PSO for the Sharpe Ratio in a financial trading system based on Technical Analysis

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Abstract. We consider four technical indicators widely used in financial practice to determine the optimal signal aggregation, trading rule definition, and indicator setting using the Particle Swarm Optimization metaheuristic applied to an important financial fitness function, that is the Sharpe Ratio. We experiment our trading system to the Italian index FTSE MIB and to a set of financial stocks belonging to the FTSE MIB over a multi-year period for training and testing. We generally achieve superior out-of-sample performance, using a standard technical analysis system as a benchmark.

Keywords: Trading system, Particle Swarm Optimization / PSO, Sharpe Ratio

1 Introduction

This paper proposes an algorithmic trading system based on Technical Analysis (TA) indicators with optimization of the signal aggregation, the trading rule definition, and the indicator setting. The parameters of the indicators, the trading rules, and the signal weights are the inputs of the system, and the fitness function to be maximized is the well-known Sharpe Ratio over the trading period. It is a complex global optimization problem, which we address by the Particle Swarm Optimization (PSO) metaheuristic (Kennedy and Eberhart in [5]), an approximate bio-inspired numerical optimizer.

TA indicators depend on one or more parameters (mainly time windows), generally assumed equal to standard values in the financial practice. The trading rules are functions whose inputs are the indicators; these functions generate signals based on market prices.

Usually, trading systems jointly consider a plurality of indicators, whose trading signals are aggregated through weighting, permetting a more informed decision-making (e.g., Corazza et al. in [2]). We consider and aggregate four popular standard indicators, that are the Exponential Moving Average (MA), the Relative Strength Index (RSI), the Moving Average Convergence/Divergence (MACD), and the Bollinger Bands (BB).

The existing studies have a main limitation: they only optimize a single category of parameters at a time, whether it be the indicators setting, or the trading rules definition, or the signal aggregation. For the first time in literature, Corazza et al. in [3] proposed to simultaneously optimize the three categories of parameters of the trading system. In that work, the fitness function to be maximized was the net capital at the end of the trading period, regardless of the risk level of the trading strategy. In the present work, we apply that proposal of simultaneous optimization to a different fitness function, that is the Sharpe Ratio.

The influence diagram of our trading system, from parameters to the fitness function, can be depicted through three intermediate levels: indicators, signals, and the aggregated signal. More precisely, the parameters of the indicators influence the computation of the indicators. Each indicator generates its own signal through its trading rule based on the trading rules parameters. The signals are aggregated into an overall signal using the weights of the indicators. Finally, the Sharpe Ratio – which depends on buying, selling, and holding decisions derived from the aggregated signal – is maximized via PSO by appropriately optimizing the three categories of parameters.

The remainder of this paper is organized as follows. The next section is devoted to describe the methodology used in this work. Section 3 presents the out-of-sample results of our optimized trading system. Some final remarks conclude the paper.

2 Methodology

2.1 Parametrization of the trading system

Our purpose is to optimize the parametrization of indicators, trading rules, and signal aggregation for a total of 23 parameters. Table 1 collects the parameters and describes their main features. For a description of the indicators, trading rules, and signal aggregation, the reader is referred to Corazza et al. in [3].

The trading system with the standard values of the parameters (reported in the last column of Table 1) serves as a benchmark for our optimized trading system.

2.2 Constrained optimization of the Sharpe Ratio

As performance measures, we consider the fitness function Sharpe Ratio over the trading period T, which is a risk-adjusted performance ratio. For its computation, we need to determine the net daily return $e(t)$, depending on: the strategy $s(\cdot)$; the stock price variation $P(t)/P(t-1)$; and the trading fee δ in the case of a strategy change:

$$e(t) = s(t-1)\ln\left(P(t)/P(t-1)\right) - \delta\left|s(t) - s(t-1)\right|,\ t = 1, \ldots, T. \quad (1)$$

The fitness function to be maximized is the Sharpe Ratio at the end of the trading period, $\rho = SR(T)$, under several constraints related to the parameters

Table 1. Parameters of the standard trading system

Parameter	Symbol	Indicator	Area	Standard value
Time window for computing MA	w_{ma}	MA	Indicator	12
Minimum period of validity of MA rule	d_{ma}	MA	Rule	1
Time window for computing RSI	w_{rsi}	RSI	Indicator	26
Threshold of RSI for entering in buy	t_{rsi}^{l,en_b}	RSI	Rule	30
Threshold of RSI for entering in sell	t_{rsi}^{h,en_s}	RSI	Rule	70
Threshold of RSI for exiting from buy	t_{rsi}^{h,ex_b}	RSI	Rule	70
Threshold of RSI for exiting from sell	t_{rsi}^{l,ex_s}	RSI	Rule	30
Short time window for computing MACD line	w_{macd}^{short}	MACD	Indicator	12
Long time window for computing MACD line	w_{macd}^{long}	MACD	Indicator	26
Time window for computing MACD signal line	w_{macd}^{signal}	MACD	Indicator	9
Minimum period of validity of MACD rule	d_{macd}	MACD	Rule	1
Time window for computing the moving average for BB	w_{bb}^{ma}	BB	Indicator	26
Time window for computing the standard deviation for BB	w_{bb}^{std}	BB	Indicator	26
(Positive) number of standard deviation for the upper BB	t_{bb}^{u}	BB	Indicator	2
(Positive) number of standard deviation for the lower BB	t_{bb}^{l}	BB	Indicator	2
Number of standard deviation for exiting from sell	t_{bb}^{c,ex_s}	BB	Rule	0
Number of standard deviation for exiting from buy	t_{bb}^{c,ex_b}	BB	Rule	0
Weight of MA signal	θ_{ma}	MA	Signal aggregation	0.25
Weight of RSI signal	θ_{rsi}	RSI	Signal aggregation	0.25
Weight of MACD signal	θ_{macd}	MACD	Signal aggregation	0.25
Weight of BB signal	θ_{bb}	BB	Signal aggregation	0.25
Threshold for the aggregated signal for entering in buy	t_{as}^{b}	–	Signal aggregation	+1/3
Threshold for the aggregated signal for entering in sell	t_{as}^{s}	–	Signal aggregation	−1/3

of the system. The global optimization problems can be rewritten as the following constrained maximization of ρ:

$$\max_{\chi \in X} \rho$$
$$\text{s.t.} \begin{cases} w_{ma},\, d_{ma},\, w_{rsi},\, w_{macd}^{short},\, w_{macd}^{long},\, w_{macd}^{signal},\, d_{macd},\, w_{bb}^{ma},\, w_{bb}^{std} \in N^{+} \\ t_{rsi}^{l,en_b} \geq 0,\, t_{rsi}^{h,en_s} \leq 100,\, t_{rsi}^{l,en_b} \leq t_{rsi}^{l,ex_s} \leq t_{rsi}^{h,en_s},\, t_{rsi}^{l,en_b} \leq t_{rsi}^{h,ex_b} \leq t_{rsi}^{h,en_s} \\ w_{macd}^{long} > w_{macd}^{short} \\ w_{bb}^{std} \geq 2,\, t_{bb}^{u} \geq 0,\, t_{bb}^{l} \geq 0,\, -t_{bb}^{l} \leq t_{bb}^{c,ex_s} \leq t_{bb}^{u},\, -t_{bb}^{l} \leq t_{bb}^{c,ex_b} \leq t_{bb}^{u} \\ \theta_{ma} \geq 0,\, \theta_{rsi} \geq 0,\, \theta_{macd} \geq 0,\, \theta_{bb} \geq 0,\, \theta_{ma} + \theta_{rsi} + \theta_{macd} + \theta_{bb} = 1 \\ t_{as}^{b} > t_{as}^{s} \end{cases}, \quad (2)$$

where X represents the parameter space.

2.3 Particle Swarm Optimization

The constrained global optimization problem (2) is formulated in terms of mixed-integer variables and it is nonlinear and nondifferentiable. Due to these complexities, exact solution algorithms are still sought in literature and we need to use an approximate solution method. Therefore, we consider to apply a metaheuristic and we choose PSO for its exploration and exploitation capabilities (Kennedy and Eberhart in [5], Wakasa and al in [7]).

Standard PSO is a solver for global unconstrained optimization problems, whereas our optimization problem is a global constrained mixed-integer one.

Consequently, we appropriately adapt the standard PSO for managing these specificities.

For dealing with integer variables, we follow a widespread approach in literature, that is the truncation method proposed in Parsopoulos and Vrahatis in [6].

For dealing with the other constraints, we reformulate our optimization problem as an unconstrained one using the exact penalty method described in Fletcher [4] and applied in the financial context in Corazza et al. in [1]. This method permits a correspondence between the optimizer of the original constrained problem and the unconstrained penalized one. The reformulated unconstrained version of the optimization problem is the maximization of the following function $\hat{\rho}$ with penalty parameter ϵ:

$$\begin{aligned}\max_{\chi \in X} \hat{\rho} = \rho - \frac{1}{\epsilon}\Big[& \max(0, -t_{rsi}^{l,en_b}) + \max(0, t_{rsi}^{h,en_s} - 100) + \max(0, t_{rsi}^{l,en_b} - t_{rsi}^{l,ex_s}) + \\ & + \max(0, t_{rsi}^{l,ex_s} - t_{rsi}^{h,en_s}) + \max(0, t_{rsi}^{l,en_b} - t_{rsi}^{h,ex_b}) + \max(0, t_{rsi}^{h,ex_b} - t_{rsi}^{h,en_s}) + \\ & + \max(0, -w_{macd}^{long} + w_{macd}^{short}) + \max(0, -w_{bb}^{std} + 2) + \max(0, -t_{bb}^{u}) + \\ & + \max(0, -t_{bb}^{l}) + \max(0, -t_{bb}^{l} - t_{bb}^{c,ex_s}) + \max(0, t_{bb}^{c,ex_s} - t_{bb}^{u}) + \\ & + \max(0, -t_{bb}^{l} - t_{bb}^{c,ex_b}) + \max(0, t_{bb}^{c,ex_b} - t_{bb}^{u}) + \max(0, -\theta_{ma}) + \\ & + \max(0, -\theta_{rsi}) + \max(0, -\theta_{macd}) + \max(0, -\theta_{bb}) + \\ & + |\theta_{ma} + \theta_{rsi} + \theta_{macd} + \theta_{bb} - 1| + \max(0, -t_{as}^{b} + t_{as}^{s})\Big];\end{aligned} \tag{3}$$

in our study we use $\epsilon = 10^{-2}$.

3 Applications

Our applications consider the closing prices of the FTSE MIB index and a set of selected stocks belonging to the FTSE MIB at the date of May 31, 2022, and traded on the market starting before January 2, 2007. We select five sectors, that are highly representative of the Italian economy. As results, we apply our methodology to the following stocks: Assicurazioni Generali S.p.A. (sector of insurance); Atlantia S.p.A. (sector of industrial products and services); Enel S.p.A. (sector of public services); Eni S.p.A. (sector of oil and natural gas); Intesa Sanpaolo S.p.A. (sector of banks).

We have conducted three different out-of-sample experiments. First, the trading period is divided into two subperiods, that is a training period and an out-of-sample testing one, but in each experiment a different length for the testing period is considered: 1 stock-month, 2 stock-months, and 3 stock-months, respectively; in Table 2, we provide the start and end dates for each in-sample and out-of-sample subperiod. Then, the trading system is optimized using the metaheuristic PSO over the training subperiod. Finally, the optimized trading system is applied to the out-of-sample testing subperiod.

The analysis is repeated 100 times, and we calculate the average value of the Sharpe Ratio as weel as other quantities of interest. In doing so, we confer a degree of statistical significance to the results, at least to some extent.

Table 2. Start and end dates for each in-sample and out-of-sample subperiod

Experiment	In sample subperiod	Out-of-sample subperiod
1	January 2, 2007 to April 29, 2022	May 2, 2022 to May 31, 2022
2	January 2, 2007 to March 31, 2022	April 1, 2022 to May 31, 2022
3	January 2, 2007 to February 28, 2022	March 1, 2022 to May 31, 2022

The results are collected in Tables 3. As for the structure of the tables, the first column indicates the length of the out-of-sample period; the third and fourth columns report the Sharpe Ratios of the standard system (which obviously are independent from the repetitions) and the mean Sharpe Ratios of the optimized system; the fifth and sixth columns present the annualized returns of the standard system and the mean annualized returns of the optimized system; finally, the last column reports the mean percentage of days in which the equity line of the optimized system is above the equity line of the standard system.

Table 3. Average out-of-sample performance of the optimized trading system over T month(s) for 100 repetitions for each asset

T	Stock	$SR(T)_{st}$	Mean $SR(T)_{PSO}$	$\bar{e}_{st}$	Mean $\bar{e}_{PSO}$	Mean % >
1	Assicurazioni Generali	−2.94	−0.28	−24.64	−6.46	48.76
1	Atlantia	−4.01	−1.87	−22.72	−8.08	71.33
1	Enel	−0.85	−0.27	−17.30	−6.38	41.24
1	Eni	−3.82	0.89	−61.99	7.09	81.90
1	Intesa Sanpaolo	6.86	2.40	111.09	46.64	23.19
1	FTSE MIB	0.35	0.35	2.87	3.29	47.90
2	Assicurazioni Generali	2.30	0.76	22.99	9.20	29.00
2	Atlantia	−3.00	0.47	−12.19	36.23	41.31
2	Enel	−1.66	−0.76	−26.51	−11.13	59.12
2	Eni	−1.89	−0.62	−34.10	−8.83	45.74
2	Intesa Sanpaolo	2.75	0.45	33.45	9.48	37.88
2	FTSE MIB	2.21	−0.64	15.59	−6.80	16.36
3	Assicurazioni Generali	6.15	1.59	74.73	28.47	18.02
3	Atlantia	1.90	1.80	22.74	46.99	32.44
3	Enel	−1.35	−1.03	−27.88	−20.35	51.95
3	Eni	−2.30	−0.72	−46.30	−15.59	72.22
3	Intesa Sanpaolo	2.54	0.74	41.39	16.83	39.11
3	FTSE MIB	1.34	0.82	9.01	12.53	59.71

It is worth highlighting that:

- When considering the 1-month long out-of-sample period, the optimized system draws or wins 5 times out of 6 compared to the standard system; the performances degrade as the length of the out-of-sample period increases. Consequently, our trading system would need to be re-optimized with appropriate frequency;
- Whatever the length of the out-of-sample period, when the optimized system loses against the standard system, the mean values of the annualized returns and of the Sharpe Ratios obtained by the optimized system are always positive, except one case This could indicate that the PSO works well in the optimization phase also in these cases, but likely paying for the choice of using a unique hyper-parametrization for all stocks;
- In some cases where the optimized system underperforms the standard system, the mean annualized returns of the optimized system are higher than those of the standard system; therefore, the optimized trading system allows an increase in the annualized return, although less than proportional to the increase in the riskiness of the strategy.

4 Concluding remarks

Our optimized trading system generally leads to superior performance over a standard TA-based trading system for a set of financial stocks belonging to the FTSE MIB on a multi-year horizon for training and testing.

Future researches will focus on: the use of a multi-objective fitness function and multi-objective PSO for constructing of an efficient risk-return frontier; the application of the capability of PSO for automatically selecting the indicators.

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