



Separability and fundamentality

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Abstract

According to High-Dimensional Wavefunction Fundamentalism (HDWF) the wavefunction field evolving in configuration space is all that exists fundamentally. The main argument in favor of HDWF is an argument from separability and locality: separability is a desirable feature of a fundamental metaphysics and HDWF is indeed such a separable metaphysics. Separability in turn is desirable because it is simple and intuitive. Tim Maudlin has recently argued that intuitiveness and simplicity cannot motivate separability. In particular, our intuitions stem from our interactions with the three-dimensional world which is non-separable. Therefore, he concludes, there is *nothing else* HDWF theorists can appeal to motivate separability. I call this Maudlin's challenge. The present paper addresses Maudlin's challenge by showing how the facts that some plurality of entities are separable entail that its constituents are fundamental, for well-motivated notions of fundamentality.

Keywords Separability · Fundamentality · Wavefunction realism · Configuration space fundamentalism

1 High-dimensional wavefunction fundamentalism

Broadly speaking, realism about the wavefunction is the view that the wavefunction represents an objective, physically significant, observer independent aspect of the world. Realism about the wavefunction comes in many varieties.¹ Consider, for the sake of simplicity, an n -particle world. One realist approach has it that the wavefunction represents a multi-field that specifies a field-value for each n -tuple of points in three-dimensional space.² According to another it represents a property-like entity—say a disposition—of the mereological sum of the n particles.³ Yet another maintains that it

¹ See, e.g., Chen (2019).

² See e.g., Hubert and Romano (2018), and Romano (2021).

³ See e.g., Monton (2006), and Suárez (2015).

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is a *sui-generis* entity that does not fit in any of our classical ontological categories.⁴ I will focus on what I shall label:

Field Realism: The wavefunction represents a *field* in $3n$ -dimensional configuration space.⁵

Strictly speaking, *Field Realism* is silent as to whether the wavefunction field—as I sometimes shall call it—describes the fundamental or a derivative level. However anyone who endorses it arguably also endorses the claim that the wavefunction field is indeed fundamental.⁶ This results in:

Wavefunction Fundamentalism: The wavefunction field defined over the $3n$ -dimensional configuration space is part of the *fundamental* ontology.

There might be room for disagreement about whether the wavefunction field *exhausts* the fundamental ontology.⁷ For example one may think that configuration-space itself is part and parcel of the fundamentalia. Maudlin (Forthcoming) can be read as suggesting that *Wavefunction Fundamentalism* entails a commitment to the fundamentality of configuration space. Indeed, he suggests that we call the view *High Dimensional Space Fundamentalism*.⁸ What matters for the rest of the paper is that the high-dimensional configuration space level exhausts the fundamental level, so to speak, and that everything in three-dimensional space is determined by it. Let me then stipulate the following:

High Dimensional Wavefunction Fundamentalism (HDWF): If something belongs to the fundamental ontology of (non-relativistic) quantum mechanics it is either part of the wavefunction field or of configuration space.⁹ Everything in three-dimensional space is (somehow) *determined* by the wavefunction field evolving in such space.¹⁰

⁴ See e.g., Maudlin (2019). This list is by no means exhaustive.

⁵ The label configuration space might be misleading in that it might suggest that it is (i) either a purely representational entity with no physical counterpart, or (ii) it has a physical counterpart but it is nonetheless derivative on “particles” and their configuration. But, as it will be clear shortly, the relevant space is supposed to be a non-derivative, physical space. Ney (2023) uses the locution “a space with the structure of configuration space” to refer to it. Albert (2023) calls it, in general, the “space of the elementary physical determinables”. I will stick to “configuration space” mainly for the sake of simplicity.

⁶ As of now, we simply work with a somewhat intuitive notion of fundamentality. I shall come back to this in §4.

⁷ See Calosi (Forthcoming).

⁸ This formulation will do for our purposes. For a more nuanced, recent characterization see Schroeren (Forthcoming).

⁹ This formulation is meant to be neutral as to whether there are proper parts of the wavefunction field and of configuration space, and whether these are fundamental. For a discussion see Calosi (Forthcoming).

¹⁰ As Ney puts it: Fundamentally, all that exists according to quantum theories on a wave function realist interpretation is the wave function evolving in its own high-dimensional space (Ney, 2021: 89).

Variants of this view have attracted significant attention in recent years, both in philosophy of physics and metaphysics.¹¹ Arguably, the most articulated, detailed, and thorough defense of the view is in Ney (2021, 2023)—but see also Albert (2023). One of the main arguments in favor of HDWF rests on a particular notion of *separability*. Separability has been motivated in terms of its simplicity and intuitiveness.¹² Recently, Maudlin has forcefully challenged both simplicity and intuitiveness and has concluded that *there are literally no other motivations in favor of separability*. One of the main goals of the paper is to suggest one motivation for separability by showing how the fact that an ontology is separable can provide an argument for its fundamentality. I take this to be important for at least two reasons. First, it provides a response to Maudlin's claim. Second, in doing so, it elucidates the relations between (different varieties of) two notions, separability and fundamentality, that have so far gone unnoticed. This is of significant interest independently of the fate of HDWF. The rest of the paper is structured as follows. In §2 I reconstruct the separability argument in favor of HDWF and discuss the notions of separability it requires. In §3 I articulate what I call the Maudlin's challenge. Roughly it asks what reasons—if any—one might have to adopt said notions of separability. Finally in §4 I suggest that if one adopts particular conceptions of fundamentality, then one can reply to this challenge.

2 Separability and the argument from separability

2.1 The argument in a nutshell

The argument from separability and locality is simple enough.¹³

The Motivational Premise: ¹⁴ Separability and locality are among the desirable features of a fundamental metaphysics. Thus, separability and locality can serve to motivate metaphysical choices for the fundamental level.

The Uniqueness Premise: HDWF is the only metaphysics of quantum mechanics that is both separable and local.

Given the two premises above, *all else being equal*, we should endorse HDWF—or so the thought goes. The argument can be refined in a number of ways, but given that it is my intention neither to criticize it nor to endorse it, this formulation will do.¹⁵ In effect, I will solely focus on separability here, but the reader should be reminded that locality is an important part of it as well.

¹¹ See e.g., Albert (1996, 2013, 2015); North (2013); Lewis (2016); Ney (2021); Calosi (Forthcoming); Hubert (Forthcoming); Maudlin (Forthcoming), Ney (2021); Wallace (2022) and, to a different extent, Ismael and Schaffer (2020).

¹² For another motivation related to the doctrine of *Humean Supervenience* see footnote 39.

¹³ See Ney (2021 80–132; 2023), and Calosi (Forthcoming). See also Albert (2023).

¹⁴ I loosely based the label on Maudlin (Forthcoming) that speaks of a *motivational demand*.

¹⁵ Ney summarizes it as follows: “wave function realism is unique in yielding quantum ontologies that not only distinguish quantum states, but do so by retaining two intuitively nice metaphysical features: separability and locality” (Ney, 2023: 8).

2.2 Notions of separability

What is separability? Ney offers many formulations that differ in letter but not in spirit.¹⁶

Separability—First Stab: A plurality of entities is separable iff (i) each of the entities possesses an “independent state”, and (ii) for any entity that is composed by entities in the plurality the state of the composed entity is “determined” by the states of its components.

Separability, as understood here, contains two conditions, an “independence” condition (i), and a “composition and determination” condition” (ii). There is historical evidence that, originally, separability did not contain any composition and determination condition. This was arguably added for the first time in (Howard 1989: 226), and has become common in the literature.¹⁷ What is important for the purpose of the paper is that both Ney’s argument and Maudlin’s response feature a composition and determination condition as in *Separability* above.¹⁸ This is the notion that is at stake here, independently of the original formulation of the principle.

More importantly, the notion of “having an independent state” that figures in the independence condition (i) is arguably subject to multiple, inequivalent readings. Here I will be interested in two such readings that result in two different notions of separability. For obvious reasons I shall call them *Strong* and *Weak Separability*:

Strong Separability: A plurality of entities is separable iff (i) (*Strong Independence (Capitol 'I')*) the state of any entity in the plurality is *not determined by any other entity*, and (ii) (*Composition and Determination*) for any entity that is composed by entities in the plurality the state of the composed entity is “determined” by the states of its components.

Weak Separability: A plurality of entities is separable iff (i) (*Weak Independence (Capitol 'I')*) the state of any entity in the plurality is *not determined by any other entity that is not a component of it*, and (ii) (*Composition and Determination*) for any entity that is composed by entities in the plurality the state of the composed entity is “determined” by the states of its components.

Strong Separability entails *Weak Separability* but the converse does not hold. Interestingly, different passages in Ney seem to back up different formulations. Here is a passage that seems to point to *Strong Separability*:

¹⁶ See e.g., Ney (2021: 81), Ney (2021: 85), Ney (2021: 127). See also Calosi (Forthcoming) and Maudlin (Forthcoming). Strictly speaking, Ney uses regions and contents of different regions in her formulations. I will stick to the formulation above for the sake of simplicity. To recover Ney’s formulation one needs to add that each x in xx is located at a distinct region, and any such x and its state is included in the content of the region at which it is located.

¹⁷ For an insightful discussion, see Ramirez (2020). Ramirez provides textual evidence that contemporary discussions on separability do include the composition and determination condition. He cites Esfeld (2004); Healey (1991); Leifer (2014); Maudlin (2011); Myrvold (2011) as examples.

¹⁸ See e.g., Ney (2021, 2023) and Maudlin (Forthcoming).

In favor of separability, there is the proposition that facts about one entity *don't depend on facts about any other entity* (Ney, 2023: 11, italics added)

Here is one that seems to just require the weaker reading:

It is not the case that what is going on *in one part* of the wave function's space *metaphysically determines what is happening in another* (Ney, 2021: 95, italics added).

Rather than choosing between the two I will simply consider both in what follows. Before we move on to some formal machinery, let me note that it is clear in context that the notions of “independence” and “determination” that figure in clauses (i) and (ii) of the notions of separability refer to the same (primitive) notion of *metaphysical determination*, to be contrasted with the notion of *causal determination* at work in locality:

Separability (...) concerns the *non-causal metaphysical determination of the features of a total system by the features of the subsystems* (Ney, 2021: 96, italics added).

2.3 Logical framework

I will use a certain logical framework to frame rigorously the discussion about separability and related notions. Such logical framework is that of (orthodox) first order plural logic with identity.¹⁹ Single signs—such as e.g., x —stand for singular terms (constant and variables), whereas double signs—such as e.g., xx —stand for plural ones. We also introduce a special predicate, $<$ whose intended interpretation is “being one of”. Thus $x < xx$ reads “ x is one of the xx ”. I sometimes say that x is a member of the xx . Standard (first-order) plural logic comprises the following principles:

Extensionality: Pluralities xx and yy are identical iff they have the same members.

Plural Comprehension: For any open formula ϕ , if there is some x such that $\phi(x)$ then there is a plurality xx such that something is a member of xx iff it satisfies ϕ .

No Emptiness: There are no empty pluralities.

We can then introduce interesting and fruitful distinctions. First, it is common to allow for somewhat degenerate pluralities that contain only one member. Following the literature I will call such a plurality an *improper plurality*. Unsurprisingly, a *proper plurality* is a plurality that is not improper, i.e., a plurality with at least two members. Then we can define a *proper subplurality* yy of xx as follows: each y in yy is identical to an x in xx but not viceversa. To this standard logical framework I add two special notions that are crucial to understand separability. The first one is a primitive notion

¹⁹ This traces back to the classic Boolos (1984). For an introduction see e.g., Oliver and Smiley Oliver and Smiley (2016) and Linnebo (2022).

of *metaphysical determination* D . The second one is a “composition operation”—which I shall indicate with Σ .²⁰ Let me spend a few words on each, starting from D . First, let us specify its relata. One may think that D takes as relata primarily “fact-like entities”. In effect, this is what will happen in the specific quantum case we will discuss at length. But one may be less restrictive and think that D can hold among different categories. For example one may think that tropes provide truthmakers and therefore determine propositions. Luckily for us, we need not enter these thorny questions here. To anticipate, I am happy to stipulate that relata of D are *primarily* fact-like entities such as “the fact that xx are in state ϕ —that is, each $x_i < xx$ is in state ϕ_i —determines the fact that y is in state φ ”. In such a case we would loosely say that the xx determine y . One can always recover the more strict reading. Furthermore, I assume that (i) D can be plural in both argument-places—thus, e.g., $yyDx$ reads “the yy determine x ”, and (ii) D is transitive. Finally, (iii) D is the relation of determination that appears in the formulation of HDWF.

Let me move on to Σ . Σ is flexible insofar as it can take any number of arguments, $0, 1, \dots, n$, for any n . Then we can define the notion of *component*, and *parthood* as follows:²¹

Component: x is a *component* of y iff y is the result of applying Σ to x and (possibly) some other object.

Parthood: x is part of y iff there is a sequence of objects $x_1, \dots, x_n, n > 0$, for which $x = x_1, y = x_n$, and x_i is a component of x_{i+1} for $i = 1, \dots, n - 1$.

It follows that *parthood* is reflexive and transitive. To ensure asymmetry so as to guarantee that parthood is at least a partial order one can impose that Σ obeys:

Anti-Cyclicity: If $x = \Sigma(\dots, \Sigma(\dots, x, \dots), \dots)$ then $x = \Sigma(\dots, x, \dots)$

From these one can recover standard mereological notions. I sometimes write $yy\Sigma x$, to be read “the yy compose x ”. In such a case I also write that $x = \Sigma(yy)$ —and I use such notations inter-changeably.²² Finally, if $y = \Sigma(yy)$ and yy is a proper plurality, that is, it contains at least two members, then $y \neq yy$. That is to say that y is distinct from the plurality of the things it is composed of, even if those things are considered collectively. This is intended to capture the slogan “ Σ -composition is *not* identity”.

2.4 Notions and formulations

We can now define several notions that will play a crucial role later on:

Σ -*Simple*: x is Σ -simple iff there is no improper plurality xx such that $xx\Sigma x$.

²⁰ Following Fine (2010). I leave Σ deliberately underspecified. For example, Σ could be mereological composition, or set-theoretic union, or what have you, depending on further principles one may require on Σ . The arguments to follow do not depend on these details.

²¹ What follows heavily draws from Fine (2010).

²² Similarly, $yy\Sigma xx$ reads “the yy compose the xx ”, by which I mean that for each x in xx there are some yy that compose x . It is a substantive question whether there are, in general, any substantive relations between Σ and D . However, the arguments in the paper do not *directly* depend upon any specific relation between them.

Σ -Composite: x is Σ -composite iff x is not Σ -simple.

Σ -Simple $_{xx}$: A plurality xx is Σ -Simple $_{xx}$ iff each x in xx is Σ -Simple.

In other words, every Σ -simple has only itself as a part. We can also define:

Subdivision: A plurality xx is a subdivision of plurality yy iff for every $x < xx$ there is a $y < yy$ such that x is part of y .

Simple Subdivision: A plurality xx is a simple subdivision of plurality yy iff xx is a subdivision of yy and every $x < xx$ is Σ -simple.

One should be careful not to conflate subdivisions and subpluralities. In effect, given the standard plural principles we assumed the two come apart. This is best appreciated by looking at Fig. 1. Figure 1 depicts different pluralities—proper part-hood going uphill along the lines. Consider the plurality $pp_1 = (x_2, x_3, x_4)$. The plurality $pp_2 = (x_2, x_4)$ is a proper plurality of pp_1 . By contrast the plurality $pp_3 = (x_5, x_6, x_7, x_8, x_9, x_{10})$ is a (simple) subdivision of pp_1 . And by *Extensionality*, $pp_2 \neq pp_3$. This will play a role in some of the arguments below.

Consider now the following substantive principle about the behavior of Σ . I am not introducing it on a whim. It will turn out to be important later on:

Σ -Simplicity: For every Σ -composite y there is a Σ -simple $_{xx}$ plurality xx such that $y = \Sigma(xx)$. Or, equivalently, $xx \Sigma y$.

It is interesting to note that some HDWF-theorists, most notably Albert, require something in the vicinity of Σ -Simplicity *explicitly* for fundamental entities. In effect, in more than one occasion, Albert (2023) endorses the claim that the fundamental space is made up of simples, or *points*. The following passage is revealing:

And the two-dimensional space is now the only one in which we can keep track of everything (...) merely by saying what’s going on at every individual one of its *points*—it’s the space (...) of the *totality of atomistic opportunities for things* (Albert, 2023: 22, italics in the original)

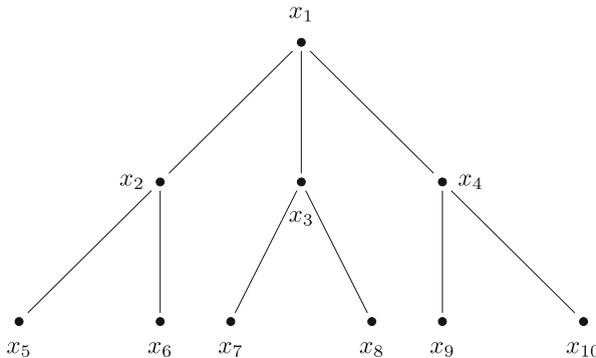


Fig. 1 Subpluralities and subdivisions

In the light of the above, the official formulations of the two notions of *separability* become:

Strong Separability (S-SEP): A plurality of entities xx is *separable* iff (i) for each x in xx there is no $y \neq x$ such that xDy , and (ii) for each Σ -composite y , there is a subplurality yy of xx such that $y = \Sigma(yy)$ —equivalently, $yy\Sigma y$ —and $yyDy$.

Weak Separability (W-SEP): A plurality of entities xx is *weakly separable* iff (i) for each x in xx there is no y such that xDy and y is not part of x , and (ii) for each Σ -composite y , there is a subplurality yy of xx such that $y = \Sigma(yy)$ —equivalently, $yy\Sigma y$ —and $yyDy$.

Given the logical relations discussed above, S-SEP entails W-SEP but the converse does not hold. The following substantive propositions can be proven:

Simple Subdivision: In the presence of Σ -Simplicity, for every weakly separable plurality xx there is a simple subdivision yy of xx such that yy is strongly separable.

To see this notice that it follows from Σ -Simplicity that every Σ -composite y is basically the result of applying Σ to a Σ -simple plurality. In other words, y is obtained by applying Σ to a plurality of simples. Take any Σ -composite in the weakly separable plurality yy and consider the plurality xx of all the Σ -simple constituents of every such y . Note that the xx constitute a simple subdivision of the yy . We now need to prove that xx qualifies as a strong separable plurality. First, every $x < xx$ is not determined by anything else. Suppose there is some $z \neq x$ such that zDx . Take any Σ -composite w in the original weakly separable plurality that has x but not z as part. It will be (partly) determined by z —given transitivity of D . Hence there will be a y that is (partly) determined by something which is not part of it, contradicting the assumption that the original plurality was weakly separable. Furthermore the xx determine every composite because given Σ -Simplicity every composite is composed by simples, and given weak separability, every composite is determined by its parts. This gives *Simple Subdivision*.

Strong Separability to Simplicity: If a plurality xx is strongly separable then it is Σ -simple.

Suppose it is not the case. Then for every x there is a proper plurality of Σ -composite xx such that $x = \Sigma(xx)$. Given the Composition clause in (Strong) Separability, $xxDy$. But $xx \neq y$. Therefore every entity x is (partially) determined by something distinct from x . And this contradicts the (Strong) Independence clause (i) in Strong Separability. As we shall see both the propositions have significant consequences.

3 Maudlin's challenge

Much of the weight of the *Separability Argument* is carried by the *Motivational Premise*. In a recent paper Maudlin asks:

The Wave Function Realist demands Separability (...) at essentially any price. But why? (Maudlin, [Forthcoming](#): 4).

Ney articulates her reasons in Ney (2021: 120-129). Apart from some historical considerations, she cites two basic reasons for demanding separability, namely that it is *simple* and *intuitive* (Ney, 2021: 127-129). Recently, Maudlin has forcefully challenged such motivations.²³ We can distinguish at least two different arguments in his discussion. The first is an *historical* argument. History of science provides us with many cautionary tales about the fate of what we thought was simple and intuitive: the case of Euclidean geometry is paradigmatic in that respect. Such geometry might indeed be simple and intuitive, but relativistic physics taught us spacetime is not Euclidean. The second argument is more theoretical in nature. We might call it the *genealogy argument*. Maudlin asks what is the *source* of our intuitions about separability—their genealogy if you will. Clearly, says Maudlin, they come from our empirical interactions with the three-dimensional world. But everyone agrees—HDWF-theorists included—that the three-dimensional world is neither separable nor local. This puts pressure on HDWF. If we all agree that our intuitions are actually false even of the three-dimensional world that actually generated them, why should we cling to them? Indeed, says Maudlin

One can't appeal to experience to justify that expectation. *And there is nothing else to appeal to* (Maudlin, [Forthcoming](#): 6, italics added).

I shall call this the *Maudlin's challenge*. Given that we cannot appeal to our experience to motivate the demand of separability, we should indeed appeal to something else. The challenge is to find that something else we can appeal to. Maudlin thinks this challenge cannot be met—there is simply nothing else to motivate such demand. We should not underestimate how radical Maudlin's claim really is. It boils down to the point there is literally *no motivation* in favor of separability. In what follows I will suggest one such motivation. Granted, it rests upon controversial assumptions. But to put it dramatically, it is not nothing. I will argue that, at least for separability, the challenge can be met by appealing to the notion of *fundamentality*.²⁴ This is not just interesting on its own. It also reveals connections between two crucial notions, separability and fundamentality, that have so far gone unnoticed.

4 The “Fundamentality” answer

4.1 Notions of fundamentality

There are two initial motivations to look at notions of *fundamentality* to meet Maudlin's challenge. First, HDWF crucially demands separability *only at the fundamental level*. In effect, it is important that the derivative three-dimensional world is *not* separable, for this partly motivates moving to higher dimensions. Second, as we saw, separability can

²³ See Maudlin ([Forthcoming](#)).

²⁴ I don't want to claim that this is the only way. Ney defends her use of metaphysically intuitive principles in Ney (2023). She rightly notes that intuitions are not incorrigible, but that alone does not entail that they are useless.

be cashed out in terms of a (metaphysical) determination relation. And it is arguably orthodoxy to define the notion of fundamentality in terms of determination.²⁵ This makes “the fundamentality answer” to Maudlin’s challenge initially promising. Yet, we should see whether we can deliver on such promise. Before we do that, I should be upfront and claim that *my aim is exploratory*. I explore an answer to Maudlin’s challenge and offer it as potentially interesting and fruitful on behalf of the HDWF-theorist. It is not my intention to endorse such an answer, nor to defend HDWF.

There are at least two broad ways in which determination can be used to define fundamentality.²⁶ Loosely speaking according to the first conception to be fundamental is to be *undetermined*, whereas according to the second to be fundamental is to be *all determining*.²⁷ In the light of this, I suggest the following:

*Fundamentality*₁: x is *fundamental*₁ iff there is no y , such that $x \neq y$, and xDy .²⁸

*Fundamentality*₂: x is *fundamental*₂ iff there is a plurality xx such that (i) x is in the xx , (ii) for every y such that if y is not in the xx or among the parts of the xx , then $xxDy$ and (iii) there is no proper sub-plurality zz of the xx such that for every y that is not in the xx -s or in the parts of the xx , $zzDy$.

Let me spend a few words on the characterizations above. The first one is a straightforward rendition of the claim that fundamentalia are undetermined. The second is a possible formulation of the idea that the fundamentalia (and their parts) determine everything else by providing a “minimal basis” for it—condition (iii) being the minimality requirement.²⁹ Note that, given the few assumptions we made on D the two notions can diverge. Indeed, something can be *fundamental*₁ without being *fundamental*₂ and viceversa.

Look at Fig. 2, where arrows represent D -relations. Then, in case (2a) x_1 is not *fundamental*₁ but is *fundamental*₂. By contrast, in case (2b), x_1 is *fundamental*₁ but

²⁵ Indeed, one may be a pluralist and think that there are different determination relations and different senses of fundamentality should be indexed by those relations. I have sympathies for this view, but discussing it goes beyond the scope of the paper. See e.g., Bennett (2017); Calosi (2020). What matters here is that the determination relation D used in the formulations of separability is among such fundamentality-tracking relations.

²⁶ Raven (2016); Bennett (2017) are usually credited for distinguishing them. For a rigorous attempt to phrase them in terms of a particular determination relation, namely *grounding*, see Leunberger (2020) and Correia (2021). For an application to philosophy of physics see McKenzie (2022). Note that my characterization departs from e.g., the one in Leunberger (2020) and Correia (2021) in significant details.

²⁷ I do not mean to suggest that these are the only relevant notions of fundamentality (ultimately) definable in terms of D . To lay my cards on the table, I am inclined to be a pluralist about fundamentality. What follows are just a notions of fundamentality that are relevant in the debate at hand.

²⁸ One might worry that *Fundamentality*₁ is too similar to the Strong Independence condition in *Strong Separability*. Indeed, in the last part of the paper I show that *Fundamentality*₁ entails such condition. Thus, they are extensionally equivalent. This is an important result on its own. Furthermore, it is important in the overall dialectic of the paper. Because now it seems clear that one can use the arguments in favor of *Fundamentality*₁—and there are many in the literature—to motivate at least the first condition of *Strong Separability*. More on this shortly.

²⁹ See e.g., Tahko (2018).

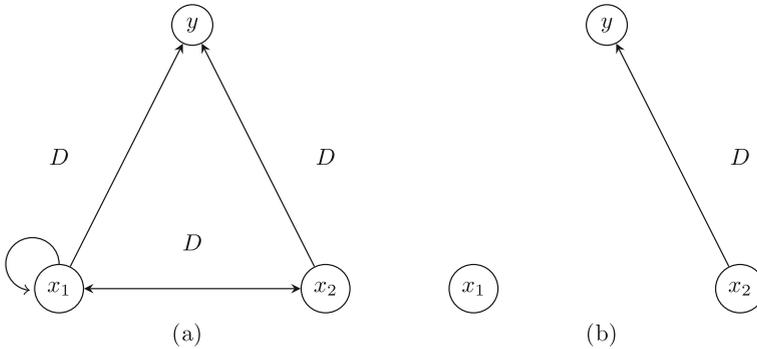


Fig. 2 Determination and fundamentality

is not fundamental_2 .³⁰ We now can define a stronger notion of fundamentality as follows:³¹

*Fundamentality*₁₂: x is *fundamental*₁₂ iff x is both *fundamental*₁ and *fundamental*₂.

We can easily define plural versions of all the notions of fundamentality above by requiring that a plurality xx is *fundamental* _{j} iff each $x < xx$ is *fundamental* _{j} . As a way of illustration, a plurality xx is *fundamental*₂—which I will write as $\text{FUND}_2(xx)$ —iff each $x < xx$ is FUND_2 .

One way to motivate the different notions of fundamentality is to look at the notion of *explanation*. Indeed, it is widely agreed that explanation tracks determination relations, and that the fundamentalia play an explanatory role. In this respect, *Fundamentality*₁ simply says that fundamentalia are *unexplained*. They provide a somewhat “ultimate” explanation in that they cannot be explained in terms of anything else. They are somewhat natural stopping points for explanations. *Fundamentality*₂ cashes out the idea that fundamentalia contribute to explain everything else. They provide a “complete” explanation. Finally, according to *Fundamentality*₁₂ fundamentalia are “unexplained explainers”. They provide a *complete explanation* that also constitute a natural *stopping point* to any explanatory requests. At least part of the appeal of the different notions of fundamentality rests on their ability to play the theoretical role required by fundamentalia being unexplained, all explainers, or both.³²

Before we move on, it is interesting to cite Albert (2023) again. This is because he goes very close to actually require a “completeness” condition that is the hallmark of *Fundamentality*₂:

The space of elementary physical determinables is what fixes the *elementary kinematic possibilities* of the world (Albert 2023: 23).

³⁰ Indeed, one may use models (2a) and (2b) above to argue that, without any constraint on D , neither *Fundamentality*₁ nor *Fundamentality*₂ are satisfactory by themselves.

³¹ I do not mean to suggest that these are the only relevant notions of fundamentality (ultimately) definable in terms of D . To lay my cards on the table, I am inclined to be a pluralist about fundamentality. What follows is just a notion of fundamentality that is relevant in the debate at hand.

³² For a review of the arguments in favor of *Fundamentality*₁ and *Fundamentality*₂, see e.g., Tahko (2023).

But even the passage I quoted earlier discussing Σ -simplicity is revealing. Recall that Albert literally writes that the fundamental space is the one in which we can “keep track of *everything*”.

4.2 The relation between fundamentality and separability

Now everything is (almost) in place to prove what I will call the *Main Conditionals*. They relate notions of separability in §2.4 with notions of fundamentality in §4.1.

*Main Conditional*₁: Strong Separability entails Fundamentality₁₂:

$$S\text{-SEP}(xx) \rightarrow \text{FUND}_{12}(xx).$$

*Main Conditional*₂: Weak Separability entails Fundamentality₂:

$$W\text{-SEP}(xx) \rightarrow \text{FUND}_2(xx).$$

*Main Conditional*₃: In the presence of *Sigma-Simplicity*, Weak Separability entails there is a subdivision yy of the xx such that the plurality yy is Fundamental₁₂.

To prove the Main Conditionals one needs a further principle, the *Determination Principle*.

The Determination Principle: For every three-dimensional entity x there is a configuration space plurality xx such that $xxDx$.

The *Determination Principle* can be motivated via the following claims—resorting to fact-talk: (i) there are configuration space facts, (ii) facts about the wavefunction are among them, and (iii) wavefunction facts are *informationally complete* in the sense of Maudlin (2007). In the case at hand, abusing terminology, the wavefunction is informationally complete because every physically relevant fact “is implied by the wavefunction” (Maudlin, 2007: 3154). If one were to endorse that D somehow backs up “implication”, one would thereby have a substantive motivation for the *Determination Principle*. Such motivation would be broader than the endorsement of HDWF.³³ We are now ready to prove the the *Main Conditionals*:

*Main Conditional*₁. Recall, one needs to prove that (i) each x in the maximally separable plurality xx is undetermined—so that every x counts as fundamental₁; (ii) the plurality xx determines everything else, and (iii) no proper subplurality of xx determines everything else, so that each x in xx is fundamental₂ in that it belongs to a minimal basis. Starting from (i). It just follows from clause (i) in Strong Separability. Clause (ii) is next. We need to show that for every y that is not in xx , $xxDy$. There are two cases: either y is a Σ -composite configuration space entity, or it is a three-dimensional entity. In the first case, given that the xx are maximally separable there is a subplurality of xx such that both $xx\Sigma y$ and $xxDy$ holds. In the second case, by the *Determination Principle* there is

³³ That does not mean that everyone would subscribe to the claims above. Bohmians that interpret the wavefunction as a multifold would probably deny (ii).

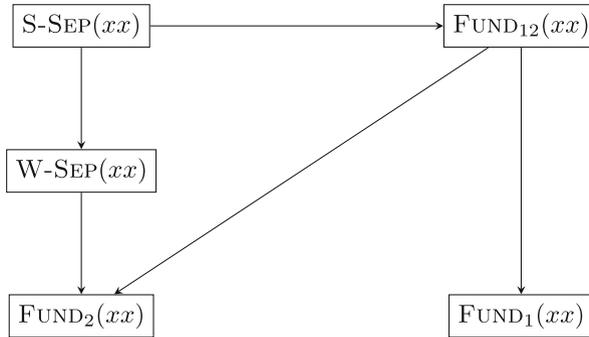


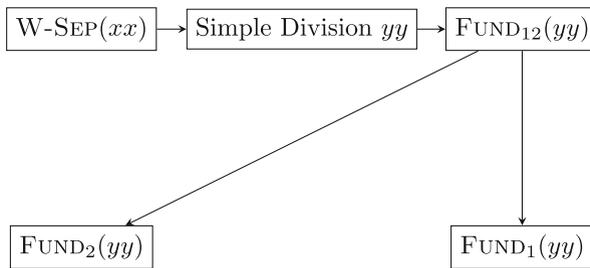
Fig. 3 Main conditional₁

configuration space plurality zz such that $zz Dy$. Suppose zz is Σ -simple. Then zz is a subplurality of xx and we are done. Suppose it is Σ -composite. Given *Strong Separability to Simplicity* there is a Σ -simple plurality such that for each $z < zz$, that Σ -simple plurality both consitutes and determine z . This and the transitivity of D will ensure that there is a Σ -simple configuration space plurality ww that is a subplurality of xx such that $ww Dy$.³⁴ Finally (iii). To see that there is no proper subplurality of xx that will fix everything consider one x in xx . According to (Strong) Separability, each x -composite, that is, each Σ -composite that counts x among its components will be determined partly by x . Hence, any proper subplurality of xx that does not include an arbitrary x cannot determine everything else. In particular it cannot determine the Σ -composite entities that are x -composite—and the entities that are determined by those x composites. This establishes *Main Conditional₁*. Figure 3 depicts logical relations graphically—arrows standing for entailment.

Main Conditional₂. Let xx be a weakly separable plurality. We basically need to prove that (i) xx and its parts determine everything else and (ii) no subplurality of xx would do. The first clause simply follows from the Composition clause of Weak Separability. To recall, for each Σ -composite y there is a subplurality yy of xx such that $y = \Sigma(yy)$ and $yy Dy$. For the second clause the proof is exactly like the proof of clause (iii) in *Main Conditional₁*. Graphically in Fig. 4.

Main Conditional₃. By *Simple Subdivision*, for every weakly separable plurality xx there is a simple subdivision yy of xx that is strongly separable. By *Main Conditional₁* the strongly separable plurality xx is fundamental₁₂. The results are summed up in Fig. 5.

³⁴ This is the Σ -simple plurality of which all the Σ -simple pluralities that determine each of the $z < zz$ are a subplurality of.

Fig. 4 Main conditional₂Fig. 5 Main conditional₃

Both *Main Conditional₁* and *Main Conditional₃* reveal interesting connections between different notions of fundamentality and the mereological structure of the separable pluralities, namely their being ultimately atomic. In effect, in a forward-looking passage, Albert brings together the atomistic structure of the fundamental space and a completeness requirement on fundamentalia:

The whole business hinges on a distinction between two different conceptions of physical space. One of these identifies physical space with the set of points at which a material particle might in principle be located—and the other (which I will argue is *more fundamental*) identifies physical space with a set of points that satisfies (among other conditions) the condition that a specification of the physical situation at every one of those *points* amounts to a *complete specification of the physical situation of the world* (Albert, 2023: 9, italics added).

At this juncture, it might be instructive to provide a simple toy illustration. Let's focus on the case discussed in *Main Conditional₁*, for this is arguably the somewhat orthodox quantum case—as we saw citing Albert extensively. I will refer to the Fig. 6—recall that (a, b, \dots, n) indicates the plurality xx whose only members are a, b, \dots, n . Figure 6 is a configuration space representation of a two-particle world, i.e., a world that only contains—in 3D-space—particle a , particle b and their Σ -composite $ab = \Sigma(a, b)$,³⁵ where the black dots represents regions of configuration space where the wavefunction intensity is high—one can picture such regions as peaks protruding out of the page. Both particles a and b can only occupy two positions, 1 and 2, on a line. Regions of

³⁵ I am using the same Σ -relation for the sake of simplicity, but the argument does not depend on its use. The example is taken from Lewis (2004: 716). It's a substantive question how to understand talk of particles in the present context. Different HDWF-theorists will arguably give different answers. According to Albert (2023) three-dimensional particles are somehow “enacted” by the wavefunction field whereas for Ney (2021) they are literally *parts* of it. What matters for the argument is that, no matter how we understand such particle-talk, their states will be the ones in the main text.

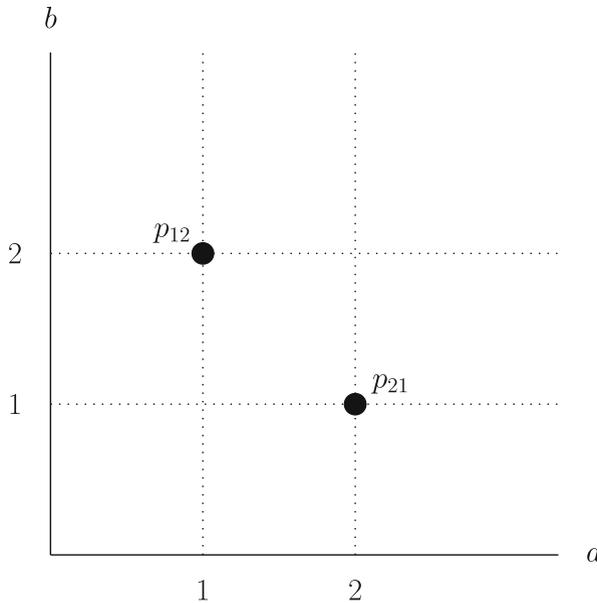


Fig. 6 A two-particle world in configuration space

configurations space obey Σ -simplicity in that they are build by applying Σ to simple points in configuration space. Among the simple points in configuration space there are points p_{21} and p_{12} . The wavefunction field intensity—as Albert (2023) explicitly claims—is characterized by an assignment of phases and amplitudes to these simple points.³⁶

The wavefunction field intensity at such simple points determines the configuration space derivative entity that is the Σ -composite region $r = \Sigma(p_{21}, p_{12})$. That is, $(p_{21}, p_{12})Dr$. We also have different three-dimensional pluralities, among which particle a , particle b and their Σ -composite $ab = \Sigma(a, b)$. The wavefunction field value at configuration space region r determines $ab = \Sigma(a, b)$ in that it fixes its quantum state (neglecting constants) as: $|1\rangle_a|2\rangle_b - |2\rangle_a|1\rangle_b$. This in turns determines a and b , both of which are assigned density matrices: $|1\rangle\langle 1| + |2\rangle\langle 2|$. Recall that D is transitive. Then the simple configuration space plurality (p_{21}, p_{12}) (together with amplitudes and phases assigned to the points) exactly because of Strong Separability, is such that neither p_{21} nor p_{12} are determined by anything and determine everything else. In explanatory terms they are *unexplained explainers* as *Fundamentality*₁₂ demands. What about the converse entailment? Does *Fundamentality*₁₂ entail *Strong Separability*? If this were the case we would have the interesting result that the two notions are indeed *extensionally equivalent*. First, note that *Fundamentality*₁ alone entails the “independence” condition (i) of *Strong Separability*. In the light of the arguments above, this is enough to conclude that *Fundamentality*₁ and the Strong Independence

³⁶ To avoid confusions, note that phases and amplitudes of the composite region are not obtained by applying Σ to the phases and amplitudes assigned to points. What is composed by Σ are the regions of configuration space.

condition (i) of *Strong Separability* are extensionally equivalent.³⁷ Call this *Partial Equivalence*. The equivalence is only partial in that *Fundamentality*₁₂ fails to entail the Composition and Determination condition (ii). This is because *Fundamentality*₁₂ requires that for every derivative entity y there is a fundamental plurality xx that determines y . However it is completely silent as to whether y and xx are Σ -related as *Strong Separability* demands. Thus *Strong Separability* is strictly stronger. *Partial Equivalence* is however important on its own. It shows that any quantum ontology that is not strongly separable is not fundamental₁, and therefore not fundamental₁₂. An example would be a pluralist ontology of spacetime regions—and density matrices associated to all such regions—such as Timpson and Wallace’s (2010) *Spacetime Statespace Realism*. Another example is *metaphysical coherentism* as advocated for Bohmian Mechanics in Lorenzetti (2022) and for Relational Quantum Mechanics in Dorato and Morganti (2022). In both cases the “allegedly fundamental” items, particles and events respectively, enter into symmetric relations of dependence that clearly violate the Strong Independence condition of Strong Separability. Thus, neither ontology counts as fundamental₁ or as fundamental₁₂.

Why are the *Main Conditionals* important? They are important insofar as they show that separability guarantees that the relevant plurality is fundamental according to different, well motivated notions of fundamentality. If one thinks that fundamentality should be cashed out in terms of any of the notions we discussed, then requiring that the ontology is separable ensures fundamentality. For the sake of illustration and a little loosely, consider once again the strongest notion of fundamentality, *Fundamentality*₁₂. According to that notion, fundamentalia are the undetermined entities that determine everything else. Requiring (strong) separability ensures exactly that. One should note that both *Fundamentality*₁ and *Fundamentality*₂ have been widely discussed and defended in the literature.³⁸ If the arguments in this paper are on the right track, one could marshal the arguments in their defense to support the demand of separability.³⁹

Let me be clear once again, at the cost of sounding repetitive. It is not my intention to defend any of the notions of fundamentality we rehearsed. The only point I want to make is that there are well-motivated notions of fundamentality that can shoulder the demand of separability on behalf of HDWF-theorists. This is enough to conclude that there is something to which HDWF-theorists can appeal to answer Maudlin’s challenge. Whether the cure is worse than the disease remains to be seen.

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³⁷ Recall that the other direction of the biconditional is established in the proof of the *Main Conditional*₁.

³⁸ The interested reader could start from Bennett (2017); Tahko (2018), and references therein.

³⁹ For an influential example of a separable metaphysics that singles out a fundamental₁₂ plurality, think of *Humean Supervenience*, and replace *supervenience* with *determination*. Ney discusses Humean Supervenience as a possible motivation for separability in Ney (2023: 10–12).

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