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Three Essays in Microeconomics

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The undersigned Vahid Mojtahed, in his quality of doctoral candidate for a Ph.D. degree in Economics granted by the Università Ca' Foscari Venezia attests that the research exposed in this dissertation is original and that it has not been and it will not be used to pursue or attain any other academic degree of any level at any other academic institution, be it foreign or Italian.

Abstract

This dissertation presents three essays in Microeconomics. In Chapter 1, I present a model of persuasive advertising over differentiated products in a duopoly. Each firm tries to increase the weight that the consumers attach to the dominant attribute of its product in which it has an advantage. Persuasion is the outcome of a communication between the firms and the consumers, where the subjective weight vector is changed accordingly. I show that if the consumers are naive or the advertising technology is effective, there is a Perfect Equilibrium, where firms earn a higher profit than by not advertising because they can further differentiate their products. I also show that duopolists regard their product as strategic substitute or complement depending on their current market share.

Chapter 2 studies an extension of the model of Rubinstein (1993) to two firms, competing in a market with consumers who are boundedly rational with respect to processing information. The cognitive bound compels customers to partition the price space. Rubinstein shows that a monopolist can earn a higher profit by exploiting consumers' lack of processing ability. I extend his model to a duopoly, and show the Nash and Correlated equilibria of the game. I prove that in competition, whether firms choose their strategies independently or dependently, firms' joint profit is lower than in a monopoly but does not vanish completely. The uncertainty regarding the consumers' cutoff point and differences across firms' prices impel firms to set their prices equal to the marginal cost.

In Chapter 3, I propose an extended market for Green Certificates as an incentive scheme for investors and producers of electricity from Renewable Energy Sources. I derive the optimal buying and selling strategies of the agents in the market and the closed-form solution of the market-clearing price by applying the Certainty Equivalence principle. The stochastic dynamic programming of the model is approximated numerically under iso-elastic demand function, and the life history of the plan is

simulated. I show that the mean and variance of prices depend on the rules set by the regulator at the beginning of the policy. The results indicate that when rational sellers follow their optimal strategy, the price fluctuation is reduced compared to the fluctuations of certificates' flow.

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All errors are the author's own.

In the Name of God
the Most Compassionate, the Most Merciful

To my loving parents.

Without their wisdom and guidance, I would not have the goals I have to strive and be the best to reach my dreams!

and to my sister, who is my best friend.

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Chapter 1

Attribute-Based Persuasive Advertising and Price Competition

1.1 Introduction

"Everyone designs who devises courses of action aimed at changing existing situations into preferred ones." Herbert Simon

This chapter presents a new approach to model persuasive advertising in a duopolistic market. In our study, ads serve as a 'favorable notice' to some attributes of a commodity, which firms believe they will increase their probability of selling. Favorable notice regarding an attribute is any word, picture, video, etc. that shifts the attention of viewers or listeners toward that attribute. Repetition of these favorable notices increases the subjective weight that audience assigns to a particular attribute. In our setting, firms utilize their advertising budget to maximize their market share. They do this through what one can consider as '*stretching the truth*' in a communication. Consider the following examples:

a) Computer experts are always debating whether it is better to buy a generic computer with Windows operating system (OS) and strong hardware configuration or an Apple computer with Macintosh OS and weaker hardware configuration. Apple advertises its advantage, which is its OS, and then charges consumers with a higher price. It persuades consumers that the price they pay for a Mac is worth its higher price compared to other brands of personal computers or laptops. Similarly, rivals of Apple advertise their hardware configuration, but they charge a relatively lower price for their products maybe because they are not as persuasive as Apple when it comes to advertising.

b) McDonald and Burger King have run different advertising campaigns with divergent slogans, and each pointing to one aspect of their products. For instance, Burger King as part of its campaign to differentiate itself by its cooking method picked the slogan “feel the fire” or “fuel your fire”. In response McDonald picked “what we’re made of” to emphasize the quality of its product and ingredients after being faced with a suit claiming that it hid the health risk of its Chicken McNuggets.

We take an initiative in modelling persuasive advertising in line with the successful marketing concept of "Unique Selling Point" (USP) proposed by Reeves (1961). Reeves, in short, asserts that the successful advertising campaign of firms professes a unique proposition to the consumers, and this persuades them, through reality distortion, to switch brands. This notion indicates that all attributes of a product can be recapitulated into two categories: a distinguished trait in one category and the rest in another with firms publicizing only the distinguished one.

Advertising campaigns by firms often take two roles simultaneously with the goal of selling more of their products. One is to provide the consumers with information about the attributes of the commodity that is being advertised, and the second, and more important, is to convince consumers that this is the product they are looking for in comparison with other alternatives. In the marketing literature, the latter is known as the *persuasive effect* of advertising while the former is known as *informative advertising*. Both types of advertising aim at changing the choice of consumers and increase the market share.

Our focus is on two firms and two categories of attributes, but it is not difficult to extend our work to many firms competing in a market with many attributes. Consumers' preferences are represented by the subjective vector weights in the products' attributes space, which affects their choice. In our analysis, consumers are boundedly rational in the sense that their choice can be manipulated by firms as a consequence of a series of communication through ads to the extent that consumers shift their attention from one feature of the product to another in favor of the firm promoting it. In our model, altering the subjective weights attached to attributes does this.

We model advertising as a communication from two or more firms to consumers by sending ads such that at the end of communication consumers' attitude might change with respect to a product. The repetition of advertisement matters because it has a persuasive effect, and not merely for the sake of maximizing dispersion insofar as everyone sees at least one.

Our consumers are aware of the existence of products. This study aims at framing cognitive biases of the consumers after receiving advertisements, or any means of communication where their perception of attributes of a product is manipulated such that their total evaluation changes, and she will be persuaded to make a choice in the interest of the communicator. Two effects are at work here; one is to shift the attention of the consumers on a certain aspect of the product and second, to enhance the objective perception of that aspect. The communicator's effort is to amplify those attributes of a product over which it has an edge, because true eloquence is an emphasis but false eloquence is exaggeration.

We need to distinguish two types of advertising strategies. One type of advertising can make a synergy. This is when one firm advertises on a given attribute and increases the weight attached to that attribute, but at the same time, it increases the weights attached to that attribute for all other firms. In the other type, firms advertise to increase only the weight attached to their own attributes. Our focus is on the latter, which requires that weights be independent across firms.

We explain persuasive advertising through a two-stage game, where in the first-stage advertising will take place, and firms try to persuade consumers. In the second-stage, they set prices to maximize their profit. We show that in the second stage of the game, firms regard their products as strategic complement and responds to a more aggressive price set by the other firm more aggressively. Nevertheless, in the first stage of the game, whether or not they consider each other's products as strategic substitutes or complement depends on their market share. In a market consisting of two firms, we define the dominant firm as the one who has more than 1/2 of the market share and subordinate firms are those who have less. Subordinate firms are more restricted in their strategy set in order to maximize their profit in both stages of the game. These conditions are based on effectiveness of advertisement and objective characteristics of the products that are discussed in more detail in the subsequent sections.

1.1.1 Related Literature

The literature of advertising is divided into many branches. One branch studies the effect of advertising on competition and deterring entry of new firms. Related to this branch are those papers that also study the effect of advertising on price competition. Advertising aims to shift the consumers' demand for firms' product. Thus, this leads to a price competition given the cross-elasticities and demand curve. For more on advertising and competition see the survey by Comanor and Wilson (1979). Our study sheds some light on this aspect, and the fact that firms may set higher prices not only because of their advertising expenses, but also due to the attributes of their products and consumers preferences before advertisement takes place. A well-known study on this topic is Schmalensee (1983). He presents a two-stage model of advertising as a way to deter entry. In his model, it is never optimal for the incumbent monopolist to advertise more in order to deter entry of the new entrant.

Another branch of the literature studies persuasive advertising. The persuasion used in advertising has common roots in Psychology and Economics. In Psychology, there have been different definitions for persuasion. What has been common in all these definitions is the effort of the communicator to mold and change the attitude of the listener by transmitting a series of messages (verbal or non verbal; reasonable or non reasonable). The term 'attitude' is also defined by many authors (Fazio, 1989; Perloff, 2003) but we follow the definition by Petty and Wegener (1998), "Attitude is the overall evaluation of an object or issue by a person, and attitude change means that a person's evaluation is modified from one value, often initial evaluation, to another".

In Economics, Kaldor (1950) was the first to hold the idea that advertising is manipulative and reduces competition, and therefore reduces welfare. Advertising would persuade consumers to believe wrongly that identical products are different, because the decision of which brand to purchase depends on consumers' perception of it rather than the actual physical characteristics of the product. Therefore, prices of heavily advertised products would rise far beyond their cost of production. The literature of advertising as a signal containing information about attributes of a product goes back to Nelson (1974). Kihlstrom and Riordan (1984), in a step ahead, modeled Nelson's idea of market competition where advertising expenditure is a signal of quality such that high quality producers advertise more as they have a higher return to advertising. As McFadden (1980) points out, attitude and perception affects market behavior as they change the choice probability of alternatives. The relation between attitudes and perception, on the one hand, and objective attributes, on the other hand, can be intervening or simply separable. In either case, objective attributes of products have an interconnection with psychological measurement.

Grossman and Shapiro (1984) were the first who studied advertising in markets with differentiated products, where consumers' only source of information is advertisements, which present accurate information about attributes of products. They also correctly point out that the reason for excessive advertising is to influence consumers' preferences, called persuasive advertising, although they did not study it in more detail. Fehr and Stevik (1998) distinguished three types of persuasive advertising. 1) To enhance the value of a product in the eye of the consumer; 2) to lead consumers to attach more importance to those differences that already exist between products; 3) a tug-of-war in which each firm attempts to attract consumers by molding their preferences to fit the characteristics of its product. Becker and Murphy (1993) present a model where advertisements enter the utility of consumers as 'goods' or 'bad'. In their simple model, advertisement increases the marginal utility of a good.

Another branch of literature on advertising focuses on *introductory advertising*, the role of advertising to introduce a product to the customers. Investing in advertisement in these models will increase the demand for the product as the fraction of the market who are aware of the product will increase. However, any type of advertisement serves more or less this function.

Our analysis has also similarities to the literature of rent-seeking contests started by seminal work of Tullock (1980). The basic model of contest focuses on non-cooperative, simultaneous competition of two or more agents trying to expend effort in order to increase their probability of winning a rent. For a complete survey of this literature, one can refer to Nitzan (1993). Skaperdas and Vaidya (2008) model persuasion as contest where the probability of persuading the audience depends on the resources expended by the players. In their model, the audience makes inferences based on the evidence presented to them, and the resources that contestants expend to produce the evidence. In our model, instead, firms make effort by advertising to enhance the perception of their commodity, hence capturing a higher market share. On a relevant path however different from advertising, Gabaix and Laibson (2006) present a model of competition among firms where each firm tries to shroud some attribute of its product. In their model, the product or service of firms is divided into "base good" and "add-ons" and firms are interested in hiding some of their add-ons attributes to exploit myopic consumers with the help of marketing schemes.

This paper is organized as following; in Section 1.2 we set the theoretical model for persuasion where firms compete to change the weights that consumers attach to attributes of each product. We express firms' market share as a function of the characteristics of the products, perception of the consumers, and the price. In Section 1.3 and 1.4, we prove the existence of Perfect equilibrium. In Section 1.5, we undertake the comparative statics to study incentives of firms in response to changes in strategies of rivals. Section 1.6 presents conclusion and final discussion.

1.2 Basic Model

Two firms compete to sell their distinct, non-homogeneous, and substitute products in the market. We label their respective products by an index $i = \{1, 2\}$, where firm i sells product i at price p_i. We normalize their marginal costs of production to zero. There is a continuum of potential consumers heterogeneous in tastes that each buys one unit of product from either of firms. The total mass of these consumers is normalized to one for convenience. Each product has two attributes. The vector of attributes[1] of firm i is denoted by $\mathbf{X}_1 = (x_1, 1-x_1)$ and $\mathbf{X}_2 = (1-x_2, x_2)$ and, for the rest of this paper, we assume that $x_i > 1/2$. This means that each firm is better than the other in one attribute. Consumers attach a vector of weights $\mathbf{W}_i = (w_i, 1-w_i)$ to the attributes of each product. It is important to assume that w_i is independent from w_j so that one firm's advertisement will only affect the perception of its own product. An additive multi attribute utility $\mathcal{U}(.) : \mathcal{W} \times \mathcal{X} \longrightarrow \mathbb{R}$ is defined for consumers over the set of attributes and their subjective weights. The attributes of each product are evaluated objectively, but firms compete by means of advertisement to change the perception of these attributes in the total evaluation of consumers such that their product will be chosen. This is known as *reality distortion*; the ability to convince consumers to believe almost everything by distorting their sense of scales of attributes such that they believe the product is as it is said. The values of weights are in the interval $[0, 1]$. Evidently, if the weight is zero for any attribute, then consumers do not care about that particular attribute.

Therefore, the ex-ante utility of consumers of buying one unit of product from firm i

[1] We can interpret this differently if each product has one prominent attribute that consumers pay more attention to, and the rest of attributes can conflate in one component.

is:

$$U_i = w_i x_i + (1 - w_i)(1 - x_i) - p_i, \qquad i \in \{1, 2\} \tag{1.1}$$

where the evaluation of attributes of a given brand excluding price is vector $\mathbf{X}_i$. The weight attached to price is one since firms cannot obfuscate consumers with respect to it.[2] In this way, for example, consumers may ask themselves how much a longer warranty or stronger engine is worth comparing to the price of brand i. Firms simultaneously send ads to consumers to change the weight that they have attached to the attributes of their products. It is intuitive that each firm tries to advertise on its best attribute and emphasize that attribute compared with other attributes. Thus, each firm's advertising strategy is shown by γ_i with the marginal cost of sending an ad normalized to one and the vector of joint advertising strategies is denoted by $\mathbf{\Gamma} = (\gamma_1, \gamma_2)$. The amount of advertising γ can be interpreted as pages of advertisement in a magazine or the number of billboards that a firm can hire.

Definition 1.2.1 *We define advertisement as a* ***communication*** *function* $\Phi_i(w_i, \gamma_i)$ *for firm* i*, twice continuously differentiable, and increasing concave in* γ_i*, according to which the weight vector of consumers* $\mathbf{W}_i$ *for product* i *is updated after each round of advertising.*

Therefore the utilities after advertising become

$$U_i = \Phi_i(w_i, \gamma_i)x_i + (1 - \Phi_i(w_i, \gamma_i))(1 - x_i) - p_i, \quad i \in \{1, 2\}. \tag{1.2}$$

The communication function, in general, may be different between firms. Firms usually make efforts in creating and sending ads that are more sophisticated to influence their viewers even more. Some firms might use TV, journals, and the

[2] Gabaix and Laibson (2006) divide the price of a commodity into two components: the price of an avoidable add-on and the price of the base good, and then they argue that firms only shroud the price of the add-ons. Since we have only one price for our product, our study is consistent with their analysis on this respect.

Internet as their means of communication since they believe they are more effective by having a larger number of viewers or deeper impact. Others believe that writing ads is a skill, and there are important aspects regarding making it, including wording, graphics coloring, placement, etc. For the sake of simplicity, we assume that firms use the same technology and means of the communications, and hence they have the same form of communication function. The effectiveness of communication function also depends on how sophisticated or naive are consumers. If consumers are manipulated easily by advertisement, they are more naive. In advertising literature, *Pre-testing*[3] is a branch of marketing research for analyzing the effectiveness of ads before their release. They can analyze the ads and direct the flow of attention during a commercial. (Young, 2005)

Concavity of $\Phi_i(w_i, \gamma_i)$ implies that the weights will increase with each firm's advertising up to a given level, but then it becomes weaker, and it cannot increase any more the weight. This is the point where consumers are fed up with the advertisement and will not believe them any more (Becker and Murphy, 1993). Becker and Murphy, among many others, argue that advertisers provide free utility raising programs to compensate viewers for exposing them to the ads.

Definition 1.2.2 *Firm i is able to **persuade**[4] consumers to buy its product among all alternatives by maximizing the probability of selecting its product meaning*

$$q_i(u_1, u_2) = \frac{e^{\Phi_i X_i - p_i}}{\sum_{i=1}^{2} e^{\Phi_i X_i - p_i}}, \qquad j \neq i. \tag{1.3}$$

[3] Also known as Copy testing.

[4] One can, without loss of generality, assume an outside option of not buying with zero utility for consumers. In this case, firms were unable to persuade consumers to buy from them and it is appropriate to be described as "charlatans". In current state of affairs, consumers view firms as those who paint the snake and add legs.

Ties are broken between firms in favor of firm one. By Equation (1.3), we see that the probability of selling one item by firm i, q_i, is monotonically decreasing in its price but its increment in γ_i is constrained by the advertisement constraints. We can interpret q_i as the market share of each firm.

As mentioned before, the crucial assumption of our model is that consumers attach two different weight vectors to the attributes of firms' products. This assumption guarantees that advertisement of one firm, on any attribute, will not change the evaluation of consumers of the other product directly. If the weights were the same $w_1 = w_2 = w$, any effort of one firm in promoting one attribute of the commodity would also increase the perceived evaluation of consumers from the other product.

The objective of firms is to maximize their profit by succeeding in selling their product. The payoff function for firm i is:

$$\max_{p_i,\gamma_i} \pi_i(\gamma_1, \gamma_2, p_1, p_2) \equiv p_i.q_i(\Gamma) - C_i = \frac{p_i e^{\Phi_i X_i - p_i}}{\sum_{i=1}^{2} e^{\Phi_i X_i - p_j}} - (\gamma_i). \tag{1.4}$$

1.3 Price Competition

Our analysis is based on a two-stage, non-cooperative duopoly. In the first stage, each firm commits itself to a level of advertisement on the attribute in which it is better. At the end of this stage, each firm observes the advertisement of others and chooses the price of its product. The strategies of firms specify actions that have to be taken in each stage.

We may now define the solution concept. We investigate the set of open-loop Nash equilibrium and therefore, use the concept of Perfect Equilibrium. A 4-tuple of strategies is a Perfect Equilibrium of the two-stage game, if, after each stage, that part of firms' strategy relating to the stage that remains form a Nash Equilibrium in that subgame. To study such Perfect Equilibrium, thereby, we begin by analyzing the final stage of the game, which is being the choice of price given the advertisement in the previous stage, and then fold back to determine firm's optimal level of advertisement in the first stage. Each firm chooses a price to maximize its own profit, taking as given the price of the other firm.

Proposition 1.3.1 *There exist a unique asymmetric price equilibrium* $(p_1^*(\gamma_1), p_2^*(\gamma_2))$ *which is the solution to the implicit non-linear system of equations*

$$p_i^* = \frac{1}{1 - q_i^*}, \quad i = \{1, 2\}, \tag{1.5}$$

where p_i^* *is the firm i's equilibrium price and* q_i^* *is given by*

$$q_i^* = \frac{e^{\Phi_i X_i - p_i^*}}{\sum_{i=1}^{2} e^{\Phi_i X_i - p_i^*}}, \qquad j \neq i. \tag{1.6}$$

Proof. The first-order condition corresponding to (3.1) is

$$\begin{aligned} \tfrac{\partial \pi_i}{\partial p_i} &= q_i + \tfrac{\partial q_i}{\partial p_i} p_i = 0 \rightarrow p_i q_i (q_i - 1) = -q_i \\ p_i &= \tfrac{1}{1 - q_i} > 0. \end{aligned} \tag{1.7}$$

The second-order condition is always negative for the dominant firm with $q_i \geq 1/2$, hence dominant firm's payoff is concave in its prices.

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2q_i(1 - q_i) + p_i q_i (1 - q_i)(1 - 2q_i). \tag{1.8}$$

The second-order condition for the subordinate firm $q_j < 1/2$, $j \neq i$ is also negative for all prices with $p_j < \frac{1}{(1/2) - q_j}$. Our extreme point in (1.7) is also the unique global maximum of the payoff functions given the constraints on price. □

The above proposition suggests that for every point decrease in price, the dominant firm, who was more successful in advertising in the previous stage of the game, can capture a higher percentage of the market compared to the subordinate firm. In other words, a firm which was more persuasive in advertising and became the dominant can capture more market shares when it comes to pricing competition. The cross price derivatives are,

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} &= \frac{\partial q_i}{\partial p_j} + \frac{\partial q_i}{\partial p_j} p_i q_i + \frac{\partial q_i}{\partial p_j}(p_i(q_i - 1)), \\ &= q_i q_j \left(2 p_i q_i + (1 - p_i)\right). \end{aligned} \tag{1.9}$$

Following (1.5) and (1.6) and the constraint that sum of the probabilities is one, we can derive the best response functions

$$\begin{aligned} Br_2(p_1) &= p_1 + \ln(p_1 - 1) + K, \\ Br_1(p_2) &= p_2 + \ln(p_2 - 1) - K, \end{aligned} \tag{1.10}$$

where $K = -2(\Phi_1 x_1 - \Phi_2 x_2) + (\Phi_1 - \Phi_2) + (x_1 - x_2)$.

The incentives of one firm to react to changes in the price of the other firm now depend on the characteristics of products and the advertisements in the previous stage of game. As we see in (1.10), if firm 2 advertises more in the first stage of the game, Φ_2 increases. This means that consumers attach higher weight to the second attribute of firm's 2 product. In response, ceteris paribus, firm 1 has an incentive to lower its price in the second stage of the game. The opposite reasoning applies to firm 1. We should bear in mind that increase in Φ_1 or Φ_2 is bounded by price and characteristic of products.

An increase in Φ_2 raises the importance of firm 2 to firm 1, and this tends to make it optimal for firm 1 to reacts more aggressively to 2's. If one firm increases the price, the other has an incentive to do the same. Nevertheless, the price cannot go beyond the upper bound, which is $2\Phi_i x_i - (x_i + \Phi_i) + 1$ else her market share will drop to zero as consumers will not buy from it any more. Another important point is that even if $x_1 = x_2$ the two products do not become homogeneous as long as consumers weigh the attributes differently. This is compatible with what we see in many markets when

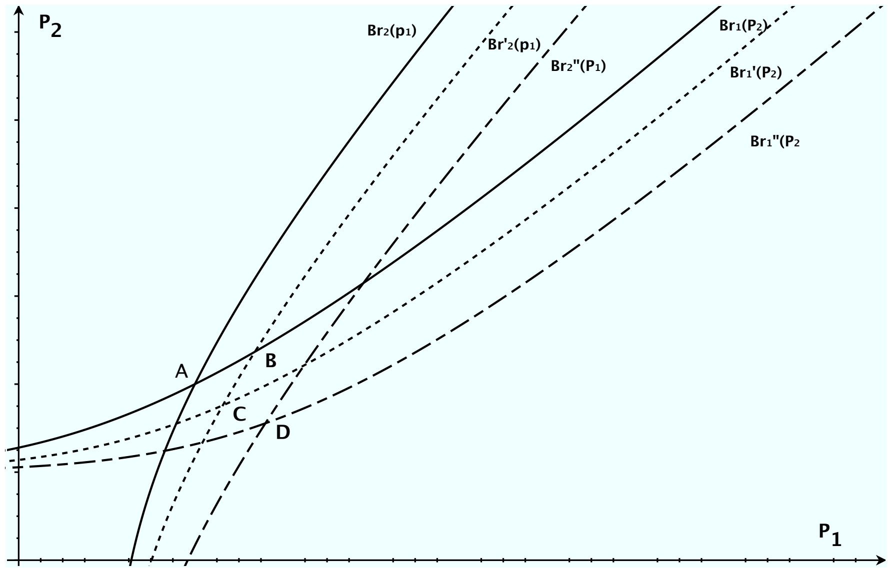


Figure 1.1: Best response to advertising

consumers perceive two products diversely even though in the eyes of a third person who has not been affected by advertising the products seem basically alike.

Neutral consumers are therefore defined by those who attach the same weight to a common attribute of all products $\Phi_i = \Phi_j$ for $i \neq j$.

The equilibrium prices p_1^* and p_2^* are function of γ_1 and γ_2. For every level of

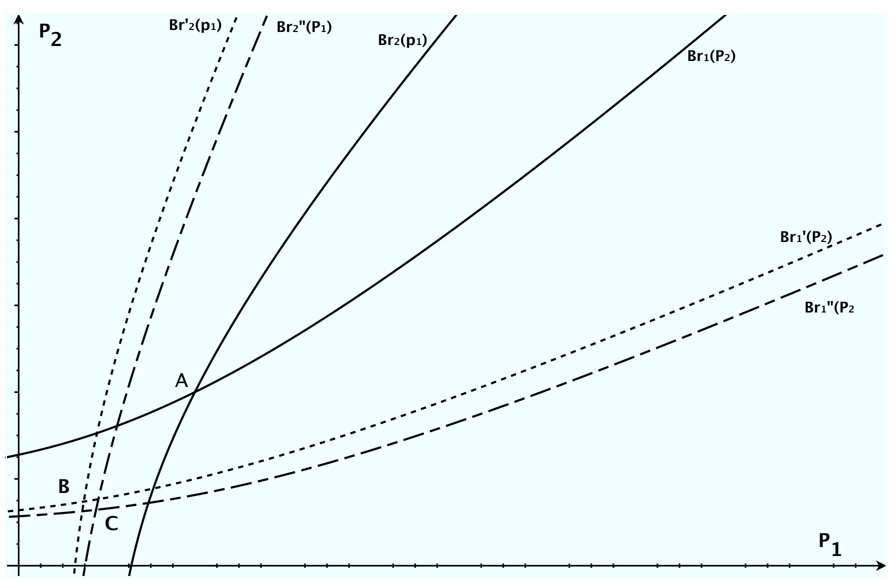


Figure 1.2: Best responses to advertising and price

advertisement and price of rival, the best reply price is non-negative. Figure 1 shows the best replies in the second stage of the game. $Br_1(P_2)$ shows the best reply of firm 1 to price of firm 2 and $Br_1(P_2)$ vice versa. Point A shows the equilibrium in the second stage of the game. If firm 2 advertises on its attribute in the first stage, the best reply of firm 1 in the second stage of the game shifts to $Br_1'(P_2)$. If the other firm does the same, then the new equilibrium is B in the second stage. $Br_1''(P_2)$ and $Br_2''(P_1)$ show the best replies in the second stage given the advertising constraint in the first stage of the game.

Figure 2 shows the best replies after advertisement and changes in prices. If each firm doubles its price in the second stage of the game, the best reply of firm one will be $Br_1'(P_2)$ and $Br_2'(P_1)$. In that case, the slopes of the best replies will change and the new equilibrium is point B. If there have been also advertisement by firm 1 in the first stage, together with doubling of price in second stage, the equilibrium point would be point C allowing firm 1 to charge a higher price.

An increase in the price of firm j increases the profit of firm i proportional to

$$\frac{\partial \pi_i}{\partial p_j} = p_i q_i q_j. \tag{1.11}$$

From (1.5), we know that higher market share for firm i entails lower price. In equilibrium, for two firms with $q_1 > q_2$; if $p_1 = p_2$ then necessarily $\Phi_1 x_1 + (1-\Phi_1)(1-x_1) > \Phi_2 x_2 + (1-\Phi_2)(1-x_2)$ with $x_1 > x_2$ to induce such outcome. The ratio of equilibrium market shares is given by,

$$\frac{q_i^*}{q_j^*} = \exp\left((\Phi_1 x_1 - \Phi_2 x_2) + (\Phi_2 - \Phi_1) + (x_2 - x_1) + (p_2^* - p_1^*)\right). \tag{1.12}$$

1.4 Advertising

We now turn to the preceding stage of the game where firms have to advertise simultaneously on the attribute in which they have an upper hand . Each firm chooses a level of advertisement γ_i.

Evaluating firm's i profit at the equilibrium price gives us the payoff functions of firms for the first-stage of the game:

$$\pi_i^\star \equiv \pi_i(\gamma_1, \gamma_2, p_1^*(\Gamma), p_2^*(\Gamma)) = p_i^* q_i^* - (\gamma_i). \tag{1.13}$$

Acting strategically, each firm sets γ_i to maximize (1.13).

Proposition 1.4.1 *A Subgame perfect Nash Equilibrium for the advertising-price game is defined by $p_i^*(\gamma_i^*)$ and γ_i^* through the implicit solution to the below equation*

$$\Phi_i'^* = \frac{1}{(2x_i - 1)(1 - q_i^*)q_i^*}. \tag{1.14}$$

Proof. The first-order condition for each firm, by the envelope theorem, is

$$\begin{aligned} \frac{\partial \pi_i^\star}{\partial \gamma_i} &= \frac{\partial u_i}{\partial \Phi_i}\frac{\partial \Phi_i}{\partial \gamma_i}\left(\frac{e^{u_i}(1+\sum_i e^{u_i}) - e^{2u_i}}{(\sum_i e^{u_i})^2}\right) - 1 = 0, \\ &= \frac{\partial u_i}{\partial \Phi_i}\frac{\partial \Phi_i}{\partial \gamma_i} q_i(1 - q_i) = 1 \\ \Phi_i' &= \frac{1}{(2x_i - 1)(1 - q_i)q_i}. \end{aligned} \tag{1.15}$$

We can identify the interior maximum only if the consumers are sensitive enough to the advertisement. This is only possible if the marginal increase to utility after advertisement $\frac{\partial u_i}{\partial \gamma_i}$ is greater than $\frac{1}{(1-q_i)q_i}$. If this condition fails, the solution exists at the boundaries. The second-order conditions show that

$$\begin{aligned} \frac{\partial^2 \pi_i^\star}{\partial \gamma_i^2} &= \overbrace{(2x_i - 1)\Phi_i'' q_i(1 - q_i)}^{<0} \\ &\quad + (2x_i - 1)^2(\Phi_i')^2 q_i(1 - q_i)(1 - 2q_i). \end{aligned} \tag{1.16}$$

The first term on the right hand side is negative by assumption $\Phi''_i < 0$ and the second term is negative for the dominant firm in the market. The subordinate firm $j \neq i$ can also achieve its global maximum if

$$\phi''_j < (2x_j - 1)(q_j - q_i)\phi'^2_j. \tag{1.17}$$

This condition asserts that as far as the negative effect of advertisement is less than $(2x_j - 1)(q_j - q_i)\phi'^2_j$, firm j can reach the maximum of its profit by advertising more. $\square$

The change in the marginal profit of firm i followed by advertisement of j, $i \neq j$, is

$$\begin{aligned} \frac{\partial \pi_i}{\partial \gamma_j} &= -\frac{\partial u_j}{\partial \Phi}\frac{\partial \Phi_j}{\partial \gamma_j}\left(\frac{e^{u_i}e^{u_j}}{(\sum_i e^{u_i})^2}\right), \\ &= -\frac{\partial u_j}{\partial \Phi_j}\frac{\partial \Phi_j}{\partial \gamma_j}q_i q_j = -(2x_j - 1)\Phi'_j q_i q_j < 0, \end{aligned} \tag{1.18}$$

and hence more advertisement of firm j reduces firm's i profit. By comparing (1.9) and (1.18), we can infer that the effect of these two actions on profitability depends on $p_i \lesseqqgtr (2x_j - 1)\Phi'_j$. We check for super/sub modularity condition for $i \neq j$

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial \gamma_i \partial \gamma_j} &= (2x_i - 1)\Phi'_i\left[\frac{\partial q_i}{\partial \gamma_j}(1 - q_i) - \frac{\partial q_i}{\partial \gamma_j}q_i\right], \\ &= (2x_i - 1)\Phi'_i(1 - 2q_i)\frac{\partial q_i}{\partial \gamma_j} = (2x_i - 1)(2x_j - 1)\Phi'_i\Phi'_j(2q_i - 1)q_i q_j. \end{aligned} \tag{1.19}$$

With only two firms in the market, we can substitute $(2q_i - 1)$ in the above equation with $(q_i - q_j)$, which shows the difference between the two firms' market shares. Since $x_i > 1/2$ for both firms, two possibilities can be considered based on the market share. First, if firm i has less than half of the market share, $q_i < 1/2$, the payoff of firm i becomes submodular. In this scenario the advertising strategy of firm j becomes strategic substitute for firm i and it responds to aggressive (higher) advertising of the other firm not aggressively since its rival's action decreases its marginal profitability in the market.

Second, when $q_i > 1/2$ the payoff function of firm i becomes supermodular and advertising strategy of firm j is strategic complement for firm i. Firm i will respond more aggressively to the advertising of firm j. This effect is also pointed out by Bulow et al. (1985) but their study is based on comparison between slopes of marginal revenue and marginal demand curves. The reason is that in an industry, the dominant firm would like to keep its front position and be the leader in the market therefore, it needs to maintain its market share and for this reason advertising of other firms threaten its superior position. The underdog, instead, will not respond to more advertisement of the topdog since the topdog will defend its position very severely and underdog can maximize its profit by saving in advertisement costs. If a firm is equally good in both attributes of a good, $x_i = 1/2$, the optimal advertising strategy will depend on the initial weight vectors of customers. If the customers attach more weight to one attribute, it will be less costly for the firm to increase the perception of customers by virtue of advertising on that attribute and as a consequence captures more market share.

1.5 Comparative Statics

In this section, we look into the comparative statics. We study how firms react to changes in the price and the advertisement level of each other, and how their profits are affected by the changes in their strategies. Differentiating along the first-order condition in the first stage, we have

$$\frac{d^2\pi_i}{dp_i^2} = \frac{\partial^2\pi_i}{\partial p_i^2} + \frac{\partial^2\pi_i}{\partial p_i \partial p_j}\frac{dp_j}{dp_i} = 0.$$

We can rewrite the above as

$$\begin{aligned}
\frac{dp_j}{dp_i} &= -\frac{\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi_i}{\partial p_i^2}} \\
&= -\frac{q_i q_j(2p_i q_i + (1-p_i))}{-q_i} = \frac{q_j q_i}{1-q_i}.
\end{aligned} \tag{1.20}$$

The nominator as calculated in (1.9) is positive and the denominator from (1.8) is negative. Equation (1.20) shows that the ratio of change in the prices of the two firms is proportional to their market shares $\frac{q_j q_i}{1-q_i}$.

It is also instructive to see how firm i reacts, changing its price, in the second stage to more advertisement of itself and the other firm in the first stage:

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial p_i \partial \gamma_i} &= \frac{\partial q_i}{\partial \gamma_i}(q_i - 1) + \frac{\partial q_i}{\partial \gamma_i} q_i + \frac{\partial q_i}{\partial \gamma_i}, \\
&= \frac{\partial q_i}{\partial \gamma_i}(2q_i), \\
&= 2(2x_i - 1)\Phi_i' q_i^2 (1 - q_i),
\end{aligned} \tag{1.21}$$

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial p_i \partial \gamma_j} &= \frac{\partial q_i}{\partial \gamma_j}(q_i - 1) + \frac{\partial q_i}{\partial \gamma_j} q_i + \frac{\partial q_i}{\partial \gamma_j}, \\
&= \frac{\partial q_i}{\partial \gamma_j}(2q_i), \\
&= -2(2x_j - 1)\Phi_j' q_i^2 q_j.
\end{aligned} \tag{1.22}$$

Since $x_i > 1/2$, we can sign the two equations above. $\frac{\partial^2 \pi_i}{\partial p_i \partial \gamma_i}$ is positive and $\frac{\partial^2 \pi_i}{\partial p_i \partial \gamma_j}$ is negative. Ceteris paribus, more advertisement of one firm in the first stage allows it to charge its consumers more in the second stage, but if its rival advertises more it has an incentive to reduce its price in the second stage to keep its market share and stay competitive.

Totally differentiating along the first-order condition in the second stage, we have (assuming that firm 2 does not advertise hence $\gamma_2 = 0$)

$$\begin{aligned}
\frac{\partial^2 \pi_1}{\partial p_1^2} dp_1 + \frac{\partial^2 \pi_1}{\partial p_1 \partial \gamma_1} d\gamma_1 + \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} dp_2 = 0, \\
\frac{\partial^2 \pi_2}{\partial p_2^2} dp_2 + \frac{\partial^2 \pi_2}{\partial p_2 \partial \gamma_1} d\gamma_1 + \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} dp_1 = 0.
\end{aligned} \tag{1.23}$$

$$\begin{pmatrix} \frac{\partial^2\pi_1}{\partial p_1^2} & \frac{\partial^2\pi_1}{\partial p_1\partial p_2} \\ \frac{\partial^2\pi_2}{\partial p_2\partial p_1} & \frac{\partial^2\pi_2}{\partial p_2^2} \end{pmatrix} \begin{pmatrix} dp_1 \\ dp_2 \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2\pi_1}{\partial p_1\partial\gamma_1} d\gamma_1 \\ \frac{\partial^2\pi_2}{\partial p_2\partial\gamma_1} d\gamma_1 \end{pmatrix}. \tag{1.24}$$

The solution to (1.24) is readily calculated to be

$$\begin{pmatrix} dp_1 \\ dp_2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} -\frac{\partial^2\pi_2}{\partial p_2^2} & \frac{\partial^2\pi_1}{\partial p_1\partial p_2} \\ \frac{\partial^2\pi_2}{\partial p_2\partial p_1} & -\frac{\partial^2\pi_1}{\partial p_1^2} \end{pmatrix} \begin{pmatrix} \frac{\partial^2\pi_1}{\partial p_1\partial\gamma_1} d\gamma_1 \\ \frac{\partial^2\pi_2}{\partial p_2\partial\gamma_1} d\gamma_1 \end{pmatrix}, \tag{1.25}$$

where Δ is the determinant of the matrix of coefficients

$$\begin{pmatrix} dp_1 \\ dp_2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} q_2 & q_1^2 \\ q_2^2 & q_1 \end{pmatrix} \begin{pmatrix} 2(2x_1 - 1)\Phi_1' q_1^2 q_2 d\gamma_1 \\ -2(2x_1 - 1)\Phi_1' q_2^2 q_1 d\gamma_1 \end{pmatrix}, \tag{1.26}$$

with $\Delta = q_1 q_2 (1 - q_1 q_2) > 0$.

The solutions to the system of equations are,

$$\frac{dp_1}{d\gamma_1} = \frac{2}{\Delta}(2x_1 - 1)\Phi_1' q_1^2 q_2^3, \tag{1.27}$$

so the price of firm 1 increases if it advertises more. Also

$$\frac{dp_2}{d\gamma_1} = -\frac{2}{\Delta}(2x_1 - 1)\Phi_1' q_1^3 q_2^2, \tag{1.28}$$

firm's 2 price will decrease. Its proportion is given by the coefficient of $d\gamma_1$. The reaction functions of the second period game are both upward sloped. Fudenberg and Tirole (1984) classify the incentives behind the first-period actions on the second-period actions as "top dog" effect if $\frac{\partial p_i^*}{\partial \gamma_i} > 0$ in contrast to "puppy dog". If a firm chooses to be a top dog, it will be more aggressive in the first stage of the game to stay tough in the second stage of the game. This suggests that if firm i is going to increase its price in the second stage, it has an incentive to advertise more to make its product more desirable in the eyes of consumers.

To study the profit variability due to advertisement, we differentiate (1.13) with respect to γ_i

$$\frac{d\pi_i^\star}{d\gamma_i} = \frac{\partial \pi_i^\star}{\partial \gamma_i} + \frac{\partial \pi_i^\star}{\partial p_i}\frac{dp_i}{d\gamma_i} + \frac{\partial \pi_i^\star}{\partial p_j}\frac{dp_j}{d\gamma_i} - 1, \tag{1.29}$$

where $\frac{\partial \pi_i^\star}{\partial p_i} = 0$ by definition of p_i^*. The first term in the left hand side of (1.29) is the direct effect of firm's i advertisement on the profit and the third term is the indirect effect. We can rewrite (1.29) given the equations (1.15), (1.28), and (1.11)

$$\frac{d\pi_i^\star}{d\gamma_i} = (2x_i - 1)\Phi_i' q_j q_i - \frac{2}{\Delta}(2x_i - 1)\Phi_i' q_i^4 q_j^3 p_i - 2. \tag{1.30}$$

Since we assumed that consumers are naive and sensitive enough to advertisement, the above equation shows us that an increase in persuasive advertisement will help firms to earn more profit. Of course, the extent of profit depends on the used technology, type of advertisement, and consumers' naïvetè.

Numerical Example

In this part, we illustrate a numerical example of how firms can compete through persuading consumers to buy their products. Suppose we have two firms $i \in \{1, 2\}$ who produce their products at zero marginal cost. Each product has two attributes being x_i and $(1-x_i)$ where x_i is normalized to [0,1]. Therefore, the vector of attributes is the same as mentioned before $\mathbf{X}_1 = (x_1, 1 - x_1)$ and $\mathbf{X}_2 = (1 - x_2, x_2)$. Firm 1 has advantage in $x_1 = x_2 + \epsilon$ and thus firm 2 has advantage in the second attribute. Let the initial weights of consumers over attributes of the two products be $w_1 = w_2 = 0.5$. The initial utilities of products are

$$\begin{cases} u_1 = w_1(2x_1 - 1) + (1 - x_1) - p_1 \\ u_2 = w_2(2x_2 - 1) + (1 - x_2) - p_2 \end{cases} \tag{1.31}$$

where the price for the two products are p_1 and p_2. The communication function is the same for both firms which is $w_i^{'} = w_i - (\gamma_{in} - 1)^2 + 1$ where w_i' is the posterior weight after advertisement. It is easy to check that the utility derived from each firm's product will be $u_1 = u_2 + 3/2\epsilon + (p_2 - p_1)$ hence if firm 1 sets its price $p_1 < p_2 + 3/2\epsilon$, it will have a higher market share ($q_1 > q_2$).

The weights as well as the probabilities of persuasion change after the communication (advertisement) takes place. We obtain

$$u_i = w'(2x_i - 1) + (1 - x_i) - p_i \tag{1.32}$$

We start from the last stage where firms will set the prices and compete with each other over prices given the advertisement in the first-stage. As shown before, the price will be determined by

$$\begin{aligned} &\tfrac{\partial \pi_i}{\partial p_i} = q_i(1 - q_1)p_i = q_1 \to p_i^* = \tfrac{1}{1-q_i^*} \quad i = \{1, 2\} \\ &q_i = \tfrac{e^{u_i(p_i^*)}}{e^{u_i(p_i^*)} + e^{u_j(p_j^*)}}, \quad i \neq j \end{aligned} \tag{1.33}$$

The profit function for the first-stage of the game can be written as a function of $p_1^*(\gamma_1)$ and $p_2^*(\gamma_2)$

$$\pi_i^* \equiv \pi_i(\gamma_1, \gamma_2, p_1^*(\gamma_1), p_2^*(\gamma_2)) = \quad p_i^* q_i^* - \gamma_i = \tfrac{q_i^*}{1-q_i^*} - \gamma_i. \tag{1.34}$$

The first-order condition implies that

$$\begin{aligned} \tfrac{\partial \pi_i}{\partial \gamma_i} &= \tfrac{\partial q_i^*}{\partial \gamma_i} \tfrac{1}{(1-q_i^*)^2} - 1 = 0 \\ &= \tfrac{-2q_i^*(2x_i - 1)(\gamma_i - 1)(1-q_i^*)}{(1-q_i^*)^2} = 1 \\ \gamma_i &= 1 - \tfrac{q_j^*}{2q_i^*(2x_i - 1)}. \end{aligned} \tag{1.35}$$

Since the second term is less than one, an interior solution exists. Thus, the amount of advertising depends on the characteristics of the product x_i, and on the market share q_i of each firm. The higher the market share or the better the prominent attribute of firm i, the less advertising is needed. The comparative statics studied in Section 1.5 applies for this example.

1.6 Conclusion

We show that in the game of attribute-based persuasion in advertising there exist an optimal advertising strategy of advertising and price. Our principal finding concerns the presence of strategic complementaries and substitutes in the agents' strategies based on some initial conditions. We have shown that firms' optimal strategies to advertise depend on their market share and the characteristic of their products. If a firm has less than half of the market share, the advertising of its rival become submodular in its advertisement, and if he has more than half of the market share, the payoffs become supermodular in the first stage of the game. In the second stage of the game, the strategies (setting prices) become strategic complements. The consumers become captive if firms advertise in the first stage, which enables them to charge prices higher than the marginal cost of production in the subsequent stage. To relax price competition, firms choose a level of advertising, which makes their product as far away from each other as possible.

1.7 Appendix

Proposition 1.3.1

$$\mathcal{H} = \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \\ \frac{\partial^2 \pi_i}{\partial p_j \partial p_i} & \frac{\partial^2 \pi_i}{\partial p_j^2} \end{vmatrix} = \begin{vmatrix} -2q_i(1-q_i) + p_i q_i (1-q_i)(1-2q_i) & q_i q_j (2p_i q_i + (1-p_1)) \\ q_i q_j (1 + p_i(q_i - 1)) + p_i q_i^2 q_j & q_i q_j^2 p_1 + q_j(q_j - 1) p_i q_i \end{vmatrix}$$

The principal minors of the above determinant are

$$\begin{aligned} \mathcal{D}_1 &= -2q_i(1-q_i) + p_i q_i(1-q_i)(1-2q_i) \leq 0, \quad \text{iff} \quad q_i \geq 1/2, \\ \mathcal{D}_2 &= p_i q_i q_j (p_i - 1) - p_i q_j (1-q_i)(1+p_i) + q_j(p_i - 1) \\ &+ p_i[p_i(q_i - 1)(2q_i - 1)(2q_j - 1) + 2(1-q_i)(1-q_j)] \geq 0. \end{aligned} \tag{1.36}$$

We know that $p_i^* = \frac{1}{1-q_i} \geq 1$ since $0 \leq q_i \leq 1$.

Proposition 1.4.1

$$\mathcal{H} = \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial \gamma_i^2} & \frac{\partial^2 \pi_i}{\partial \gamma_i \partial \gamma_j} \\ \frac{\partial^2 \pi_i}{\partial \gamma_j \partial \gamma_i} & \frac{\partial^2 \pi_i}{\partial \gamma_j^2} \end{vmatrix},$$

with

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial \gamma_i^2} &= (2x_i - 1)\phi_i'(\frac{\partial q_i}{\partial \gamma_i} q_j + \frac{\partial q_j}{\partial \gamma_i} q_i) + (2x_i - 1)\phi_i'' q_i q_j, \\ &= (2x_i - 1)^2 \phi_i'^2 q_i q_j (q_j - q_i) + (2x_i - 1)\phi_i'' q_i q_j, \\ \frac{\partial^2 \pi_i}{\partial \gamma_j^2} &= -(2x_j - 1)\phi_j'(\frac{\partial q_i}{\partial \gamma_j} q_j + \frac{\partial q_j}{\partial \gamma_j} q_i) - (2x_j - 1)\phi_j'' q_i q_j, \\ &= (2x_j - 1)^2 \phi_j'^2 q_i q_j (q_j - q_i) - (2x_j - 1)\phi_j'' q_i q_j, \\ \frac{\partial^2 \pi_i}{\partial \gamma_i \partial \gamma_j} &= (2x_i - 1)\phi_i'(\frac{\partial q_i}{\partial \gamma_i} q_j + \frac{\partial q_j}{\partial \gamma_i} q_i), \\ &= -(2x_i - 1)(2x_j - 1)\phi_i' \phi_j' q_i q_j (q_j - q_i), \\ \frac{\partial^2 \pi_i}{\partial \gamma_j \partial \gamma_i} &= (2x_i - 1)\phi_i'(\frac{\partial q_i}{\partial \gamma_i} q_j + \frac{\partial q_j}{\partial \gamma_i} q_i), \\ &= -(2x_i - 1)(2x_j - 1)\phi_i' \phi_j' q_i q_j (q_j - q_i). \end{aligned} \tag{1.37}$$

The principal of the above determinant is

$$\mathcal{D}_1 = (2x_i - 1)^2 \phi_i'^2 q_i q_j (q_j - q_i) + (2x_i - 1)\phi_i'' q_i q_j \tag{1.38}$$

On the right hand side we know that ϕ'' is negative and if $q_i > q_j$ the whole term in (1.38) becomes negative and therefore our function is concave. Firm i achieves a maximum by advertising equal to γ_i^* as shown in (1.14). The other firm with less market share can achieve a local maximum given the following condition

$$\phi_j'' < (q_j - q_i)(1 - 2x_j)(\phi_j')^2. \tag{1.39}$$

Chapter 2

On Price Recognition and Competition with Boundedly Rational Consumers

2.1 Introduction

"Engineering, medicine, business, architecture and painting are concerned not with the necessary but with the contingent - not with how things are but with how they might be - in short, with design." Herbert Simon

The purpose of this paper is to present a model where the presence of boundedly rational consumers in the market allows rational firms to exploit consumers and gain a higher profit. Competition forces, desire to surpass rivals and gain market share, take away some of the incentives of firms to take advantage of consumers. For this reason, they cannot jointly achieve the profit that a monopolist could extract. Any effort of firms to collude implicitly even by correlating their strategies, fails due to the competition forces as explained below.

There is vast empirical evidence in economic theory pointing to boundedly rational decision makers. The list of examples is so long that we suggest the interested reader to read the survey by Mallard (2011). Across all studies on this topic, more is dedicated to boundedly rational consumers rather than firms. The reason is attributed to the fact that firms are interacting more with the market, and they are the price setters in an environment of intensive competition with rivals. This is the reason that more and more firms set complicated price policies, which are hard for consumers to understand. In many sectors, the cost of a good is related to the degree of consumers' rationality. For instance, the cost of tutoring slow learning students during the exam preparation period is higher than promising students comparing to the rest of the year. The same point applies to advisory or warranty services like Apple Care.

Standard economic theory assumes that agents act as if they perform exhaustive searches over all possible decisions and pick the best. It was only after Simon (1956) that this assumption was substituted with agents, who are 'satisficers' and have procedural rationality instead of substantive rationality. Optimization can be, for a variety of reasons such as cognitive limits and information processing, costly or impossible, and a decision maker may instead adopt a suboptimal solution such as heuristics to her problem. Firms often sketch a complex, multi-dimensional price policy that is difficult for consumers to grasp in its entirety. For instance, banks offer services, which consumers have to pay different fares or tariffs for disparate compartments such as withdrawing cash, transferring money among domestic or international accounts, and, etc., For instance; they may price some parts very high, but price some segments low depending on whether it is a domestic or international transaction, and present it to consumers as a price scheme. Many studies show that consumers use heuristics to simplify the decision-making process (e.g. applied to above example) and help them to choose the best, to the extent which is possible for them, alternative.

We extend the model of Rubinstein (1993) where a monopolist can extract additional profits from heterogeneous customers whose diversity stems from their different abilities in processing information. Rubinstein shows two contrastive ways of modeling customers' bounded rationality. First, when customers have different abilities in partitioning the price space, the type that can partition more can escape falling in a high-price trap in a certain state of the world set by the monopolist. Second, borrowing the concept of 'perceptrons' from neural networks, he resembled consumers to perceptrons of different orders. A sophisticated consumer can process a higher number of price vector components for making his purchase decisions.

We explore a model where there are two states of the world: low and high. All agents active in the market have the same priors about probabilities of the states of the world, but only firms know the realization of the real state of the world. Consumers need one unit of good and have different willingness to pay for it in each state. We focus our attention, without loss of generality, only on boundedly rational consumers because other types of consumers, i.e. rational consumers, can escape the trap set by the firms as described by Rubinstein (1993). Consumers are boundedly rational in the sense that they can divide the price space into two partitions. This prevents them from extracting valuable information from price policies about the state of the world. We will further explain this in Section 2.2.1. Firms have different but constant marginal costs for different states. We focus our study on a duopoly where the firms have the same cost structure. The marginal costs of firms are higher than consumers' willingness to pay in the high state. Hence, the firms have an incentive to sell only in the low state of the world where they earn more profit. They have the obligation to sell to all customers in both states of the world once they announce their prices. We show that there are equilibria where firms are not able to earn any positive profits since they set their prices independently to compete with each other to capture the whole market share. Three possible equilibria emerge when firms set their prices independently. Under two of them firms employ a random device to exploit consumer's bounded rationality and avoid selling them in the high state of the world. Even under correlated equilibrium which is a form of implicit collusion, still they cannot earn a higher profit by selling at prices higher than the marginal cost in the low state. However, they benefit from consumer's cognitive constraint by not selling in the high state of the nature

Related literature

As we step away from one of the central assumptions of economic theory, the rationality of decision makers, we face agents who are not able to identify, assimilate, or process information in an optimizing way. The models of cognitive bounded rationality are divided into many categories, ranging from agents who are satisficers, or that use rule of thumbs in decision making, to agents who have constraints in processing abilities or gathering information. Among all papers in the literature that are relevant to the topic of bounded rationality and competition, the following ones are most relevant to our work. Our focus is mainly on organization of useful information that adds to complexity of decision. In many papers below, the partitioning of information affects the accuracy of inferences and hence the optimality of decisions.

Luppi (2006) characterizes the equilibrium price when two firms compete a la Bertrand and consumers have an exogenous cutoff. By exogenous cutoff, we mean that the consumers partition the price space before the firms announce their price policies. She shows that the firms will set equal prices above their marginal cost, contrary to the standard Bertrand model. This is due to consumers' cognitive inability to recognize prices beneath the cutoff point so that firms do not have incentives to undercut prices below the cutoff. A cutoff point is a threshold according to which the price space is separated into categories. Luppi also suggests an endogenous price partitioning so that consumers minimize their expected cost by detecting the state of nature.

Dow (1991) studies consumers who wish to find the lowest price in the market, but they have constraints on their memory of search history. In his paper, consumers divide the set of search histories into many categories, and then they can only remember the category of their actual search. These simplify the inference about the real history of search and help them to make better decisions. Jehiel (2005) presents a new solution concept for multi-stage games where players bundle similar nodes at which other players have to move into categories called *analogy classes*, and they have only expectations about the average behavior in each category. In this way, players simplify what they need to know about other players' behavior.

Salop and Stiglitz (1977) model a different information constraint not involved with limited memory. In their model, consumers are informed asymmetrically about the distribution and location of prices of a good and one group incur costs to be informed. They conclude that this leads to price dispersion among other some other possible results. On the same line, Chen et al. (2010) show the effect of limited consumer memory on price competition between firms. In their paper, consumers summarize information about prices into categories and can remember only the average price in each category. This put a bound on consumers' ability to compare prices in the market. Firms, knowing about this, exploit consumers and earn a higher profit. Similar to the work of Rubinstein, if consumers categorize information about prices into finer categories, competition among firms is elevated and the price is pushed down making it hard to exploit consumers.

In a slightly different arrangement from the literature of constraints on information gathering, Spiegler (2006) shows a model where agents are procedurally rational.

They sample prices of firms, which employ a distribution over their prices, and buy from the firm with the lowest price in the sample. Firms taking into account the behavior of consumers will increase the variance in their price setting, or obfuscate, and exploit consumers' bounded rationality even in a competitive market.

This paper proceeds as following. In Section 3.2 we describe the basic assumption of our model. Our model generalizes the model of Rubinstein (1993) to the price competition between firms. We concentrate only on the behavior of boundedly rational agents in the market since they are the targets of firms' intricate pricing policy. We explain how the bound on consumers' cognitive ability in processing information prevent them from accepting firms' offers. Section 2.3 states the main results. We characterize the prices which sustain the equilibria and show that firms can avoid selling to boundedly rational consumers by devising a pricing policy that force consumers not to buy from them in the high state of the nature. In Section 2.4, we characterize the correlated equilibrium, when firms' strategies are correlated with each other, yet they cannot recover the monopolist's profit jointly together. In Section 3.4 we conclude.

2.2 A Basic Model

We now present our model, where assumptions of Rubinstein are tailored to our needs. There are two firms $I = \{1, 2\}$ producing one homogeneous good in two states of the world $S = \{H, L\}$. The high state is associated with the high marginal costs of firms and the low state with the low marginal costs. Firms are competing over a unit mass of consumers who each desire one unit of good. The prior belief about the probabilities of states of the world is μ^H and $\mu^L = 1 - \mu^H$. This is common knowledge among agents of the market, but only the sellers know the realization of the real state of the world.

Consumers must receive an expected strictly positive surplus given the prices announced by the two firms. This assumption is necessary to rule out those equilibria in which consumers behave arbitrarily different without having any economic reason. Their valuation for the good in the high state of the world is v^H, and in the low state of the world is v^L. In the low state of the world, both firms produce at constant costs c^L. However, in the high state of the world, the firms' marginal costs are $c^H > c^L$. The consumers' surplus from consuming one unit of good considering the prices, p_i, of firms in either of states is $v^s - p_i$. (Hereafter we use subscripts to index the firms and superscripts to index the states unless otherwise is mentioned.) The firm with lowest price captures the whole market share, but the demand is divided equally among firms, if they set identical prices. It is assumed that $c^H > v^H > v^L > c^L$. This assumption appears naturally in many sectors like education and consultancy services, where providing services for boundedly rational consumers costs more than the accepted price in the market. For instance, cell phone providers propose a price policy that charges clients with less capability of grasping the policy a higher price in some states compared to others. (i.e., weekend and night minutes versus during the day)

The game carries on hereunder: a) Sellers for each state of the world submit their price policies, which are drawn from a probability measure with finite support. At these prices, sellers are committed to providing all consumers with the good at their declared prices in both states of the world. b) Consumers select their partition given their cognitive constraint to have only one cutoff point in price space. Consumers set this cutoff point arbitrarily. In this way, they can simplify their decision-making procedure based on inferences they made in every partition. c) The state of the world is revealed only to sellers, and the sellers commit themselves to the prices, which are determined by the probabilistic device under a). d) The customers then decide to buy from either of the sellers based on their informational capabilities to process the prices, the posted prices, and the declared pricing policies.

This is a two-stage game, where firms announce prices and in the next stage, the customers can reject or accept the offers. The choice of consumers is affected by their cognitive ability which we will explain later. It is easy to see that in state H sellers cannot gain from selling to consumers since their costs of producing the good is uniformly higher than consumers' reservation value.

Bear in mind that the firms' strategy is a lottery in any state but not a mixed strategy. In mixed strategy, firms are indifferent towards all strategies that are in the support of their mixed strategy. However, in our case, firms strictly prefer the strategy of employing a random device in any state to a pure one, i.e. setting one price for each state of nature. Firms cannot prevent consumers from buying from them e.g. due to market regulations, or consumer protection policy. They can only prevent consumer from buying from them through price policies that they offer.

2.2.1 Consumers' Computational Bound

We now explain the bound on consumers' rationality in more detail. Since consumers do not know which state of nature has happened, they try to infer the state of the world based on the prior and the likelihood of signals they receive. Recall that a price policy p_i of firm i is a set of prices for each state of the world. When consumers do not know the realized state of the world, they may accept a price, which is too high, and incur a loss. In the real world, consumers tend to partition the possible prices into categories and make inferences in each of them, and finally assign an action to each category. The finer they partition the price space, the more accurate their inference about the state of the world so that their choice leads to a more optimal action.

Assume that consumers have one cutoff, on which their decision is based. They can

divide the price space into just two connected sets and decide to “Buy", “Not to buy" given a cutoff point t^* or “Always buy", “Never buy" given any price. This is in line with the way that consumers think a price as "fair" or "expensive", and make their decisions to buy or not. The consumers are homogeneous with respect to their abilities in partitioning the price space using only one cutoff point. Notice that once the cutoff point is decided, all firms which charge less than cutoff point are the same in the eyes of consumers. This is important because price competition below the cutoff point is meaningless; and the timing of game matters concerning to cutoff decision. If the cutoff point is decided before the firms announce their price policies, firms can exploit the consumers by setting their price just below the cutoff point. Whereas if firms announce their policies first, the choice of the cutoff point of consumers depends on firms' announced prices, and the result will be different from the previous case.
In our model, the consumers' cutoff point is not exogenous. They decide about the cutoff point between stage 1 and 2, which is after the announcement of price policies of firms and before the realization of the state of the world. The placement of cutoff is uncertain and can be between any two possible prices, inside the price policies, in the price space.

2.3 Equilibrium

In this section, we analyze the equilibria when firms set their prices independently. Suppose firms can set their prices lower than the reservation value $x_i = v^L - \epsilon_i^L$, or higher than the reservation value $y_i > v^L$, and a price $z_i = v^H - \epsilon_i^H$ for state H. We call y_i a mine-price since if consumers buy at that price, they incur a great loss. The strategy of firm i is a price policy P_i consisting of price(s) p_i^s for each state and the choice of a random device α_i. Based on above potential prices, the consumers select their cutoff point in between any two prices of firms. Note that since the sellers incur losses in state H by selling to consumers, the optimal strategy for them is to set price z_i, as close as possible to the reservation price, with probability one. We can fix the ϵ_i^H for the two firms to ϵ^H since competition in the high state does not benefit them. The following proposition shows the equilibrium when firms do not devise a transparent price policy. In propositions 2.3.2 and 2.3.3 we show that they can do better.

Proposition 2.3.1 *There exist a Nash equilibrium in price policies P_1 and P_2 where both firms set their prices to $p_i^L = c^L$ in state L and $p_i^H = v^H - \epsilon^H$ in state H and make a payoff of $\pi_i^* = \frac{1}{2}\mu^H(p^H - c^H)$.*

Proof. Suppose that firms announce prices x_1 and x_2 for state L, and z for state H. Three cases can emerge, as shown in Figure 2.1, and we study them in the following. In the first case, consumers will put the cutoff point between x_2 and z. The special case of $x_1 = x_2$ also falls in case I. For any price that falls in the high category, they infer that the state is H and buy. For prices below the cutoff point, since there were no mine-price announced by firms, they will infer that state is low and buy from either firms with equal probability. In the second and third cases, the consumers may put the cutoff point between x_1 and x_2 depending whether $x_1 > x_2$ or not, for any price that falls in low category, they are going to infer that the state is low and buy from the firm with lowest price and reject the offer of other firm. Thus, the other firm has incentives to bring down her price below the cutoff point. Since firms do not know the choice of cutoff point of consumers, and firms set prices simultaneously, each firms' optimal strategy is to set price equal to marginal cost. □

The uncertainty regarding the choice of cutoff point of consumers and the force of competition make firms bring down their prices to the marginal cost, and prevent them from being able to exploit consumers as before like the case of the monopolist. With above strategies of firms and since $c^H > p^H$ firms will only make losses in short term. Both firms cannot prevent consumers from buying them in the high state.

Hence the payoff of firms is half of the loss from selling in state H. This result is the same as standard Bertrand price competition, where firms undercut each other's prices to capture the whole market share. In this scenario, the price policy shrink to a price for each state of the world thus does not result in any loss for consumers, and they are able to perfectly infer the states.

Since our focus is on the competition in the presence of boundedly rational consumers, we removed rational consumers from the market and consequently our firms are making a loss in the equilibrium in Proposition 2.3.1. Obviously, in the real world markets, firms have enough rational customers from whom they are going to make profit.

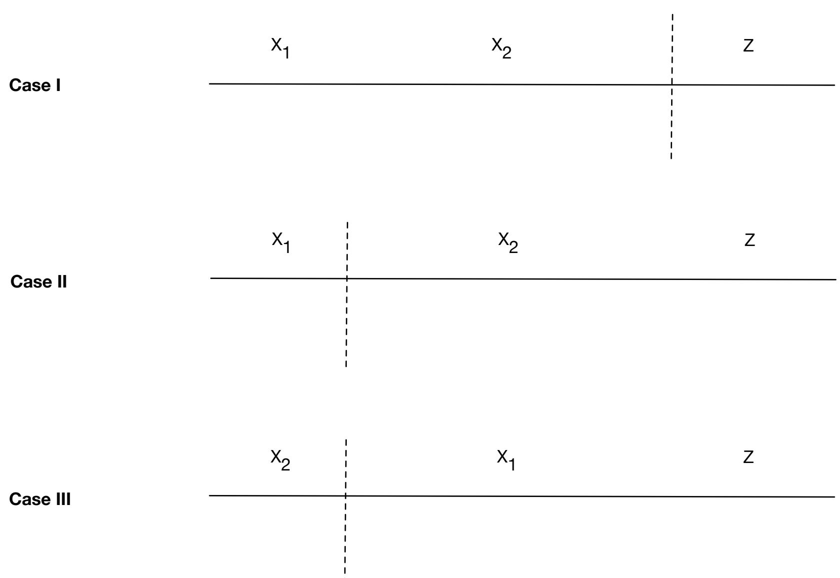


Figure 2.1: Firms use simple price policies.

Now we show that firms can do better if they use a random device in state L and exploit the cognitive bound of consumers. Under the monopoly case the seller is able to set price in state H equal to $v^H - \epsilon^H$, and employ a random device in state L over prices $v^L - \epsilon^L$ and $\frac{v^L+v^H}{2}$ and maximize her profit because the consumers were not buying as they were afraid to incur a loss when they partition the price space into two sets. The idea was that the consumers were either putting the cutoff point between $v^L - \epsilon^L$ and $\frac{v^L+v^H}{2}$, or between $\frac{v^L+v^H}{2}$ and $v^H - \epsilon^H$ and condition their decision based on their expected payoffs. The monopolists was choosing ϵ^L, ϵ^H, and the random device in such a way that $\mu^L\epsilon^L > \mu^H\epsilon^H$. Notice that the mine-price can be any price higher than the reservation price in state L but for the sake of simplicity in calculation, we follow Rubinstein (1993) in assuming that it is equal to $y_i = \frac{v_L+v_H}{2}$. Under competition, firms will not reduce (increase) their price (ϵ^H) in the high state, but they can reduce (increase) their price (ϵ_i^L) in the low state. The next two propositions establish this idea.

Proposition 2.3.2 *There exist an asymmetric Nash equilibrium in price policies P_1 and P_2 where firm one employs a price policy, p_1^*, described by setting c^L with probability $\alpha_1 \in \max\{\frac{2\epsilon^L+(v^L-v^H)}{(v^L-v^H)-2\epsilon^L}, \frac{\mu^L(v^L-v^H)+\mu^H\epsilon^H}{\mu^L(v^L-v^H)}\}$ and $\frac{v^L+v^H}{2}$ with probability $1-\alpha_1$ in state L and $v^H - \epsilon^H$ with probability one in state H, and the other firm sets her policy p_2^*, by setting c^L in state L and $v^H - \epsilon^H$ in state H with probability one, and earn the payoff of $\pi_i^* = 0$, $\forall\ i \in I$.*

Proof. Now, suppose one firm uses a mine-price, y_1, and a random device over his prices in state L. We have to consider four cases depending on the placement of the cutoff point. Figure 2.2 depicts the placement of the cutoff point in different cases. For the first three cases, suppose firm 1 sets x_1 and y_1, and firm 2 sets x_2 with $x_1 < x_2$ for the low state and z for the high state. First, suppose the consumers put the cutoff point between y_1 and z. Under this case, it does not matter whether $x_1 \lesseqqgtr x_2$. If the price falls in the high category, they can infer that it is state H and will buy since the price is fair for that state. If the price falls in the low category, they will not be able to infer the state of the world. The fear of making a loss by buying at mine-price y_1, a price higher than their reservation price, make them reject the offer because (see

Appendix)

$$\alpha_1 > \frac{2\epsilon^L + (v^L - v^H)}{(v^L - v^H) - 2\epsilon^L}.$$

Second, if the cutoff point is set between x_2 and y_1, whenever the price falls in the low category, the consumers infer that the state is low and buy the good. The case where $x_1 = x_2$ may also happen and will not affect the result of case II. If the price falls in the high category, they cannot infer the state of the world.

$$\alpha_1 > \frac{\mu^L(v^L - v^H) + \mu^H \epsilon^H}{\mu^L(v^L - v^H)}.$$

Firm 1 can choose α the maximum between the two above probabilities such that later is better for customers. With this strategy, firms can avoid selling in state H and the subsequent loss.

Third, suppose the cutoff is between x_1 and x_2. Firm 2 must reduce his price to a level lower than the cutoff point. In this case, together with the fourth case the firm who sets the lower price between the two firms will capture the whole market share. The uncertainty that these two cases bring to firms, make them set their prices equal to the marginal cost and share the market. If it was not because of these two cases, firms would have set their price as close as possible to the reservation price of consumers in state L like the second case and earn a higher profit. □

Note that the competition over the mine-price will not affect the result of the game and only changes the probabilities of charging the mine-price. Once again, competition does not allow firms to exploit the bound on rationality of consumers completely and make a joint profit equal to the monopolist's.

Our result points to a real-world phenomenon; firms tend to fabricate price policies to exploit boundedly rational consumers in markets and earn a higher profit compared to the standard Bertrand competition. The interesting outcome of the above fabrication is that firms who have not devised such tricky policies can also benefit from the opaque situation and not sell in the high state of the world. The main reason is that consumers do not distinguish between firms who charge a simple price and those who concoct a price scheme. They are satisfied by escaping a possible loss in after purchasing a good in the market. To make such discernment they need to be able to do an exhaustive search. Instead, they respond to a market and all prices that exist in the market.

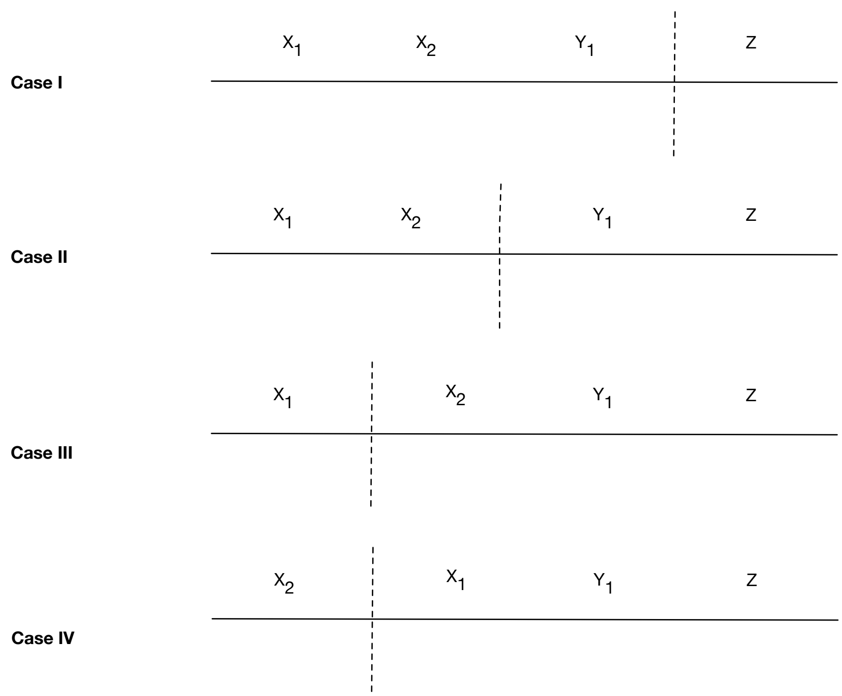


Figure 2.2: Firm 1 adopts a complex price policy and firm 2 uses a simple price.

Proposition 2.3.3 *There exist a symmetric Nash equilibrium where firm i employ a price policy p_i^* to charge customer c^L with probability $\alpha_i \in (0,1]$ and $\frac{v^L+v^H}{2}$ with probability $1-\alpha_i$ in state L, and both charge customers $v^H - \epsilon^H$ with probability one in state H and earn a payoff of $\pi_i^* = 0 \ \forall \ i \in I$.*

Proof. We have to consider six possible cases. Assume the following order for the first five cases $x_1 < x_2 < y_1 < y_2 < z$[1] and in the last case $x_2 < x_1$. The first case is

[1]For a general discussion we assume that there are two different mine-prices y_1 and y_2 but assuming only one mine-price will not affect any result of this part and make the derivation of results easier.

when the consumers put the cutoff point between y_2 and z. For any price that falls in the high category, they infer that the state is high and they will buy. If the price falls in the low category, they cannot infer the state of the world and they reject the offers of firms. In the second and third cases, they put the cutoff points between y_1 and y_2. In these two cases, consumers cannot infer the state of the world. Both firms have incentives to turn the situation to their own favor by setting ϵ_i^L, α_1, and α_2 in such a way that consumers prefer to buy in case II from firm one and in case III from firm two in the low state of the world and not to buy in the high state. Both firms can achieve this goal by increasing ϵ_i^L, reducing price x_i, or increasing α_1 and α_2. Hence, prices again drop down to marginal costs. Consumers will not buy if (See Appendix)

$$\alpha_1 > \frac{2\epsilon^L + (2-\alpha_2)(v^L - v^H)}{(v^L - v^H) - 2\epsilon^L}.$$

In the fourth case, the consumers put the cutoff between x_2 and y_1. They will buy from either of firms for prices that fall in the low category, but will not buy in the high category. Note that, consumers were indifferent to buy from either firm in the low state in this case if the cutoff was set exogenously above x_2, and firms did not have an incentive to compete over prices. Consumers will reject the offer if

$$\alpha_1 > \frac{2\mu^L(v^L - v^H) + \mu^H\epsilon^H - \alpha_2(v^L - v^H)\mu^L}{\mu^L(v^L - v^H)}.$$

Firm choose the maximum between the above two probabilities. Taking into account this decision of the other firm, it is enough for firms to choose any positive probability $\alpha_i \in (0, 1]$ so that fourth case is more favorable to the consumers, and they will not buy in the high state.

In the next two cases, consumers put the cutoff point between x_1 and x_2. Similar to our argument in proposition 2.3.2, this again sparks the competition between the two firms to reduce the prices to marginal costs so that at least capture half of the market share. In this scenario, firms are able to avoid selling to consumers in state H but cannot exploit the cognitive bounds of consumers. The payoffs of consumers in this scenario will be zero. □

Proposition 2.3.3 is more robust compared to the previous proposition. If we were presuming a level of rationality for consumers to distinguish between firms who set simple price and those who set complex price scheme, now under this proposition this will not be an issue any more. The consequence of every firm in the market charging a mine-price is that the probability of it decreases. In other words, the market becomes opaque and firms can exploit consumers by lowering the probability of charging the mine-price.

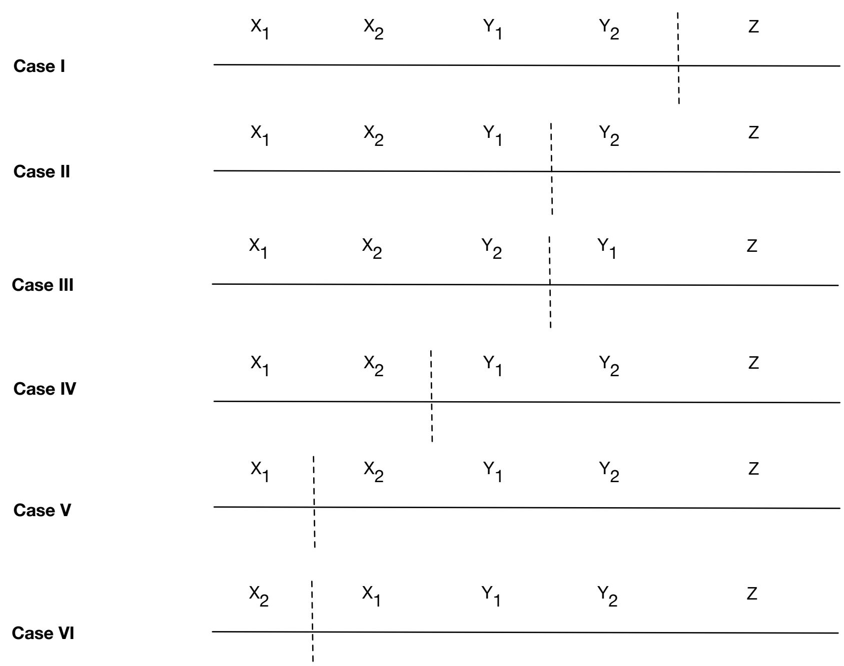


Figure 2.3: Both firms adopt complex price policies.

2.4 Correlated Equilibrium

In the previous section, we assumed that firms set their prices independently. As it turns out, given the lack of coordination, they are not able to exploit the bound on consumers' rationality. We now turn our attention to pricing policies that sustain equilibria when firms select set their prices by an implicit coordination device. Aumann (1974) proposed the idea that firms' strategies may be correlated; a new set of equilibria as an extension to the Nash equilibrium emerges, which are called *correlated equilibria.* Formally, a correlated equilibrium is defined as a joint probability distribution Γ, which is a common knowledge, over the set of strategies of all players which is the set of all positive prices P below the reservation price such that if, before taking any action, players receive a recommendation (e.g., from a moderator) randomly drawn from Γ, then no player has an incentive to deviate from the recommendation provided that all other players follow theirs'.

One may expect that, besides the equilibria described in Propositions 2.3.2 and 2.3.3 that are the result of independent strategy decision of firms, there are other correlated equilibria, which yield higher profits. For instance, firms may look for an equilibrium with a positive profit where both firms set their prices $v^L - \epsilon^L$ with probability α_i and $\frac{v^L+v^H}{2}$ with probability $1 - \alpha_i$ in state L and $v^H - \epsilon^H$ in state H if they will be able to coordinate with the help of an implicit, non obligatory agreement.

We confirm that firms cannot make any positive profit even if they coordinate through some communication device before choosing their strategies. Recall that the maximum payoff that they might achieve is $\frac{1}{2}Pr(L|p_1,p_2)(v^L - \epsilon^L)$ if both firms choose $v^L - \epsilon^L$ in state L, which is the maximum price they charge and consumers will still buy. Nevertheless, the outcome of the Bertrand game is very similar to the game of Prisoner Dilemma. Here a firm will defect if it lowers the price from $v^L - \epsilon^L$ and capture the whole market. This sparks the price war between firms and price is driven down to the marginal cost. Correlated equilibrium cannot help us to reach payoffs like the one mentioned above. The sufficient conditions for the correlated equilibrium are *strategic incentive constraints*,

$$\sum_{p_{-i} \in P_{-i}} \Gamma(p)\big(\pi_i(p) - \pi_i(p_{-i}, e_i)\big) \geq 0, \quad \forall i \in I, \forall p_i \in P_i, \forall e_i \in P_i, \tag{2.1}$$

where p_i is the recommendation of the mediator and e_i is any other action. Equation (2.1) represents the inequalities that a mediator's correlated strategy must satisfy to ensure that all firms rationally follow his recommendation. Vector Γ is a correlated equilibrium if it maximizes the expected sum of firms' profit, and it satisfies (2.1) and the following probability constraints

$$\sum_{e \in P} \Gamma(e) = 1 \text{ and } \Gamma(C) \geq 0, \quad \forall p \in P \tag{2.2}$$

Proposition 2.4.1 *The game presented above does not support any correlated equilibrium where firms can charge any higher price than marginal cost even with the help of a coordination device. The only correlated equilibrium is when firms randomize among Nash equilibria of the original game with no communication.*

Proof. The mediator cannot recommend or, in other words, put any positive probability on strategies from which a player is going to deviate. The recommendation of the mediator is divided into a) charging the same price, preferably as close as possible to the reservation value of the consumers in the low state v^L, which is the monopolist's price, and share the market equally or b) deviating and undermining the price of the other firm which leads to giving the whole market to it.

We can simplify our analysis by considering any two generic prices in the price space in the low state $[c^L, v^l)$ where the consumers accept the offer. Suppose these prices are $x^+ = v^L - \epsilon$ and $x^- = v^L - \epsilon'$ with $\epsilon' > \epsilon$ but ϵ' very close to ϵ.

Without loss of generality we can start our investigation for correlated equilibrium by assuming that x^+ is the monopolist's price. Notice that none of the players know the recommendation of the mediator to the other firm, and each receive different partial information about the outcome of the mediator's randomization. If firm 1 knew that the mediator has suggested firm 2 to play x^+, then player 1 is not willing to choose x^+ when it is suggested to it by the mediator. The mediator can randomize among (x^+, x^+), (x^+, x^-), (x^-, x^+), and (x^-, x^-) with probabilities γ_1, γ_2, γ_3, and γ_4 respectively. We intend to look for any correlated price strategy such that both firms do not have incentives to deviate. Hence the normal form of the game in the low state can be shown by the matrix in Figure 2.4.

	x^-	x^+
x^-	$(v^L - \epsilon')/2, (v^L - \epsilon')/2$	$v^L - \epsilon', 0$
x^+	$0, v^L - \epsilon'$	$(v^L - \epsilon)/2, (v^L - \epsilon)/2$

Figure 2.4: The payoff matrix for the game with correlated strategies in the low state.

The mediator intends to maximize the expected sum of payoffs

$$\max \left(\gamma_1(\sum^I \pi_i(x^+, x^+)) + \gamma_2(\sum^I \pi_i(x^+, x^-)) + \gamma_3(\sum^I \pi_i(x^-, x^+)) + \gamma_4(\sum^I \pi_i(x^-, x^-)) \right) \quad (2.3)$$

subject to the incentive constraints,

$$\begin{aligned}
&\gamma_1 (\pi_1((x^+, x^+) - \pi_1((x^-, x^+)) + \gamma_2 (\pi_1((x^+, x^-) - \pi_1((x^-, x^-)) \geq 0 \\
&\gamma_3 (\pi_1((x^-, x^+) - \pi_1((x^+, x^+)) + \gamma_4 (\pi_1((x^-, x^-) - \pi_1((x^+, x^-)) \geq 0 \\
&\gamma_1 (\pi_2((x^+, x^+) - \pi_2((x^+, x^-)) + \gamma_3 (\pi_2((x^-, x^+) - \pi_2((x^-, x^-)) \geq 0 \\
&\gamma_2 (\pi_2((x^+, x^-) - \pi_2((x^+, x^+)) + \gamma_4 (\pi_2((x^-, x^-) - \pi_2((x^-, x^+)) \geq 0 \\
&\gamma_1 \geq 0, \quad \gamma_2 \geq 0, \quad \gamma_3 \geq 0, \quad \gamma_4 \geq 0, \quad \text{and} \quad \textstyle\sum_{i=1}^4 \gamma_i = 1,
\end{aligned} \quad (2.4)$$

One can see that to satisfy the system of equations all probabilities must be zero except γ_4. Suppose for instance, the mediator wants to implement (x^+, x^+) as an equilibrium with probability γ_1. If he recommends x^+ to firm 1, its conditionally expected profit from playing disobedient price x^- is higher than its conditional expected profit from playing mediator's recommendation. Given the symmetry, the same applies to the player 2. Therefore, the first and third incentive constraints can not be satisfied for any strictly positive γ_1, γ_2, and γ_3. The second and fourth constraints can be satisfied only if γ_4 is set equal to one, and the Equation (2.3) is maximized.
The mediator can not suggest (put any strictly positive weight on) any strategy in which any firm charges the higher price between the two possible prices x^+ and x^-. Extending the set of possible prices, for example to $x^{--} = x^- - \epsilon'$, we can see that among all possible price, the mediator will put weight on the lowest possible price in the price space which is the marginal cost of firms. That is the best the two firms can achieve even if they correlate their strategies with the help of a coordination device such as a mediator. So the only self-enforcing strategies that firms could put into action with no formal contract would be randomizations between the Nash equilibria of the original game without the communication. □

Despite our expectation, in correlated equilibrium firms are not able to recover monopolist's profit jointly together, and the equilibrium is just in support of the Nash equilibria that we studied in Section 2.3.
Correlating strategies is a form of implicit collusion, which happens more among incumbent firms in mature markets. Mature markets are those in which firms have had a history of interactions, and greedy strategies carry out poorly. Under current setting, each firm aims for the whole market share and deviates from any implicit coordination. Only an enforceable agreement can keep prices at a pre-agreed level, else firms try to steal from each other's business by undermining each other's prices which resembles the game of Prisoner Dilemma.

2.5 Conclusion

We analyze a model of price competition with boundedly rational consumers, who can cut the price space into two connected sets by placing one cutoff point on the price space based on firms' announced prices. In the same manner as Bertrand, firms set their prices equal to the marginal costs due to the competition. As a consequence of firms deciding their prices independently, they cannot exploit any more consumers by fixing their prices higher than the marginal cost. The uncertainty regarding the placement of the cutoff point pushes firms to compete over prices and lower down their prices to the marginal cost. If the cutoff point was decided exogenously as discussed in Luppi (2006) then all the prices below the cutoff point were accepted by the consumers, and firms would have ended up setting prices above the marginal costs and equal to the cutoff point.

In addition, we show that firms can do better by exploiting the cognitive limitation of consumers in doing complex calculations. They reach a higher profit if they set a price policy that forces consumers not to buy in the high state of the world, where if firms sell, they incur a loss. Furthermore, even if firms coordinate on prices, they cannot exploit the consumers any more. This is attributed to the nature of the Bertrand competition which like Prisoner Dilemma, collaboration is dominated by betrayal.

2.6 Appendix

Proposition 2.3.2

The consumers reject the offer in first case if

$$\mu^L \left[(1-\alpha_i)(v^L - y_1) + (v^L - x_2) + \alpha_i(v^L - x_1)\right] < 0,$$

and therefore

$$\alpha_i > \frac{2\epsilon^L + (v^L - v^H)}{(v^L - v^H) - 2\epsilon^L}.$$

In the second case consumers reject the offer if

$$\mu^L(1-\alpha_i)(v^L - y_1) + \mu^H(v^H - z) < 0,$$

and therefore

$$\alpha_i > \frac{\mu^L(v^L - v^H) + \mu^H \epsilon^H}{\mu^L(v^L - v^H)}.$$

A firm should choose α_i in such a way that the later is better for the consumers.

Proposition 2.3.3

Given the strategies of firms in the second or third case, consumers reject the offers if

$$\mu^L \left[\alpha_i(v^L - x_1) + \alpha_j(v^L - x_2) + (1-\alpha_i)(v^L - y_1) + (1-\alpha_j)(v^L - y_2)\right] < 0.$$

For tractability and without loss of generality, we can assume that $y_1 = y_2 = \frac{v^L + v^H}{2}$ therefore

$$\alpha_i > \frac{2\epsilon^L + (2-\alpha_j)(v^L - v^H)}{(v^L - v^H) - 2\epsilon^L} \tag{2.5}$$

In the fourth case, where the cutoff point is set between x_2 and the mine-price, the consumers reject the offer if

$$\mu^L(1-\alpha_i)(v^L-y)+\mu^L(1-\alpha_j)(v^L-y)+\mu^H(v^H-z)<0,$$

and therefore

$$\alpha_i > \frac{\mu^L(v^L-v^H)(2-\alpha_j)+\mu^H\epsilon^H}{\mu^L(v^L-v^H)}. \tag{2.6}$$

The firms should choose the maximum between the above two probabilities so that consumers prefer to buy in state L. From above equation we can see that the random device that firm i chooses depends on the random device chosen by firm j. We can derive the same value for random device α_j of firm j. For the second and third cases, we have

$$\alpha_j > \frac{(2-\alpha_i)(v^L-v^H)+2\epsilon^L(1+\alpha_i)}{v^L-v^H}. \tag{2.7}$$

In the fourth case

$$\alpha_j > \frac{\mu^L(v^L-v^H)(2-\alpha_i)+\mu^H\epsilon^H}{\mu^L(v^L-v^H)}. \tag{2.8}$$

Firm i takes into account that firm j also uses a random device. We can have four possible cases but given the symmetry we focus only on two. Suppose firm j choose a random device with a probability greater than those in (2.7) and (2.8) for a very small positive value of τ. Firm i takes the decision of firm j into its consideration. For the first case from (2.5) and (2.7), we have

$$\alpha_i > \frac{2\epsilon^L+(v^L-v^H)(\frac{2(v^L-v^H)-(2-\alpha_i)(v^L-v^H)-2\epsilon^L(\alpha_i+1)}{v^L-v^H}-\tau)}{(v^L-v^H)-2\epsilon^L},$$
$$\alpha_i > \frac{-2\epsilon^L(\alpha_i)+(v^L-v^H)\alpha_i-(v^L-v^H)\tau}{(v^L-v^H)-2\epsilon^L},$$
$$(v^L-v^H)\tau<0,$$

which is true for all values of α_i. For the second case from (2.6) and (2.8), we obtain

$$\begin{aligned}
&\alpha_i > \frac{\mu^L(v^L-v^H)(\frac{2\mu^L(v^L-v^H)-\mu^L(v^L-v^H)(2-\alpha_i)-\mu^H\epsilon^H}{\mu^L(v^L-v^H)}-\tau)+\mu^H\epsilon^H}{\mu^L(v^L-v^H)},\\
&\alpha_i > \frac{\mu^L(v^L-v^H)\alpha_i-\tau\mu^L(v^L-v^H)}{\mu^L(v^L-v^H)},\\
&\tau\mu^L(v^L-v^H) < 0,
\end{aligned}$$

which is always true since $(v^L - v^H)$ is negative. As a result, for any positive value of random device chosen by firms, the firms can achieve their goal of preventing the consumers to buy from them in the bad state of the world.

Chapter 3

A Market-Based Governance for Tradable Green Certificates

3.1 Introduction

Currently, under different environmental plans, governments are trying to promote usage of renewable resources (i.e. Wind, hydro-power, solar, biomass, geothermal, and bio-fuel) for producing electricity and combat climate change. A Directive[1] of the European Parliament and of the Council of 27 September 2001 is dedicated to the promotion of electricity produced from Renewable Energy Sources (RES) in the internal electricity market of EU15. By this white-paper, the EU15 must achieve the target of 12% of overall energy (electricity, transport, heating) consumption produced from renewable energies by 2010 and 20% by 2020. In addition, under article 5 of the report, each member state is required to set national targets for the consumption of electricity from renewable sources.

Article 18 of this directive clearly emphasizes the importance of deploying the strength of market forces and internal markets to make production of electricity from

[1] Directive 2001/77/EC

RES competitive and attractive to citizens. The political and technical prerequisites for introducing a successful support scheme is out of the scope of this paper, and our focus is on the financial and economic motives and efficiency of such plans.

Among the incentive policies that have been proposed and implemented in most countries is the Feed-In Tariff (hereafter FIT) policy. The objectives and key features of a good FIT have been under question since Germany introduced this plan for the first time in 1991. As a part of FIT policy, investors in RES receive a fixed price, usually higher than the market prices of electricity from the conventional sources, for the produced electricity throughout a fixed period of time (i.e. 20 years) during which their access to the grid is guaranteed. As a result, they can easily evaluate their payoffs using a Net Present Value (NPV) evaluation. For more details about this policy, one can refer to Mendonca et al. (2010). This policy has been very successful in increasing the installed capacity of electricity plants from RES as there is no risk for investors.
On the other hand, the risk that authorities face concerning this option is over determining the price, which is going to be fixed for the next 15-20 years of the contract. Governments strive to evade the fiscal burden of environmental subsidies by implementing a mechanism which attempts to help both their environmental policy (carbon taxing, subsidizing RES, and diversifying their energy production portfolio) and avoid political or bureaucratic pressures. For this reason authorities have moved toward using market-based governance instead of direct administrative intervention.

The alternative to a FIT policy is a Tradeable Green Certificate (TGC) market as is currently in force in Sweden. The rules of this policy allow investors in RES to sell their electricity in the spot market and provide the investors with certificates for the

Green electricity that they have produced, which can be sold in a separate market known as the market for Tradable Green Certificates (TGC). Thus, the investors are provided with another source of income that can compensate for revenue fluctuation in the electricity spot market.
Since the introduction of RES plants, most countries started with FIT schemes as a guarantee for sharp and fast growth of capacity, but this cannot be continued due to its costs and rigidities. A second step for boosting the supply of electricity from RES relies on the help of a market for TGC or other measures such as grants, tax exemptions, investment support, and reduced VAT, which have been implemented across many EU countries. Each member state is allowed to choose its support scheme itself. Quota obligation and tax measures alone give little incentives for investment in new technologies such as PV.[2]

The TGC market is similar in spirit to the Emission Trading Scheme (ETS) in issuing certificates for firms and imposing penalties for those who do not comply with the quota or cap. The difference between the two schemes is about deciding to either punish the polluters or rewarding those who are contributing to the substitution of the polluted industries with greener ones. ETS has been criticized on divergent aspects, but the most important ones are about 'windfall profits' and 'grandfathering'. Producers of electricity from renewable resources can be disparate with respect to their installed capacity, and the technology that they deploy. They are entitled to receive certificates proportional to the amount of the electricity that they produce. The regulators set this proportion, that may differ among different types of technologies. The reason is that different technologies have different capacity factor[3], or they are at different maturity stages in the market. Some have already been widely spread

[2] Photovoltaic Solar Energy: Development and current research, 2009.

[3] Specifies the ratio of actual output of a power plant over a period of time against its output if it had operated at full nameplate capacity for the entire time.

and incorporated in the market successfully with reasonable profitability while others, maybe with a bright future, are still under evolution, and they are costly to be used by investors in the market without the support of subsidies.
We tackled this issue by pooling different technologies in our model and implementing necessary instruments for the regulator to shift incentives toward a targeted technology without disrupting the market mechanism.

The main difference between FIT and the market for TGC is about who takes the risk. In case of FIT, it is the government who takes the risk but in the market for TGC, it is the private investors or producers who are bearing the risk and therefore need a risk premium, yielding a price higher than in the spot market.

The policy maker can avoid problems with the allocation of permits as it is the case in cap and trade schemes. The Green Certificates is based on the capacity of the plants initially invested by those who take the risk. This also helps to subsidize (or incentivize) new technologies by controlling the number of Green Certificates that each technology is allowed to receive. The market provides sufficient incentives for initial investors in renewable technologies but as the production increases and technologies become mature the profit of new investors decrease.

We study a market for storable Green Certificate with disturbances, which are not serially correlated, originating from stochastic production of electricity. The demand and supply of Green Certificates are derived from the optimizing behaviour of individual agents (buyers of Green Certificates and producers of electricity from renewable resources). The rational expectation equilibrium price is explicitly solved.

We propose a closed-form solution for the selling and buying strategies of buyers and sellers in a TGC market. The supply strategy is derived from the decision of investors in RES plants to maximize their income during the policy lifetime when they are entitled to receive certificates proportional to the electricity that they pro-

duce. This is another source of income for them, separately from the electricity spot market. The demand in this market is set by the regulator through imposing a quota proportional to the electricity consumption of households or businesses. The buyers' strategy is derived from the decision of buyers to minimize their expenditure during the policy. Finally, we obtain the price as a result of market clearing condition and simulate the price to shed more light on the behaviour of agents in the market.
We derive the rational expectation equilibrium for risk neutral producers under uncertainty about the number of certificates that they receive in each period of a finite policy. We assume that the number of certificates in hand at each period is a function of the previous period's certificate that was received and of a realization of a shock.

3.1.1 Related Literature

As pointed out by Lemmings (2003), generally producers or potential investors in RE are exposed to two types of risk: a) caused by swing in volume of electricity that they can produce; b) lack of information about the shape of demand and supply curve in order to evaluate the expected return on investment.
While electricity is a perishable good and cannot be stored to be sold in the future, the certificates (Green Tags) are durables with maturity of one year or more. This makes them an asset that can be traded in the market and buyers and sellers can use different trading strategies (i.e. trend following or value trading) which are common in financial markets [Vogstad, 2003].

The riskiness of supply strategy in the market for certificates depends strongly on the microstructure of the market and the learning behaviour of the sellers. As a market designer, the aim is to motivate the investors or support the existing producers by minimizing the risk through reducing price fluctuations. Enabling banking is among the tools that can be considered for reducing the price fluctuation in the market.

The dynamics of Tradable Certificate prices, production, and inventory follows the same ones of commodity spot and future markets discussed by many scholars.[Pindyck, 2001; Kemp, 1963; Chambers and Bailey, 1996; Scheinkman and Schechtman, 1983; Sarris, 1984; Kawai, 1983]

Tradable Green Certificate is similar to any storable commodity whose provision is uncertain and agents who hold them speculate about their future prices. In the market for a storable good, demand at any date depends not only on the current price but also on buyers' expectations about future prices.

Scheinkman and Schechtman (1983) propose a simple model of single storable commodity with non-random demand but stochastic production. They proved the existence of a unique and stable rational expectation equilibrium for risk neutral producers who produce and store the good based on forecasts of future price distributions.

Stokey (1981) shows that if the seller makes a binding commitment about his future sales then this strategy will maximize his profit. Moreover, it is shown that if the market opens at fixed intervals during the policy, this translates as a commitment that the total supply available to buyers will remain limited over the near future. As the period gets very long, it is as if the seller can commit about his entire future sales strategy. Another important result of Stokey is that if the length of the selling period increases, seller's profit approaches the maximum attainable.

Sarris (1984) presents a model to study a storable commodity with and without presence of a futures market and asses its impact on variability of cash prices. His results indicate that a future market tends to reduce the period to period variance of price if the supply-of-storage curve becomes more elastic.

Kawai (1983) studies the effect of commodity futures market in a stochastic rational expectation framework on the price formation process. He concluded that futures market, in the short run, will not affect price fluctuation.

In the following, we find the Rational Expectation Equilibrium (REE) in a competitive market defined as a pair of functions, one describing how buyers' expectations are formed and one describing the sellers' sales strategy. The inventory or stock of certificates in the hands of buyers is viewed as the state variable of the system. At each date, buyers form expectations about the total stock of the good that will be sold at each date in a finite future. These expectations are conditioned on the current stock at each date. It is assumed that buyers know the inverse demand function for the certificates, so that their expectations about the path of the stocks determine their expectations about the path of price of certificates over the finite future.
The sellers are assumed to know the function describing how buyers' expectations are formed. Given this function, they choose a sale strategy that maximizes the present discounted value of their future profit stream. If this selling strategy fulfils buyers' expectations, the result is an REE.

In Section 3.2, we assume that the shocks are independently and identically distributed across different technologies and time periods. The producers decide how much to sell or store based on their forecasts of future prices to maximize their discounted profits. In order to derive closed-form solution for the market-clearing prices, we assume that agents form rational expectations with perfect foresight about the future flow of certificates. The equilibrium prices are a function of certain state variables such as the total stock and inventory left from the previous period and the quota set by the government. We show that under our assumptions, the stochastic process of equilibrium variables, e.g. stocks, prices, and sale, converges to some stationary distributions. In Section 3.3, for the numerical part, instead we assumed a central planner and a common shock for all technologies with an iso-elastic demand function. Section 3.4, presents the final remarks and concludes.

3.2 The Basic Model

As a general overview of the model, consider risk neutral producers who make decisions in period t about the supply of GC for period *t+1*. At the time of supply decision, the period *t+1* sales price p^{t+1} is unknown, but once the decision is made the quantity to be supplied s^{t+1} is certain. Since the Green Certificates is assumed to be storable, there is an inventory holding. The price taking sellers are assumed to maximize their expected profits.

A buyer is an agent who makes a demand decision at time when he faces actual market spot price without being exposed to market uncertainty. The price taking buyer minimizes her expenditure subject to the quota obligation and finds his demand for Green Certificates at time t, where demand function is assumed to be linear.

The source of uncertainty for the producers is regarding the shocks in the certificates flow due to weather which affect the production of electricity from RES and consequently, the number of certificates producers receive. We deal with idiosyncratic shocks when agents use different technologies in the market. Thus in addition, we assume that there are no private insurance markets against bad weather. Producers and buyers smooth their respectively profit and expenditure during the policy by storing the certificates. An agent accumulates certificates in the *bad states*, when today's price is lower than the discounted expected future prices, and sells in the *good states*, when today's price is higher than the discounted expected future prices.

3.2.1 Sellers' Strategy

Suppose there are $\mathcal{J}$ different technologies, which are entitled to receive certificates and there are $\mathcal{I}$ risk-neutral investors who have invested in RES, and they are producing electricity from these technologies at times $\{0, 1, \cdots, T-1\}$. The marginal cost of producing electricity from RES is generally close to zero[4], and there is only a high sunk cost of capacity installment in the beginning. Therefore, it is profitable for firms to produce electricity as long as the extra revenue from selling the additional unit, its price, exceeds the marginal cost of producing that unit, which is zero.

As it will be described in the following, there is uncertainty over the future profit from the investment since production of electricity is subject to fluctuation due to change in weather. The producers have two sources of income from: a) selling the electricity in the electricity spot market; and b) selling the Green Certificates in the TGC market. It is optimal for producers to sell all their produced electricity in the spot electricity market since their marginal cost of production is zero. Our focus is on maximizing the profit of producers in the TGC market. Moreover, we assume that producers are price takers and at time t they face the inverse demand function $p_c^t(S^t) = \alpha - k_1 S^t$ where $S^t = \sum_{i=1}^{\mathcal{I}}(s_i^t)$ with α and k_1 being positive, p_c^t being the price of GC, S^t total supply of GC in the market, and s_i^t supply of producer i each at time $t \in \{0, 1, \cdots, T-1\}$.

The potential investor or the existing producer would like to maximize his profit by selling the certificates during the period of policy

$$\max_{s_{ij}^t} \{E^0 \sum_{t=0}^{T} \beta^t \pi_{ij}^t(s_{ij}^t)\}, \tag{3.1}$$

where E^0 is the mathematical expectation operator conditional on the available information at time 0, $\beta \in (0, 1)$ is the common discount factor for all agents, and π_{ij}^t

[4]The exceptions may be those technologies that use bio-fuel and biomass as inputs.

is the profit of seller i who uses technology j at time t.

We assume that producers receive certificates following an auto-regressive process. Hence, producers receive on average a given number of certificates proportional to their capacity, but also the number of certificates that they receive in the year $t+1$ is related to what they received in the previous year and subject to a shock in weather. Therefore, we have

$$y_{ij}^{t+1} = \rho y_{ij}^{t} + (1-\rho)\overline{y}_{ij} + \epsilon_{ij}^{t+1}, \quad |\rho| < 1, \quad \epsilon_{ij} \sim (0, \sigma_{ij}^2) \tag{3.2}$$

where $\overline{y}_{ij}$ is the average number of certificates they receive in each year as mentioned before, and variance of $V(y_{ij}) = \frac{\sigma_{ij}^2}{1-\rho^2}$, ρ is the persistence parameter, ϵ_{ij} are the independently and identically distributed random shocks with $Cov(\epsilon_{ij}^t, \epsilon_{il}^t) = 0$, $j \neq l$ for all t, and $Cov(\epsilon_{ij}^t, \epsilon_{ij}^r) = 0$, $t \neq r$. For brevity, we might drop the indexes whenever it is not confusing.

These certificates can be accumulated and sold during but before the termination of the policy. Let the *stock of certificates* for individual i evolves according to

$$c_{ij}^{t+1} = c_{ij}^{t} + y_{ij}^{t} - s_{ij}^{t}, \tag{3.3}$$

where c_{ij}^{t+1} and c_{ij}^{t} are the stock of certificates in the next and current period. The marginal cost of banking certificates is assumed to be zero since they are not entitled to the usual inventory costs of physical commodities. This allows the sellers to bank their certificates as long as the plan lasts and at no extra costs. Given that sellers are allowed to bank their certificates, speculations become relevant, so producers develop rational expectations over prices. Although we allow for the individual sellers to go short or borrow certificates, this is not possible at the aggregate level since the market as a whole cannot hold a negative stock at any point of time. A negative stock at any time means that the market can borrow from the future and that is not allowed.

However, at the individual level, a seller can go short by borrowing from another. We can write the Bellman optimality equation:

$$V(c_i^t) = \sup_{s_i^t}\{\pi(s_i^t, c_i^t) + \beta E^t[V(c_i^{t+1})]\}, \tag{3.4}$$

where E^t denotes the expected value at time t given the information available by that time. This means that our system is a Markov decision process. We maximize the equation above subject to:

$$\begin{aligned} & c_{ij}^{t+1} = c_{ij}^t + y_{ij}^t - s_{ij}^t \\ & \textstyle\sum_i^I c_{ij}^t \geq 0, \\ & c_{ij}^0 = 0, \quad c_{ij}^T = 0. \end{aligned} \tag{3.5}$$

The following Euler equations must hold:

$$\begin{cases} s_i^t > 0, & \text{if} \quad p_c^t > \beta E^t(p_c^{t+1}), \\ s_i^t = 0, & \text{if} \quad p_c^t \leq \beta E^t(p_c^{t+1}). \end{cases} \tag{3.6}$$

We can solve (3.1) given the constraints in (3.5) backwardly. In the last period of the policy, it is optimal to sell everything. Subsequently, recursion on the Euler equation implicitly defines a rule for selling certificates. Unfortunately, under other types of demand function such as iso-elastic demands, the selling rules do not have an analytical solution, so they must be approximated by numerical methods.

To derive the results analytically, we consider the perfect certainty version of the model described above, which implies that the shocks are known in advance, and they are always equal to their expected values. Results in most cases would be very similar for versions of the model with quadratic profit and normal uncertainty (zero mean and finite variance of ϵ_{ij}^t). The optimal policy for selling certificates for agent

i is (see Appendix A)

$$s_i^t = \max\{\Phi\left(c_i^t + y_i^t(\frac{1-\rho^{T-t+1}}{1-\rho}) + \overline{y}_{ije}(1-\rho^{T-t})\right) + \Upsilon P_c^t, 0\}, \tag{3.7}$$

where Φ and Υ are given by,

$$\Phi = \frac{2\beta^{T-t}(1-\beta)}{2(1-\beta^{T-t+1}) - (1-\beta)^2 \sum_{t=0}^{T-t}(t+1)\beta^t};$$

$$\Upsilon = \frac{\sum_{t=0}^{T-t}(t+1)\beta^t(1-\beta)^2}{k_1\mathcal{I}\left(2(1-\beta^{T-t+1}) - (1-\beta)^2 \sum_{t=0}^{T-t}(t+1)\beta^t\right)}.$$

This is the *Competitive Rational Strategy* because agents try to maximize their life time profit as price taking agents. Equation (3.7) shows that the selling is divided into two parts: A part depends positively on the market price, and the other part proportional to his stockpile, capacity, and current certificates' flow. Equation (3.7) shows the optimal amount of selling for an individual agent when uncertainty regarding certificates' flow is resolved by assuming that the individual seller receives a guaranteed number of certificates equal to the expectation of the stochastic process (the certainty equivalent property) shown in (3.2). At time t, the higher the number of accumulated certificates c^t, the more an individual seller sells in the market. The reason is that in the previous periods, agents have banked their certificates because they had more states where the discounted future price was lower than current price (bad states). Nevertheless, as the time has passed to t, some of the uncertainty has been resolved[5] and they expect that in the future they will face states where the current price will be more than the discounted expected future price (good states). The opposite reasoning applies when they have banked few certificates up to time t. Some sellers may sell at minimum prices but as the price increases, the seller is

[5] At time 0 with $T-1$ periods to go you have more uncertainty regarding future flows of certificates compared to time $T-2$ when you have only one period ahead of you with uncertainty.

willing to sell more certificates in the market. If the sellers have more certificates in their inventory or they have plants with higher capacities, they will sell more certificates in the market.

3.2.2 Buyers' Strategy

The demand in our market is driven by the quota set on final consumers of electricity (households), who buy the GC as a fixed percentage of their electricity consumption. The quota is exogenously set by the regulator. Allowing for the market to be cleared more than once a year or allowing for the demand to roll to other years will change the slope and elasticity of demand.

The buyers, whether be households or utility companies, in the TGC market are obliged to buy a given number of certificates every τ ($\tau < T$) years proportional to their consumption of electricity, hence each of them has a quota which must be satisfied by period τ. They would like to minimize their expenditure during this time subject to the constraint of buying the quota. Suppose there are $\mathcal{B}$ buyers in the market with β as their discount factor. The buyers have rational expectation over future prices so if $\beta E^t(p_c^{t+1}) \geq p_c^t$ they will buy more today, $d_b^t > 0$, and bank it in their inventory. Similarly if $\beta E^t(p_c^{t+1}) < p_c^t$ they will postpone their purchase to $t+1$. At time τ they have to pay a penalty p_p for the difference between their quota obligation and certificate that they have accumulated in their inventory up to τ .

The instantaneous expenditure function of an individual buyer b is:

$$\xi_b^t(d_b^t, k_b^t) = (p_p - p_c^t)d_b^t, \tag{3.8}$$

subject to

$$k_b^{t+1} = k_b^t + d_b^t, \quad k_b^0 = 0, \quad k_b^\tau = \overline{QO}_b, \tag{3.9}$$

where d_b^t is the number of certificates that an individual buyer buys at time t, k_b^t is the inventory of certificates, k_b^0 is the inventory at the beginning of the policy, and at time τ she has to hand in $\overline{QO}_b$ certificates to comply with her obligation. The buyers would like to minimize their expenditure from buying the certificates during the whole period of policy, so they want to

$$\min_{d_b^t} \{E^0 \sum_{t=0}^{\tau} \beta^t \xi(k_b^t, d_b^t)\}.$$

Bellman optimality equation yields:

$$V(k^t) = \inf_{d_b^t} \{\xi(d_b^t, k_b^t) + \beta E_t[V(k_b^{t+1})]\}, \tag{3.10}$$

The optimal policy for buying certificates for an agent is (see Appendix B)

$$d_b^t = \max\{\Delta\left(\overline{QO}_b - k_b^t\right) - \Lambda p_c^t, 0\}, \tag{3.11}$$

where Δ and Λ are given by

$$\Delta = \frac{2(1-\beta)\beta^{\tau-t}}{2(1-\beta^{\tau-t+1}) - (1-\beta)^2 \sum_{t=0}^{\tau-t-1}(1+t)\beta^t};$$

$$\Lambda = \frac{(1-\beta)^2 \sum_{t=0}^{\tau-t-1}(1+t)\beta^t}{k_1 B\left(2(1-\beta^{\tau-t+1}) - (1-\beta)^2 \sum_{t=0}^{\tau-t-1}(1+t)\beta^t\right)}.$$

Equation (3.11) shows that as the inventory k_b of an individual buyer increase, she tends to buy less until she matches the quota obligation $\overline{QO}_b$. If that is the case, she does not need to buy any certificate. This is shown by the maximum between zero and the first term in the braces. The buying strategy is negatively related to the prices so when prices increase, buyers prefer to buy fewer certificates at time t and wait for future to fulfil their obligations.

3.2.3 Market Clearing Price

Scheinkman and Schechtman (1983) proved the existence of equilibrium when the objective functions are bounded and the shocks are i.i.d. From equation (3.11) and (3.7), we can derive the market clearing prices $\{p_t^c\}_{t=1}^T$

$$p_c^t = \max\left\{\frac{1}{\Lambda+\Upsilon}\left(\Delta(\overline{QO}-K^t)-\Phi\left(C^t+Y^t(\frac{1-\rho^{T-t+1}}{1-\rho})+(1-\rho^{T-t})\sum_j(\overline{Y}_j)\right)\right),0\right\}, \tag{3.12}$$

where $\overline{QO}$ is the total obligation of the buyers in the market, K^t is the stock of certificates bought by buyers up to time t, C^t the stock of certificates held by sellers, Y^t the total flow of certificates at time t,and $\overline{Y}_j$ is the capacity of all plants in the market using technology j.

As the capacity of plants increases during the policy, and in order to minimize price oscillation, the regulator can increase quota coefficient proportional to the newly installed capacity.

Any increase in availability, i.e. increase in stocks and current supply, decreases the expected price for next periods. The equilibrium prices also shed lights on the rational expectation formation when p^t and p^{t+1} are substituted in the inter-temporal arbitrage condition. A higher sell at time t is translated into a lower stockpile for time $t+1$ and as a result agents expect a higher price for the certificates at time $t+1$. In contrary, a higher expected future price, spur sellers, as shown in equation (3.6), to bank more certificates now and wait to sell more for future. These two contrary effects lead the market into the equilibrium prices.

The rational expectation TGC market endogenizes future prices by connecting their formation to the optimizing behaviour of commodity stockholders, producers,[6] and consumers. Furthermore, it enables producers to revise their price expectations given exogenous shocks in the market. With linear inverse demand function, the prices can

[6]In our model stockholders, speculators and producers are the same but one can assume speculators and producers as two different types of agents in the market.

become negative. This can happen when all agents meet their obligations meaning that $\overline{QO} - K^t$ goes to zero, price can become negative. In such case, the market-clearing price is the maximum between the first term and zero in equation (3.12). The availability in the market which consists of the stockpile of certificates C^t, average number of certificates per period $\overline{Y}$, and current flow of certificates Y^t never inclines toward zero. With a low $\overline{QO}$ set by the regulator, it is very likely that the market-clearing price hits zero. Therefore, for the success of such market, it is necessary that the regulator sets a high enough quota which prevents the market prices from collapsing. When the regulator knows the number of buyers, sellers, and the capacity of plants in the market, equation (3.12) can provide a reliable estimate of how high should be the overall quota in the market to guarantee an acceptable price for the producers of electricity to continue producing, or potential investors to invest in RES.

In the next Section, we solve another model by relaxing two of our simplifying assumptions regarding certainty equivalent and linear demand functions for the following reasons: a) although CE helps us to easily derive analytical solution, it is a very strong presumption on the information measure of agents who make buying or selling decisions under uncertainty regarding certificates' flow; b) the linear demand function has the advantage of simplicity, but it permits zero or negative prices for large supply values. Therefore, it is more appropriate for us to consider an alternative demand function such as iso-elastic demand. However, we are not able to solve the model analytically with an iso-elastic demand function, and we must turn to numerical methods in order to approximate the value function.

3.3 The Numerical Solution

Although the linear rational expectation model has been very useful in deriving closed form solutions for theoretical studies of markets for commodities, it is incapable of reasonably capturing the storage process that is essential to dynamic characteristics of such markets. Some flaws of the linear inter-temporal arbitrage equation are as following: a) it fails to explain the vast disparity in the spot-future commodity price spread; b) stock levels can become negative whenever there is a short supply; c) it fails to capture the unobservables in the Euler equations[7], and d) it struggles when models have inequality constraints.[see Miranda and Glauber (1993); Miranda (1998)]
Therefore, we need to turn to numerical methods to be able to evaluate different policy rules and learn about the importance of given assumptions.
In this Section, we assume that the inverse demand function is iso-elastic with $p_c^t = \alpha/(S^t)^{k_1}$ with $k_1 < 0$. It should be mentioned that in the case of iso-elastic demand, the non negativity of prices is always guaranteed. So, in contrast to the previous Section with linear inverse demand function, we do not need to ensure this property by selecting the values of the model parameter carefully. Iso-elastic demand has another desirable property regarding the behaviour of consumers. This demand function result when consumers always spend a constant budget share on a given commodity, which provides for the reciprocity of price and quantity. As further all consumers have demand function of the same shape, solving for the aggregate problem is easy and market demand function of the sane shape results. However, this demand function is not desirable for dealing with monopoly[8] because the monopolist can keep its revenue constant by choosing the quantity as low as possible, which results in very high price due to the reciprocity between quantity and price in the demand function.

[7] Note that the exogenous shock at time $t+1$ is not yet observed at time t or before.

[8] Agliari and Puu (2002) propose the modified iso-elastic inverse demand: $p = \frac{\alpha}{(S)^{k_1}+w}$ with a positive small w such that the maximum price will be α/w when quantity tends to zero.

The instantaneous profit function of firms is written as

$$\pi^t = \left(\frac{\alpha}{(S^t)^{k_1-1}}\right). \tag{3.13}$$

We denote all aggregate variables with capital letters. The objective function is now written as

$$\max_{S^t}\left\{E^0\sum_{t=0}^{T}\beta^t(\frac{\alpha}{(S^t)^{k_1-1}})\right\}, \tag{3.14}$$

subject to our set of constraints

$$\begin{aligned}
&C^{t+1} = C^t + Y^t - S^t,\\
&Y^{t+1} = \rho Y^t + (1-\rho)\overline{Y}^t + \epsilon^t, \quad |\rho| < 1, \epsilon \sim (0, \sigma^2)\\
&C^t > 0, \quad C^0 = 0, \quad C^T = 0.
\end{aligned} \tag{3.15}$$

Define the maximized value of (3.14) V^t as

$$V(C^t) = \sup_{S^t}\{\pi(S^t, C^t) + \beta E^t[V(C^{t+1})|C^t, S^t]\}, \tag{3.16}$$

where E^t represents the conditional expectation under distribution of shocks. Our objective function is not concave since

$$\frac{\partial^2\pi^t}{\partial(S^t)^2} = -k_1\alpha(-k_1+1)(S^t)^{-k_1-1} > 0. \tag{3.17}$$

We cannot apply the Certainty Equivalence property to solve the dynamic programming problem because our objective function as shown in (3.17) is not concave (Zeldes, 1989). However, the problem can be solved numerically and from the point of view of a benevolent central planner[9] who maximizes everyone's welfare. Since the equilibrium price function $\Xi(S)$ is not known a priori and deriving it is a non trivial functional equation problem. $\Xi(S)$ must simultaneously satisfies an infinite number

[9]Stokey (1989) shows that the competitive rational expectation equilibrium is Pareto optimal.

of conditions, meaning that for every feasible and realizable supply .

We are interested in deriving the optimal level of stock c^{t+1} for time $t+1$ given the level of stock c^t at time t subject to a realization of shock in certificates' flow. Hence, we iterate on the value function (3.4) starting from the last period T after which the policy is terminated[10] and firms are no more entitled to certificates. All certificates must be sold before this date else they vanish. The program is written in such a way that it searches for each state, time, and realization of possible shocks and chooses the one that maximizes the sum of the current profit and the discounted expected value of next period's value function as shown in Bellman equation (3.4). The state space is discretized into an $\mathcal{S}$ elements grid using a technique proposed by Bertsekas (1976). Note that the aggregate stock of the market cannot be negative (as mentioned before) since the market as a whole cannot borrow from future. This assumption puts a constraint on the feasible set of states. In the current setting, firms can bank their certificates up to the end of policy. This gives them the highest flexibility in adjusting their stockpile of certificates based on their expectations about future prices.

All certificates must be sold during this period of time. The technique to discretize the exogenous shocks to flows of certificates is described in more detail in Tauchen and Hussey (1991) who provide a simple way to discretize VAR (Vector Auto Regressive) processes relying on Gaussian quadrature. We assumed five states of the world for flows which accounts for 95% of the Gaussian distribution. These states include the mean and two standard deviations around the mean meaning $\{-2\sigma, -1\sigma, \mu, 1\sigma, 2\sigma\}$, where $\mu = \overline{Y}^t$. The transition probability from one possible state to another is also calculated through the method suggested by Tauchen and

[10]In case of an infinite market, the Contraction Mapping Theorem guarantees the existence of a solution under mild regularity conditions.

Hussey (1991).

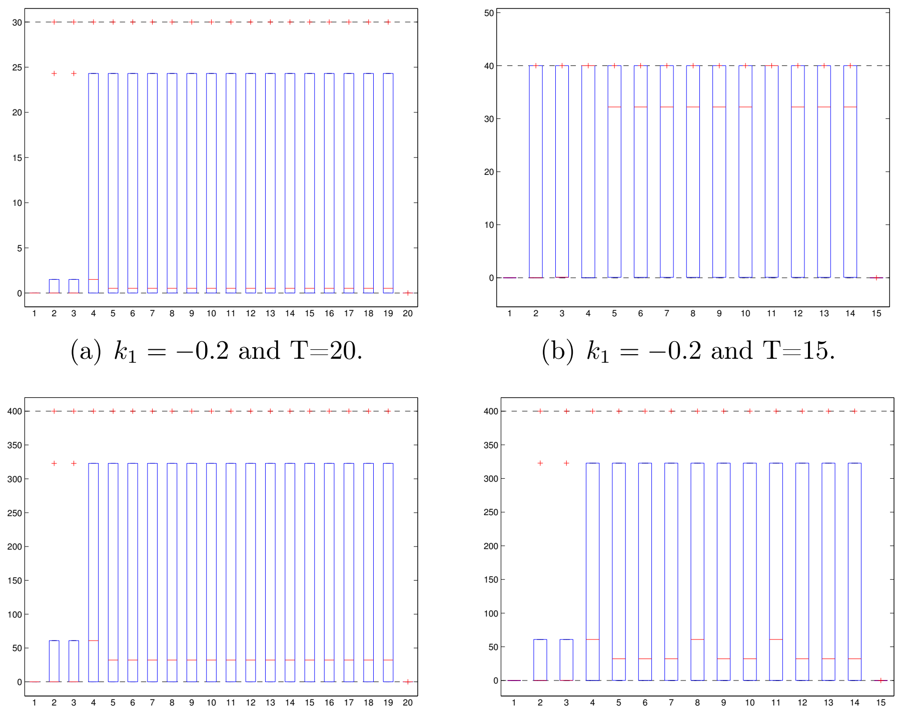
After deriving the decision rule, we simulate the life time history of policy for 10000 times starting from an initial level of stocks. For this purpose, we use a T-years Markov chain in each iteration which governs the realizations of shocks and the transition from one state to another.
The parameters of the numerical problem are summarized in Table 3.1. We choose α such that the demand function passes through $S = 5$ and $p_c = 5$.

Table 3.1: Parameters of the life time simulation of TGC Market

Demand Elasticity ($1/k_1$)	-0.2(-0.33)	Mean certificates' flow ($\overline{Y}$)	5
Time of policy (T)	15(20)	Discount factor (β)	0.95
Demand parameter (α)	625	Persistence coefficient (ρ)	0.8
Standard deviation of shock (σ_ϵ)	1	Mean of shocks	0

Other parameters in Table 3.1 such as $\overline{Y}$, σ_ϵ, or time of policy T are chosen based on subjective judgement. The discount factor β is chosen for an interest rate of 5% which is common in the literature. The numbers in the parenthesis in Table 3.1 for demand elasticity and policy's time show the altered parameters for which we simulated the model. The demand elasticity depends on the regulator's rules on the demand side of the market, specifically the buyers' target date of submitting the certificates τ. Ceteris paribus, the longer the τ, the more elastic is the demand function since the buyers have more flexibility in acquiring their certificates from the market, and the lower will be the price during the policy. On the supply side, the time of policy affects sellers strategy in the market. The intuition is that the shorter the policy time; the more certificates will be sold in the market, and the lower will be the price on average.

Figure 3.1 shows the price of GC in the market during the policy along its inter quantile range resulted from 10000 rounds of simulation. Subfigures in Figure 3.1



(a) $k_1 = -0.2$ and T=20. (b) $k_1 = -0.2$ and T=15.

(c) $k_1 = -0.33$ and T=20. (d) $k_1 = -0.33$ and T=15

Figure 3.1: Certificates Price.

show the price and its variation in simulations for altered values of time and demand elasticity in the model. The average price during the policy is invariant although the flow of certificates is subject to variations and therefore, the income of producers who have invested in RES is stable. The variation shown by the box plots takes into account that in our simulation, we have had a very wide fluctuation in the flow of certificates due to the randomly selected Markov chain in each simulation. Nevertheless, despite this variation, the resulted average price during the policy shows a lower variance compared to certificates' flow. The reason is simply because the agents in the market (here sellers) behave as if they have rational expectations about future prices. When the price increases, they tend to sell more and store fewer certificates. The inter-temporal storage arbitrage equation can smooth the effects of temporary surfeits or shortages.

At the beginning of the policy, the firms have more uncertainty about the life time profit, but this is approximately offseted by the fact that they also have more periods left to spread out any unexpected change in the current flow of certificates. Increasing the time of policy does not diminish the effect of certificates' uncertainty on the level of supply.

Figure 3.2 shows the empirical cumulative distribution of the GC flow, which is the source of randomness in the producers' income and market price. Comparing Subfigures in Figures 3.2 and 3.3 illustrates that sellers by following the optimal selling strategy can smooth the price of the GC in the market and help to stabilize their income. In other words, the Subfigues in Figure 3.3 have second-order stochastic dominance over the counterparts in Figure 3.2.

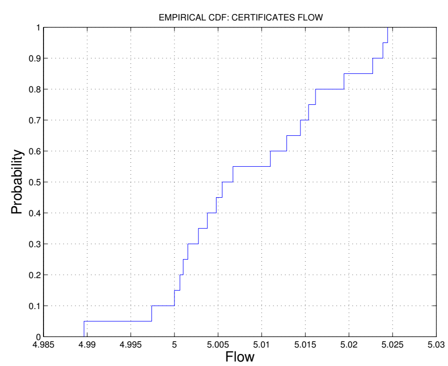
Figure 3.3 establishes two points: a) an increase in the policy time, raises the probability of higher prices b) an increase in the demand elasticity, in absolute values, decreases the probability of higher prices. For instance, comparing sub-figures a) and b) in Figure 3.3 shows that $\Pr(p_c < 35)$ in the first case is 0.45, but in the second case, it increases to almost 0.65 when the time of policy is decreased by five years. This is counter-intuitive since we were expecting that shortening the time pushes the sellers to oversell, and prices will be lower on average during the policy. The reason is due to the fact that having a longer time in policy enables the sellers to smooth their selling more and therefore, the probability of having a high price in the market is lower compared to a shorter policy. Instead, in a shorter policy, agents are less flexible in smoothing their selling when they are hit by shocks and the probability of having a high price in the market is greater.

The same argument applies for an increase in the demand elasticity. While with elasticity 0.33 the $\Pr(p_c < 10)$ is one, the same probability for elasticity 0.2 is 0.2. This result is self-evident because lower elasticity means that the certificates are more important in the eyes of consumers, and they are willing to pay a higher price for them when the time τ to comply is short.

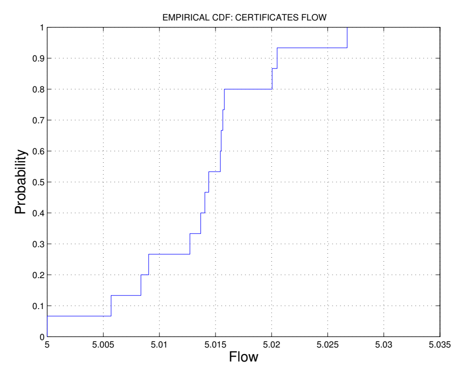
Table 3.2 summarizes the statistics of the simulations when policy time and demand elasticity of the model is varied.

Table 3.2: Mean and standard deviation of price.

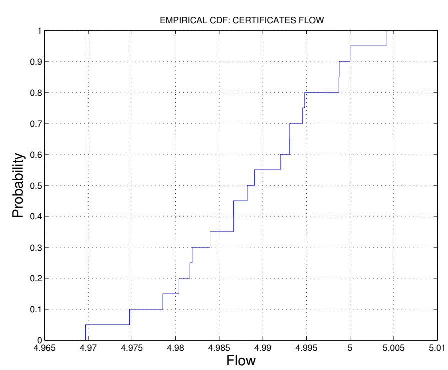
Parameters	$k_1 = -0.2$ T=20	$k_1 = -0.2$ T=15	$k_1 = -0.33$ T=20	$k_1 = -0.33$ T=15
Mean of Price	29.39	26.01	180.66	163.87
Standard Deviation of Price	12.35	12.99	70.39	76.12



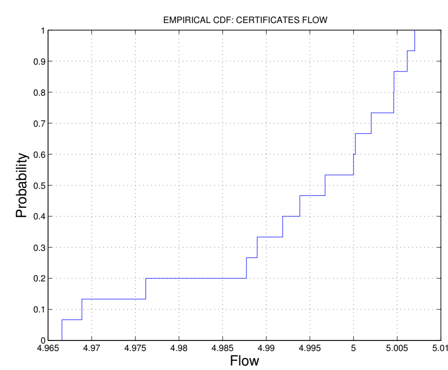
(a) $k_1 = -0.2$ and T=20.



(b) $k_1 = -0.2$ and T=15.

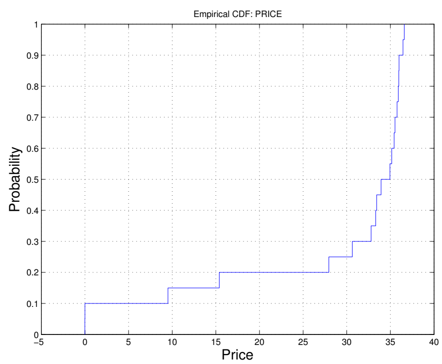


(c) $k_1 = -0.33$ and T=20.

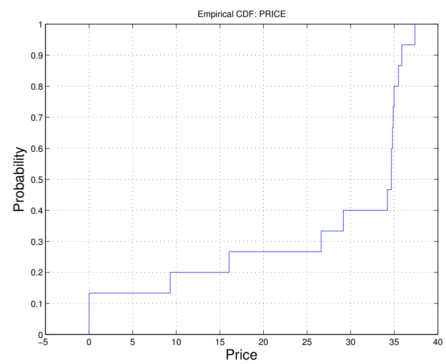


(d) $k_1 = -0.33$ and T=15.

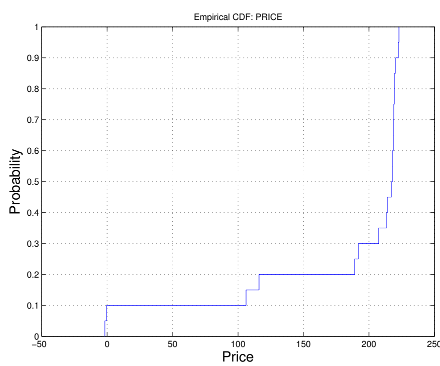
Figure 3.2: Empirical CDF of Certificates Flow.



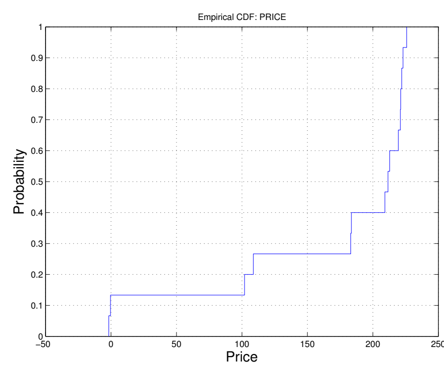
(a) $k_1 = -0.2$ and T=20.



(b) $k_1 = -0.2$ and T=15.



(c) $k_1 = -0.33$ and T=20.



(d) $k_1 = -0.33$ and T=15.

Figure 3.3: Empirical CDF of Certificates Price.

3.4 Conclusions

We derived the rational expectation partial equilibrium in the tradable Green Certificate market. The GC is issued by the regulator to provide investors with another source of income to offset any price variation in the electricity spot market due to uncertainty in weather and hence production of electricity. In addition, we obtained an optimal strategy for selling and buying of certificates in the market which maximizes the life-time profit of sellers. The possibility of banking the certificates by the sellers makes the prices less variable compared to the flow of certificates in the market. Since the demand is set by the government, the extent of price depends on the quota level. As the equilibrium prices may collapse due to surplus, the government should set an overall quota which is higher than the total number of issued certificates. An approximation for quota level can be derived from equation (3.12). Any expectations regarding an increment in the quota will be translated into higher prices in the next periods. The success of such a market depends on the stability of prices, which is the repercussion of the decisions of the regulator regarding the timing of the policy on sellers and buyers' side at the beginning of policy. In the numerical part, we only studied two of such parameters namely the time of policy and the demand elasticity. We concluded that setting a shorter time for submitting the quota which is translated into a higher elasticity, results in higher average prices in the market that is more attractive for producers or investors. Even so, shortening the time of the policy, increases the variation in prices (the risk for producers) which is less desirable.

3.5 Appendixes

3.5.1 Appendix A

Competitive Sellers

$$\dot{\pi}_(s^t) = \beta E^t \dot{\pi}(s^{t+1}) \qquad \text{(Euler Equation)}$$

We substitute the derivative of the profit function inside the Euler equation and obtain

$$\begin{aligned}
\dot{\pi}(s^t) &= \beta E^t \dot{\pi}(s^{t+1}) \\
\alpha - 2k_1 I s_i^t &= \beta E^t(\alpha - 2k_1 I s_i^{t+1}) \\
\alpha(1-\beta) - 2k_1 I S^t &= -2k_1 I \beta E^t(s^{t+1}) \\
s^t &= \tfrac{(1-\beta)(\alpha)}{2k_1 I} + \beta E^t(s^{t+1})
\end{aligned} \tag{3.18}$$

At time T the stock of certificates must be zero as the policy is ended, and certificates will vanish so $c^T = 0$ therefore from (3.3), we have $s^{T-1} = c^{T-1} + y^{T-1}$ (since we are calculating optimal policy for an individual seller and a specific technology, we simply write c^t rather than c_{ij}^t unless the distinction is necessary)

$$\begin{aligned}
s^{T-2} &= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-2}(s^{T-1}) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-2}(c^{T-1} + y^{T-1}) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-2}(c^{T-2} + y^{T-2} - s^{T-2} + y^{T-1}) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I} . \tfrac{1}{1+\beta} + \tfrac{\beta}{1+\beta}(c^{T-2} + y^{T-2}(1+\rho) + \overline{Y})
\end{aligned} \tag{3.19}$$

Given the optimal selling calculated above, for time $T-3$ we will have:

$$\begin{aligned}
s^{T-3} &= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-3}(s^{T-2}) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-2}\left(\tfrac{\alpha(1-\beta)}{2k_1 I} . \tfrac{1}{1+\beta} + \tfrac{\beta}{1+\beta}(c^{T-2} + y^{T-2}(1+\rho))\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(1 + \tfrac{\beta}{1+\beta}\right) + \tfrac{\beta^2}{1+\beta} E^{T-3}\left(c^{T-3} + y^{T-3} - s^{T-3} + y^{T-2}(1+\rho)\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(\tfrac{1+2\beta}{1+\beta}\right)\left(\tfrac{1+\beta}{1+\beta+\beta^2}\right) + \tfrac{\beta^2}{1+\beta+\beta^2} E^{T-3}\left(c^{T-3} + y^{T-3} + y^{T-2}(1+\rho)\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(\tfrac{1+2\beta}{1+\beta+\beta^2}\right) + \tfrac{\beta^2}{1+\beta+\beta^2}\left(c^{T-3} + y^{T-3}(1+\rho+\rho^2) + \overline{Y}(1+\rho)\right)
\end{aligned}$$

And for $T-4$ we will have:

$$\begin{aligned}
s^{T-4} &= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-4}(s^{T-3}) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I} + \beta E^{T-4}\left(\tfrac{\alpha(1-\beta)}{2kI}\left(\tfrac{1+2\beta}{1+\beta+\beta^2}\right) + \tfrac{\beta^2}{1+\beta+\beta^2}\left(c^{T-3} + y^{T-3}(1+\rho+\rho^2)\right)\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(1 + \tfrac{\beta(1+2\beta)}{1+\beta+\beta^2}\right) + \tfrac{\beta^3}{1+\beta+\beta^2} E^{T-4}\left(c^{T-4} + y^{T-4} - s^{T-4} + y^{T-3}(1+\rho+\rho^2)\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(\tfrac{1+2\beta+3\beta^2}{1+\beta+\beta^2}\right)\left(\tfrac{1+\beta+\beta^2}{1+\beta+\beta^2+\beta^3}\right) + \tfrac{\beta^3}{1+\beta+\beta^2+\beta^3} E^{T-4}\left(c^{T-4} + y^{T-4} + y^{T-3}(1+\rho+\rho^2)\right) \\
&= \tfrac{\alpha(1-\beta)}{2k_1 I}\left(\tfrac{1+2\beta+3\beta^2}{1+\beta+\beta^2+\beta^3}\right) + \tfrac{\beta^3}{1+\beta+\beta^2+\beta^3}\left(c^{T-4} + y^{T-4}(1+\rho+\rho^2+\rho^3) + \overline{Y}(1+\rho+\rho^2)\right)
\end{aligned}$$

From the above calculations, we derive the optimal strategy:

$$s^{T-t} = \frac{\alpha(1-\beta)}{2k_1 I}.\frac{\sum_{z=0}^{t}((z+1)\beta^z)}{\sum_{z=0}^{t}\beta^z} + \frac{\beta^t}{\sum_{z=0}^{t}\beta^z}\left(c^{T-t} + y^{T-t}(\sum_{z=0}^{t}\rho^z) + \overline{Y}(\sum_{z=0}^{t-1}\rho^z)\right) \tag{3.20}$$

or

$$s^t = \frac{\alpha(1-\beta)}{2k_1 I}.\frac{\sum_{t=0}^{T-t}((t+1)\beta^t)}{\sum_{t=0}^{T-t}\beta^t} + \frac{\beta^{T-t}}{\sum_{t=0}^{T-t}\beta^t}\left(c^t + y^t(\sum_{t=0}^{T-t}\rho^t) + \overline{Y}(\sum_{t=0}^{T-t-1}\rho^t)\right)$$

or by substituting the geometric series inside:

$$s^t = \frac{\alpha(1-\beta)^2}{2k_1 I(1-\beta^{T-t+1})}.(\sum_{t=0}^{T-t}((t+1)\beta^t)) + \frac{\beta^{T-t}(1-\beta)}{1-\beta^{T-t+1}}\left(c^t + y^t(\frac{1-\rho^{T-t+1}}{1-\rho}) + \overline{Y}(\frac{1-\rho^{T-t}}{1-\rho})\right)$$

In order to obtain the supply function, we multiply this equation by the number of firms and substitute from the demand function,

$$S_c^t = \Phi I\left(c^t + y^t(\frac{1-\rho^{T-t+1}}{1-\rho}) + \overline{Y}(\frac{1-\rho^{T-t}}{1-\rho})\right) + \Upsilon P_c^t \tag{3.21}$$

where Φ and Υ are given by,

$$\Phi = \frac{2\beta^{T-t}(1-\beta)}{2(1-\beta^{T-t+1}) - \sum_{t=0}^{T-t}(t+1)\beta^t(1-\beta)^2}$$

$$\Upsilon = \frac{(1-\beta)^2\sum_{t=0}^{T-t}(t+1)\beta^t}{k_1\left(2(1-\beta^{T-t+1} - \sum_{t=0}^{T-t}(t+1)\beta^t(1-\beta)^2)\right)}$$

3.5.2 Appendix B

Competitive Buyers

The Euler equation states that

$$\dot{\xi}_(d^t) = \beta E^t \dot{\xi}(d^{t+1}). \quad \text{(Euler Equation)}$$

Sellers are acting in a competitive environment, and they are price takers, so buyers face an inverse supply function similar to what has been calculated before:

$$p_c = \alpha + k_1 D_c^t. \tag{3.22}$$

The utility firms or households would like to minimize their expenditure on green certificates, and they have to satisfy the quota given their consumption of electricity

$$\min_{d_b} \xi = (P_p - p_c^t) d_b^t, \tag{3.23}$$

subject to

$$k_b^0 = 0, k_b(\tau) = \overline{QO}_b.$$

Our law of motion for an individual buyer is

$$k_b^{t+1} = k_b^t + d_b^t. \tag{3.24}$$

The Euler equation is rewritten as:

$$d_b^t = \frac{(\beta - 1)(\alpha + P_p)}{2k_1 B} + \beta E^t (d_b^{t+1}). \tag{3.25}$$

In the last period, an individual buyer must meet his quota obligation $\overline{QO_b}$. As a result, from (3.24) we have $\overline{Q_b} = k_b^\tau + d_i^\tau$. Substituting this in (3.25) we obtain

$$\begin{aligned} d_b^{\tau-1} &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-1}(d_b^\tau) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-1}(\overline{QO}_b - k_b^\tau) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B(1+\beta)} + \tfrac{\beta}{1+\beta}(\overline{QO}_b - k_b^{\tau-1}). \end{aligned}$$

Given the optimal buying strategy at time $\tau - 1$ and the Euler equation, for time $\tau - 2$ we obtain

$$\begin{aligned} d_b^{\tau-2} &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-2}(d_b^{\tau-1}) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-2}\left(\tfrac{(\beta-1)(\alpha+P_p)}{2k_1B(1+\beta)} + \tfrac{\beta}{1+\beta}(\overline{QO}_b - k_b^{\tau-1})\right) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B}\tfrac{(1+2\beta)}{1+\beta+\beta^2} + \tfrac{(\beta^2)}{1+\beta+\beta^2}\left(\overline{QO}_b - k_b^{\tau-2}\right). \end{aligned}$$

For time $\tau - 3$, we follow the same procedure as the last step and we obtain

$$\begin{aligned} d_b^{\tau-3} &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-3}(d_b^{\tau-2}) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B} + \beta E^{\tau-3}\left(\tfrac{(\beta-1)(\alpha+P_p)}{2k_1B}\tfrac{(1+2\beta)}{1+\beta+\beta^2} + \tfrac{(\beta^2)}{1+\beta+\beta^2}(\overline{QO}_b - k_b^{\tau-2})\right) \\ &= \tfrac{(\beta-1)(\alpha+P_p)}{2k_1B}\tfrac{(1+2\beta+3\beta^2)}{1+\beta+\beta^2+\beta^3} + \tfrac{\beta^3}{1+\beta+\beta^2+\beta^3}(\overline{QO}_b - k_b^{\tau-3}) \end{aligned}$$

The optimal policy for an individual buyer b is

$$d_b^{\tau-t} = \frac{(\beta-1)(\alpha+P_p)}{2k_1B}\left(\frac{\sum_{z=0}^{t-1}(z+1)\beta^z}{\sum_{z=0}^{t}\beta^z}\right) + \frac{\beta^t}{\sum_{z=0}^{t}\beta^z}\left(\overline{QO}_b - k_b^{\tau-t}\right),$$

or

$$d_b^t = \frac{(\beta-1)(\alpha+P_p)}{2k_1B}\left(\frac{\sum_{t=0}^{T-t-1}(t+1)\beta^t}{\sum_{t=0}^{T-t}\beta^t}\right) + \frac{\beta^{\tau-t}}{\sum_{t=0}^{\tau-t}\beta^t}\left(\overline{QO}_b - k_b^t\right).$$

Substituting the geometric series inside

$$d_b^t = \frac{(\beta-1)(\alpha+P_p)}{2k_1B}\left(\frac{(1-\beta)\sum_{t=0}^{\tau-t-1}(t+1)\beta^t}{(1-\beta^{\tau-t+1})}\right) + \frac{(1-\beta)\beta^{\tau-t}}{(1-\beta^{\tau-t+1})}\left(\overline{QO}_b - k_b^t\right). \tag{3.26}$$

The demand of an individual buyer is

$$d_b^t = \Delta\left(\overline{QO}_b - k_b^t\right) - \Lambda p_c^t, \tag{3.27}$$

where Δ and Λ are given by

$$\Delta = \frac{2(1-\beta)\beta^{\tau-t}}{2(1-\beta^{\tau-t}) - (1-\beta)^2\sum_{t=0}^{\tau-t-1}(1+t)\beta^t};$$

$$\Lambda = \frac{(1-\beta)^2\sum_{t=0}^{\tau-t-1}(t+1)\beta^t}{k_1B\left(2(1-\beta^{\tau-t+1}) - (1-\beta)^2\sum_{t=0}^{\tau-t-1}(t+1)\beta^t\right)}.$$

The aggregate demand in the market will be:

$$D^t = \Delta' B\left(\overline{QO}_b - k_b^t\right) - \Lambda' p_c^t. \tag{3.28}$$

where Δ' and Λ' are given by

$$\Delta' = \frac{2(1-\beta)\beta^{\tau-t}}{2(1-\beta^{\tau-t+1}) - (1-\beta)^2\sum_{t=0}^{\tau-t-1}(1+t)\beta^t};$$

$$\Lambda' = \frac{(1-\beta)^2\sum_{t=0}^{\tau-t}(t+1)\beta^t}{k_1\left(2(1-\beta^{\tau-t+1}) - (1-\beta)^2\sum_{t=0}^{\tau-t-1}(t+1)\beta^t\right)}.$$

Market Clearing Price

We derive the market-clearing price by equations (3.21) and (3.28)

$$p_c^* = \frac{\mathcal{B}\Delta(\overline{QO}_b - k_b^t) - \mathcal{I}\Phi\left(c_i^t + y_i^t(\frac{1-\rho^{T-t+1}}{1-\rho}) + \overline{Y}(\frac{1-\rho^{T-t}}{1-\rho})\right)}{\Lambda + \Upsilon}. \quad (3.29)$$

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Estratto per riassunto della tesi di dottorato

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Titlo della tesi: Three Essays in Microeconomics

Abstract. In this dissertation, we study three issues in Microeconomics. The topics range from marketing to game theory and environmental economics. The first chapter presents a novel model for persuasive advertising. Two firms compete in a market where they optimally decide their advertising and pricing strategies. The second chapter studies the competition of duopolists when the customers are boundedly rational with respect to processing information. Finally, the third chapter considers a market as an incentive scheme for promoting investment in Renewable Energy Resources and analyses the price and factors that affects price stability.

Estratto. In questa tesi, analizziamo tre problemi relativi alla Microeconomia. Gli argomenti spaziano dal marketing, alla teoria dei giochi fino alla economia dell'ambiente. Il primo capitolo presenta un nuovo modello sulla persuasione nella pubblicità. Due imprese competono in un mercato in cui decidono razionalmente le loro strategie pubblicitarie e di prezzo. Nel secondo capitolo si guarda alla competizione di imprese in un duopolio nel caso in cui i clienti abbiano razionalità limitata rispetto alla capacità di processare informazione. Infine, nel terzo capitolo si considera un mercato come schema di incentivi per promuovere investimenti nelle Renewable Energy Resources (Energie e Risorse Rinnovabili) e si analizzano i fattori che impattano sulla stabilità dei prezzi ed i prezzi stessi.