dependence** with something similar, yet different. In this case the option is twofold:

Boundary-Internal Parts Dependence. A boundary is dependent on the *internal parts* of the whole it is part of.

Boundary-Interior Dependence. A boundary is dependent on the *interior* of the whole it is part of, that is, on the *mereological fusion* of the internal parts of the whole.[23,24]

Consider the particular case we have been discussing. Let *ii* be the internal parts of *w*, and let *i* be the interior of *w*. Then, we have:

$$b \lhd ii \tag{13}$$

$$b \lhd i \tag{14}$$

This would solve the puzzle. Whether **Boundary-Internal Parts Dependence** and (or) **Boundary-Interior Dependence** are less (or more) plausible than **Boundary-Whole Dependence** is an interesting question that, I am afraid, deserves further independent scrutiny. Enough has being said to get the discussion started.

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Notes

1 See Smid (2015).

2 Let me clarifiy things a little. Consider an object o, its mereological complement—the fusion of those things that do not overlap o, that I will write as $\sim o$—and their boundary b. b is the boundary of *both* o and $\sim o$. Yet I will assume, for the sake of simplicity, that it is only part of *one* of them, either o or $\sim o$. This thesis is a substantive thesis that traces back to Bolzano. For a discussion see Casati and Varzi (1999, 86-89).

3 So that dependence is also irreflexive.

4 As for one example, see Casati and Varzi (1999) and references therein.

5 For more on this, see §3.

6 Clearly, this character is *not* deflationist about dependence.

7 For an introduction see Correia (2008).

8 For a defense of such an ideological choice see, for example, Lando (2017).

9 I assume that $\lhd$ can be flanked by both singular and plural terms on both argument places.

10 Let $\phi(x)$ be an open formula. The Comprehension Principle states that there is a plurality xx of things that satisfy the formula: $\forall y(y \prec xx \leftrightarrow \phi(y))$. See, for example, Oliver and Smiley (2013).

11 This is also the reason why I slightly changed the formulation of the claims involved in the puzzle, to keep track of the difference between singular and plural terms.

12 Tahko and Lowe (2015) discuss *identity dependence*. This seems but a particular case of Essential Dependence.

13 This is straightforward in the simple case of Modal Existential Dependence, but it can be easily verified for all of the others with the help of mild and widely agreed assumptions, for example, the assumption that essence entails necessity.

14 Or at least less plausible. Or, if you find it plausible, it is good to recognize what it is that you are buying into.

15 A possible argument in favor of **Strong Distributivity of Dependence** runs as follows. One can endorse the following similar—yet different—principle, **Strong Distributivity of Dependence for Pluralities**: $\forall x(x \prec yy \rightarrow yy \lhd x)$. Informally the principle says that a given plurality depends individually on each of its members. It is easy to see that **Strong Distributivity** follows from **Strong Distributivity of Dependence for Pluralities** and transitivity of $\lhd$. However the principle in question amounts to endorsing some sort of essentialism—namely, essentialism for pluralities. I have the impression that anyone who is skeptical about mereological essentialism will be skeptical about essentialism for pluralities as well. To be fair, Linnebo (2017) suggests that **Strong Distributivity of Dependence for Pluralities** is commonly accepted by plural logicians, when he writes: “In any world in which a plural term denotes at all, it denotes the same objects”(Linnebo 2017, §2.3). A detailed discussion goes beyond the scope of the paper. I will rest content pointing out a solution that can be endorsed by essentialists of various sorts. See §4. Thanks to an anonymous referee here.

16 Note that, under the assumption that $\lhd$ is asymmetric, **Strong Distributivity** entails **Weak Distributivity**. The converse does not hold. Hence the names.
17 Thanks to Jeroen Smid for discussion here.
18 Those philosophers that do not endorse **Weak Distributivity**.
19 This is not meant to be a full-fledged argument. Rather it is meant to offer a reason in favor of **Weak Distributivity of Dependence** for dialectical purposes.
20 Modulo endorsing one of the general suggestions I reviewed in §1.
21 Recall that arguably, Mereological Essentialists will endorse **Strong Distributivity of Dependence**. Given that **Strong Distributivity** entails **Weak Distributivity**, they will be Distributionalists. In any event, they will have to face the puzzle.
22 Nothing new under the sun. First define *Overlap*: $x \circ y \equiv \exists z(z \sqsubseteq x \wedge z \sqsubseteq y)$. Then define *Fusion*: $F(z, xx) \equiv \forall y(y \prec xx \rightarrow y \sqsubseteq z) \wedge \forall y(y \sqsubseteq z \rightarrow \exists w(w \prec xx \wedge y \circ w))$. For the sake of simplicity, in the text I take fusions to be unique.
23 A little more precise. Let $\propto$ be the topological primitive of *connection*. Then, define *Internal Part*: $x \sqsubseteq_i y \equiv x \sqsubseteq y \wedge \forall z(z \propto x \rightarrow z \circ y)$. The Comprehension Principle gives us the internal parts *ii* of *w*: $x \prec ii \leftrightarrow x \sqsubseteq_i w$. The *interior* *i* is defined as $i \equiv \iota z(F(z, ii))$. Given this framework we can also define *b* as follows. As in footnote 2, let $\sim x$ be the mereological complement of *x*. Then, the *exterior* of *x* is: $e(x) \equiv i(\sim x)$. The *closure* of *x* is: $c(x) \equiv \sim (e(x))$, and finally: $b(x) \equiv \sim (i(x) + e(x))$ where $x + y$ is just the binary fusion of *x* and *y*, a particular case of mereological fusion, as defined in footnote 22. The construction follows Casati and Varzi (1999: 54-62).
24 Note that **Boundary-Internal Parts Dependence**, together with **Aggregativity of Dependence**, entails **Boundary-Interior Dependence**.

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