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ORIGINAL PAPER

Creating a winner's curse via jump bids

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Abstract We show that *jump bids* can be used by a bidder to create a winner's curse and preserve an informational advantage that would otherwise disappear in the course of an open ascending auction. The effect of the winner's curse is to create allocative distortions and reduce the seller's expected revenue. Two novel features of equilibrium jump bids are derived. First, the jump bid may fail to hide completely the value of the common value component. Second, a bidder with a higher type might jump bid less frequently than a bidder with a lower type.

Keywords Auctions · Efficiency · Jump bids · Winner's curse

JEL Classification D44 · D82

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1 Introduction

Jump bidding refers to the practice of calling a price strictly higher than the current highest standing bid in an open ascending auction.[1] The use of jump bids is widespread both in auctions[2] and in other markets not explicitly regulated by auction rules such as corporate takeovers.[3] The existing work on the topic is typically based on signaling models in which a bidder places a costly jump bid to reveal that he has a favorable type.[4]

An alternative explanation to signaling has been introduced by Ettinger and Michelucci (2015). It is based on the somehow opposite motive that a jump bid can be used to limit the amount of information that can be aggregated by hiding the exact drop out prices of the bidders who do not match a jump bid. In that paper we considered a setting where the identity of a bidder's opponent holding the highest ex post valuation depended on the exact drop out prices. In that context, we showed that a jump bid may reduce the expected price paid by a bidder by pooling drop out prices for which the identity of the opponents with highest ex post value differs.

This paper enriches the hiding/manipulating information motives for jump bidding by looking at a setting for which the incentive to hide information comes from a different source: a winner's curse argument. We have in mind situations in which a subset of the bidders have better information about some common value elements of the object on sale (perhaps because they are insiders/incumbents) than others (entrants), and where this informational asymmetry might disappear or narrow because of the information that can be aggregated in the open ascending auction.

We show that the better informed bidder may call a price in order to prevent this information revelation process. The reason is that by preserving an informational advantage the informed bidder *forces* the less informed one to take into account of a potential winner's curse. This fact dampens the expected willingness to pay of the less informed bidder and may decrease the expected price paid by the better informed one.

Compared to our previous work, this paper offers the following contributions. First, it provides an alternative reason why manipulating information via a jump bid can be part of an equilibrium strategy.[5] Second, it adds to the literature two features of equilibrium jump bids. The first one is that the jump bid may fail to completely hide

[1] In the paper we use the term "jump bidding" and "calling a price" interchangeably.

[2] See, for instance, Cramton (1997), Plott and Salmon (2004), Börgers and Dustmann (2005), Mark et al. (2007) for FCC auctions; and Easley and Tenorio (2004), He and Popkowski Leszczyc (2013), and Grether et al. (2015) for online auctions.

[3] See Burkart and Panunzi (2008) for a review of takeovers in finance (there a jump bid determines the so called takeover premium).

[4] See for instance, Fishman (1988) and Avery (1998) for the seminal contributions, and Hörner and Sahuguet (2007), Bulow and Klemperer (2009), Roberts and Sweeting (2013) for more recent ones. We already summarized these papers in Ettinger and Michelucci (2015), so that reader can refer to our earlier work for a more comprehensive literature review.

[5] Note that also in signaling models, the rationale for signaling typically differs depending on the set-up analyzed. The same holds for "hiding" models.

the value of the common value component. The second one is that the probability to jump bid might decrease with the type of the bidder who places the jump bid.[6]

2 Auction rules

We compare two variants of the English auction: the *standard* clock auction format and the *dynamic* clock auction in which bidders are allowed to call a price strictly higher than 0 at the beginning of the auction (see Avery 1998).

In the *standard* clock auction, the price starts from zero, and it is increased at a constant pace by an exogenous device such as a clock. Bidders are considered active only if they are currently pressing a button. At any point in time, i.e., at any price $p \geq 0$ indicated by the clock at a specific instant of time, each active bidder may decide to leave the auction. The identity of the bidders who quit is publicly revealed so that a bidder knows exactly against whom he is competing at any time during the auction. The auction ends when the penultimate bidder quits. The last active bidder wins the object and pays the price at which the penultimate bidder exited. We use the following tie-breaking rule. If the k last active bidders (with $k \geq 2$) leave the auction at the same price, p, the good is sold at price p with a probability $1/k$ to each of the k last active bidders.

In the *dynamic* clock auction, there are two stages. In the first stage, bidders privately communicate to the auctioneer the jump bid that they want to place. The second stage works as a standard clock auction format. If no price has been called, the auction starts at price 0. If at least one strictly positive price has been called in the first stage, the auctioneer communicates the identity of the bidders who have called the highest price, p, and the remaining bidders independently communicate to the auctioneer whether they want to be active when the clock auction starts at price p.[7] Before the ascending auction starts from price p, the set of active bidders is revealed publicly by the auctioneer.

3 The setting

We consider the following framework with three bidders.[8] Bidders' valuations are:

- $v_1 = s$.
- $v_i = s + t_i, \quad i = 2, 3$.

Bidders' valuations depend on the value of s; s is privately observed by Bidder 1 and Bidder 2. Bidder 2 and Bidder 3's valuations are ex-ante symmetric. However, Bidder 3 has an informational disadvantage, he does not know the realization of s. He only knows that s is distributed according to a uniform distribution on the interval

[6] This can be seen as a counterpart of the non monotonicity of jump bids for signaling motives shown by Hörner and Sahuguet (2007) (although for rather different strategic reasons).

[7] Any bidder who has called a strictly positive price in the first stage commits to be active at that price at the start of the second stage.

[8] Three is the minimum number of bidders to have a jump bid for the motive we propose in this paper.

[0, 1]. The valuations above assume that Bidder 2 and Bidder 3 have extra motivations for buying the good, so that Bidder 1 never has the highest valuation for the good. This helps to clarify that the motive for concealing information is new. It implies that Bidder 2's real rival is always Bidder 3, which does not allow the use of the envelope argument provided in Ettinger and Michelucci (2015). For $i = 2, 3$, Bidder i receives private information t_i. t_2 and t_3 are independent of the value of s, and i.i.d.

We analyze a simple case where $t_i \in \{t_l, t_h\}$, with $1 \leq t_l < t_h$.[9] Extensions to the model are examined in Sect. 5. We consider a discrete rather than a continuous type space for the t_i's to simplify the exposition and computations.[10] We restrict attention to equilibria with non weakly dominated strategies and the solution concept is Perfect Bayesian Equilibrium.

4 The analysis

We assume that $1 \leq t_l < t_h < t_l + 1$, and each type is equally likely.[11] The equilibrium analysis of the game where jump bids are not allowed is standard. The equilibrium actions are presented below.

Proposition 1 *In any equilibrium of the clock auction without jump bids, Bidder 1 leaves the auction at a price equal to s; Bidder 2 leaves the auction at a price equal to $s + t_2$; Bidder 3 leaves the auction at a price equal to $q + t_3$, q being the price at which Bidder 1 leaves the auction. Bidder 2 (resp. Bidder 3) obtains the good, if $t_2 \geq t_3$ (resp: $t_3 \geq t_2$), at a price equal to $s + t_3$ (resp: $s + t_2$) and makes a profit equal to $t_2 - t_3$ (resp: $t_3 - t_2$). If $t_2 = t_3$, Bidder 2 and Bidder 3 tie. This is resolved with a random draw.*

Bidder 1 and Bidder 2 have a unique weakly dominant strategy; they stay active up to their respective valuations for the good. By observing Bidder 1's behavior, Bidder 3 can perfectly infer his valuation for the good and stay active up to v_3. The auction process allows the piece of information that is not known by all the bidders at the beginning of the auction to be perfectly revealed. The allocation is efficient and the expected revenue is $1/2 + (3t_l + t_h)/4$.

The opportunity to jump bid may modify the equilibrium analysis and affect the outcome of the auction, as the following proposition illustrates.[12]

Proposition 2 *There exists an equilibrium of the* dynamic *clock auction in which:*

- *Bidder 2 always calls price $p = 1$ and then stays active up to $s + t_2$.*

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[9] Note that $1 \leq t_l$ guarantees that $s \leq t_l$, $\forall s$, i.e. that the private value component is more important than the common value component with probability one.

[10] The results are qualitatively robust to the introduction of intermediary types but the continuous case is much more complex to study and we could not solve it.

[11] The condition $t_h < t_l + 1$ is not necessary for the existence of the equilibria we mention but we added it because it induces that Bidder 3 cannot infer the value of s by observing the value of v_2 for any value of v_2. There exist values of v_2 that can be obtained either with $t_2 = t_l$ and *high* values of s or with $t_2 = t_h$ and *low* values of s.

[12] Equilibrium strategies are specified in full in the "Appendix".

- *Bidder 1 immediately leaves the auction after the jump bid by Bidder 2 at price* $p = 1$.
- *Bidder 3 never calls a price. After Bidder 2 calls price* $p = 1$*, Bidder 3 stays active up to* t_l*, if* $t_3 = t_l$*, and up to* $1 + t_h$*, if* $t_3 = t_h$.

Proof In the "Appendix".

Bidder 1's strategy is easy to understand. In any case, the good is worth less than 1 for him so that he prefers staying out after the jump bid to price $p = 1$.

The strategies of Bidder 2 and Bidder 3 in this equilibrium build on the winner's curse. Bidder 2 knows the value of s and Bidder 3 does not. Without jump bids, Bidder 3 discovers the value of s by observing at which price Bidder 1 exits. Since bidders can place jump bids, Bidder 2 calls a price sufficiently high so that Bidder 3 cannot discover the value of s.

If $t_3 = t_h$, the winner's curse is not an issue for Bidder 3 since $t_3 < t_2$ is not possible. Therefore, when $t_3 = t_h$, Bidder 3 stays active up to his highest possible valuation, $1 + t_h$, since he does not fear becoming a victim of the winner's curse. If $t_3 = t_l$, the winner's curse is an issue for Bidder 3. He knows that $t_3 \leq t_2$ and that Bidder 2 will leave the auction at a price equal to $s + t_2$. In order to avoid buying the good for a price higher than his valuation for it, he leaves the auction at a price equal to his lowest possible valuation for the good: t_l.

Now, let us consider Bidder 2's motives. If $t_2 = t_l$, it is clear that calling price $p = 1$ is profitable: if he does not jump bid, he obtains no profit; while if he jump bids, he obtains a strictly positive profit when $t_3 = t_l$. If $t_2 = t_h$, calling price 1 gives an extra profit s when $t_3 = t_l$ while, when $t_3 = t_h$, Bidder 3 is indifferent between jump bidding or not.

Corollary 1 *In the considered equilibrium of the* dynamic *clock auction, the allocation is efficient and the expected seller's revenue is* $1/4 + (3t_l + t_h)/4$*, which is strictly lower than what is obtained in any equilibrium of the standard clock auction.*

The jump bid only affects the equilibrium allocation when $t_2 = t_3$, but in this case whether Bidder 2 or Bidder 3 obtain the good does not affect efficiency. Bidder 2 wins when $t_2 = t_3 = t_l$ and Bidder 3 wins when $t_2 = t_3 = t_h$. The jump bid also reduces the price paid by Bidder 2 when $t_3 = t_l$ by s, hence the expected revenue loss for the seller of $E(s)/2 = 1/4$.

Remark 1 An equilibrium without any jump bid in which bidders behave as in an equilibrium of the clock auction without jump bid also exists. In order to build up such an equilibrium, we could propose an equilibrium belief of Bidder 3 such that he believes that if Bidder 2 calls a price p and Bidder 1 does not stay active after the jump bid, $s = min(p, 1)$ with probability 1 (since no jump bid is ever realized this belief cannot be contradicted by the actual distribution of s conditional on Bidder 2's calling a price).

5 Two variations of the baseline set-up

We consider two variations of the basic set-up we considered. We first introduce an intermediate value for t_i, t_m. This allows to show that the probability to jump bid may be decreasing in the type (i.e. the valuation) of the bidder calling the price. Then, we relax the assumption $t_l \geq 1$, which made the jump bid *costless*. This allows to show that, despite the jump bid, the auction process may still reveal a part of the information that the bidder calling the price would like to prevent from being revealed.

5.1 Allowing for more than two types' realizations for t_i's

We analyze the case where $t_i \in \{t_l, t_m, t_h\}$, with $1 \leq t_l < t_m < t_h < 1 + t_l$, and each type is equally likely. The equilibrium analysis of the game where jump bids are not allowed is the same as in the previous section except for the obvious modifications implied by the additional type. The auction is still efficient and the expected revenue is $1/2 + (5t_l + 3t_m + t_h)/9$. The equilibrium of the dynamic auction with jump bids is presented below.[13]

Proposition 3 *If* $1/2 > t_h - t_m > t_m - t_l$,[14] *there exists an equilibrium of the* dynamic *clock auction in which:*

- *If $t_2 \in \{t_l, t_m\}$ and for any value of s, or if $t_2 = t_h$ and $s \geq \underline{s}$, with $\underline{s} = t_h - t_m$, Bidder 2 calls price 1 and then stays active up to $s + t_2$. If $t_2 = t_h$ and $s < \underline{s}$, Bidder 2 does not call a price and stays active up to $s + t_2$.*
- *Bidder 1 immediately leaves the auction after a jump bid by Bidder 2 at price $p = 1$ and when no price is called, stays active up to s.*
- *Bidder 3 never calls a price. When Bidder 2 calls price $p = 1$, Bidder 3 stays active up to t_l, if $t_3 = t_l$, up to $t_m + \tilde{s}$ with $\tilde{s} \equiv 2t_h - 2t_m$, if $t_3 = t_m$, and up to $1 + t_h$, if $t_3 = t_h$. If no jump bid is placed, Bidder 3 leaves the auction at a price equal to $q + t_3$, q being the price at which Bidder 1 leaves the auction.*

Proof In the "Appendix".

We focus on the differences with the baseline set-up. Bidder 1's strategy is unchanged. Bidder 3's strategy is unchanged if $t_2 \in \{t_l, t_h\}$. If $t_3 = t_m$, Bidder 3 fears the winner's curse (in case $t_2 = t_h$) but, if he leaves the auction too early, he may miss an opportunity to derive a strictly positive profit (in case $t_2 = t_l$). Bidder 3 can safely stay active up to t_h since if Bidder 2 leaves the auction for a price lower than t_h, it must be the case that $t_2 \leq t_m$. Further, if $s < \tilde{s}$ and $t_2 = t_h$, Bidder 2 does not call a price. Therefore, Bidder 3 knows that after a jump bid, Bidder 2 cannot have a high type if he leaves at a price below $t_h + \tilde{s}$. Therefore, he can stay active up to $t_h + \tilde{s}$ without fearing the winner's curse. Now, after a jump bid, if Bidder 2 leaves at a price higher than $t_h + \tilde{s}$, the probability that $t_2 = t_h$ is at least as high as the probability that $t_2 = t_l$. Further, the loss that Bidder 3 with type t_m makes if he wins and $t_2 = t_h$ (i.e. $t_h - t_m$) is larger

[13] Again, we introduce the actions played along the equilibrium path and specify strategies in the "Appendix".

[14] We may obtain equilibria with the same properties with less restrictive assumptions but these assumptions ease the exposition. What is needed for the addition of t_m to be meaningful is that $t_h - t_m > t_m - t_l$, otherwise Bidder 3 of type t_m would display the same type of aggressive strategy as when he is type t_h (that is being active till $t_m + 1$). $t_h - t_l < 1/2$ is imposed only to guarantee that $\tilde{s} \leq 1$, as $s \in [0, 1]$ (see below for the definition of $\tilde{s}$).

than his profit if he wins and $t_2 = t_l$ (i.e. $t_m - t_l$). Therefore, Bidder 3 prefers to leave the auction at price $t_h + \tilde{s}$.

Let us consider how Bidder 2's strategy is affected. If $t_2 = t_l$, it is again clear that calling price $p = 1$ is profitable. If $t_2 = t_m$, without jump bids, Bidder 2 obtains an expected profit of $(t_m - t_l)/3$, while with a jump bid he obtains $(t_m - t_l + s)/3 + \max(0, s - 2t_h + 2t_m)/3$. Thus, calling price 1 is profitable. If $t_2 = t_h$, there is a trade-off. Calling price 1 gives an extra profit s, when $t_3 = t_l$; while, when $t_3 = t_m$, calling price $p = 1$ is counterproductive for low values of s because Bidder 3 stays active up to $2t_h - t_m$, which is higher than $t_m + s$ when $s < 2t_h - 2t_m = \tilde{s}$. Hence, there exists a level of s, $\underline{s}$, for which Bidder 2 with type t_h is indifferent between calling a price or not and for $s < \underline{s}$, no price is called.

Corollary 2 *The probability of observing a jump bid by Bidder 2 is strictly lower when $t_2 = t_h$, than for $t_2 = t_l$ and $t_2 = t_m$.*

The corollary above might appear surprising. Typically in signaling games higher types can mimic lower types, which here would imply that if a lower type finds it profitable to place a jump bid, so should a higher type. This argument does not apply in our case despite the fact that, conditional on having called price $p = 1$, Bidder 2 would prefer Bidder 3 to believe that t_2 is high. Similarly, Bidder 2 would prefer Bidder 3 to believe that the expected value of s is lower after a jump bid, which again is not the case. The explanation is that the jump bid reveals some information about s that Bidder 3 can use to bid more aggressively (when $t_3 = t_m$), and that Bidder 2 is more affected by this change in behavior caused by the jump bid when $t_2 = t_h$ and $s < \underline{s}$.[15]

Corollary 3 *In the considered equilibrium of the* dynamic *clock auction, the allocation may be inefficient, if $2t_h - t_m < 1 + t_l$, and the expected revenue is strictly lower than in any equilibrium of the standard clock auction.*

Proof **Inefficiency:** Consider the case $2t_h - t_m < 1 + t_l$. If $(t_2, t_3) = (t_l, t_m)$, Bidder 2 calls price $p = 1$ and stays active up to $s + t_l$, Bidder 3 leaves the auction at price $t_m + 2t_h - 2t_m = 2t_h - t_m$. Since $2t_h - t_m < 1 + t_l$, there exist values of s sufficiently close to 1 such that $2t_h - t_m < s + t_l$. For these values of s, Bidder 2 wins the auction although $v_3 > v_2$.

Expected revenue: The only case in which the price may be higher in the equilibrium of the *dynamic* auction is when $(t_2, t_3) = (t_h, t_m)$ and $s \in [t_h - t_m, 2t_h - 2t_m)$. Bidder 2 wins the auction and pays $2t_h - t_m > t_m + s$. This represents an expected increase in revenue of $\frac{t_h - t_m}{9} \frac{t_h - t_m}{2} = \frac{(t_h - t_m)^2}{18}$ as compared to what is obtained in the same situation in a standard clock auction. However, when $t_3 = t_l$, there is a price decrease of s in a *dynamic* clock auction that represents an expected loss in revenue equal to $\frac{E(s)}{3} = \frac{1}{6} > \frac{(t_h - t_m)^2}{18}$. Thus, the expected revenue is strictly lower in the considered equilibrium of the *dynamic* clock auction. □

[15] To see why, recall that there is a trade-off of costs and benefits between placing a jump bid or not when $t_2 = t_h$, while there is no such trade off when $t_2 = t_l$ or $t_2 = t_m$ because those types of Bidder 2 can never profitably win against a Bidder 3 of type $t_3 = m$ if jump bids are not used.

5.2 Allowing for $t_l < 1$

Let us go back to the baseline set-up, with only two types. In Sect. 4, we assumed that $t_l > 1$. Thus, Bidder 2 had the option to call price $p = 1$ hiding the values of s for any values of t_2 and s without any direct cost since Bidder 3 stayed active at least up to $t_l > 1$.[16] Now, if we assume that $t_l = \frac{1}{2}$.[17] The equilibrium that we proposed in Sect. 4 no longer stands since when $t_2 = t_l$ and $t_l + s < 1$, Bidder 2 does not want to call price $p = 1$. Nonetheless, an equilibrium exists in which Bidder 2 computes a jump bid that preserves the informational advantage necessary to induce a winner's curse and discloses the minimum amount of private information.[18]

Proposition 4 *If $t_l = \frac{1}{2}$, there exists an equilibrium of the* dynamic *clock auction in which:*

- *If $s \in [0, 1/2)$, Bidder 2 calls price $p = 1/2$ and then stays active up to $s + t_2$; if $s \in [1/2, 1]$, Bidder 2 calls price $p = 1$ and then stays active up to $s + t_2$.*
- *Bidder 1 immediately leaves the auction after a jump bid by Bidder 2.*
- *Bidder 3 never calls a price. When Bidder 2 calls price $p = 1/2$, Bidder 3 stays active up to t_l, if $t_3 = t_l$, and up to $t_h + 1/2$, if $t_3 = t_h$. When Bidder 2 calls price $p = 1$, Bidder 3 stays active up to $t_l + 1/2$, if $t_3 = t_l$, and up to $t_h + 1$, if $t_3 = t_h$.*

Proof In the "Appendix".

In this equilibrium, Bidder 2 always calls a price, but the price he calls depends on the value of s. Again, we observe that even though Bidder 2 would prefer to reveal as little information as possible regarding s, he does reveal some information about s with his jump bid. After the jump bid, Bidder 3 knows whether $s < 1/2$, or $s \geq 1/2$. Because $t_l = 1/2$, it is no longer costless for Bidder 2 to call price $p = 1$, when $t_2 = t_l$. However, Bidder 2 manages to partition the interval [0, 1] on which s lies and to raise his payoff with the jump bids, since he pays $t_l + \frac{\mathbb{1}_{s\geq 1/2}}{2}$ rather than $t_l + s$, when $t_3 = t_l$. Thus, the expected gain from jump bidding is $\frac{1}{2}E(s - \frac{\mathbb{1}_{s\geq 1/2}}{2}) = \frac{1}{8}$. Intuitively, the more coarsely the interval [0, 1] is partitioned, the less information is communicated to Bidder 3, which is good for the purpose of imposing a winner's curse. However, there is some restriction on how the interval [0, 1] can be partitioned because the size of the elements of the partition cannot be larger than t_l.[19]

As in the case we considered in 4, the jump bid does not affect the efficiency of the auction but it reduces expected revenue ($(3t_l + t_h)/4 + 3/8$ rather than $(3t_l + t_h)/4 + 1/2$).

[16] Let us mention that an equilibrium jump bid with partitions such as the one we propose in the current subsection would also exist in the baseline set-up. However, notice that because it would reveal some information about s that Bidder 2 can incorporate in his bidding, it would yield a strictly lower expected profit for Bidder 2 as compared to the equilibrium we presented.

[17] We could consider any $t_l \in (0, 1)$.

[18] The analysis of the game where jump bids are not allowed is unchanged.

[19] Note that, if $t_2 = t_l$, Bidder 2 does not call more than $1/2 + s$. Also, calling less than s is useless since with such a low jump bid, Bidder 1 stays active after the jump bid and s is revealed during the auction process in any case. Then, in equilibrium, Bidder 2 with a type t_l only calls a price in the interval $[s, s + t_l]$. This explains why the size of the elements of the partition cannot exceed t_l.

6 Conclusion

The use of jump bidding strategies is widespread in many markets ranging from standard auctions to takeover contests. This paper suggests a novel strategic use of jump bidding; creating a winner's curse in an environment where it would not arise otherwise. Interestingly, for sensible values of the parameters of the model, we observe that the bidder calling a price is less likely to do so when he has a more favorable private type.

Appendix

Proof of Proposition 2

Consider the following strategies:

- Bidder 1. Never calls a price, stays active up to s (whether no price is called or a price lower than s is called), and leaves the auction if a price higher than s is called.
- Bidder 2. Always calls price $p = 1$ and stays active up to $s + t_2$ afterwards. If a bidder calls a price higher than $p = 1$, stays active up to $s + t_2$.
- Bidder 3. Never calls a price. If no price is called, leaves the auction at a price equal to $q + t_3$, q being the price at which Bidder 1 leaves the auction if it is in the interval [0, 1]. If Bidder 1 does not leave the auction at a price lower than 1, Bidder 3 stays active up to $1 + t_3$.

  If a price p is called in the first stage:

  (a) If Bidder 2 calls a price p strictly lower than 1, Bidder 3 stays active after the jump bid. Then, if Bidder 1 stays active after the jump bid, Bidder 3 behaves as in the case without jump bid. If Bidder 1 does not stay active after the jump bid, Bidder 3 stays active up to $t_3 + p$. If Bidder 2 calls a price p strictly higher than 1, Bidder 3 stays active after the jump bid up to $1 + t_3$. If Bidder 2 calls a price $p = 1$, Bidder 3 stays active after the jump bid. Then, if Bidder 3 has type $t_3 = t_l$, he stays active up to t_l; else, he stays active up to $t_h + 1$.

  (b) If Bidder 1 calls a price $p < 1$, Bidder 3 stays active and then leaves the auction at a price equal to $q + t_3$, q being defined as before. If Bidder 1 does not leave the auction at a price lower than 1, Bidder 3 stays active up to $1 + t_3$. If Bidder 1 calls a price $p \geq 1$, Bidder 3 stays active up to $1 + t_3$.

We also suggest a belief function for Bidder 3 which is coherent with these strategies.

Bidder 3's belief conditional on observing that Bidder 2 calls price 1 and Bidder 1 does not stay active after must be the same as its prior since Bidder 2 always calls price 1 and Bidder 1 never stays active after such a jump bid at the equilibrium. With such a belief and assuming that Bidder 2 stays active up to his valuation for the good, Bidder 3 cannot obtain more than what he obtains following the proposed strategy (if $t_3 = t_l$, he cannot derive any profit and leaving at price t_l is a best response and if $t_3 = t_h$, leaving at price $t_h + 1$ is also a best response).

Now, since at the equilibrium, Bidder 2 never calls a price different from 1, we can propose many beliefs following a jump bid to a price different from 1: they will not be in contradiction with the actual distribution of s conditional on the jump bid. We assume that Bidder 3 believes that if Bidder 2 calls a price $p \neq 1$ and Bidder 1 does not stay active after the jump, $s = \min(p, 1)$. This is no incoherent with Bidder 1 and 2's strategy and Bidder 3's strategy is coherent with this belief.

If Bidder 3 observes that Bidder 1 leaves the auction at a price p, he believes that $s = \min(p, 1)$.

If bidders choose these strategies, their behaviors coincide with what we describe in Proposition 2. Now we need to show that these strategies constitute an equilibrium.

Bidder 1: Staying active beyond (or calling a price higher than) his valuation is weakly dominated. Further, considering Bidder 2 and Bidder 3's strategies, Bidder 1 cannot make a profitable deviation with a jump bid lower than his valuation.

Bidder 2: Whether a price is called or not, in the second part of the auction, staying active up to his valuation for the good is a weakly dominant strategy. Therefore, in order to find a profitable deviation, we need to focus on the jump bidding part of the strategy, assuming that after any possible jump bid, he will stay active up to his valuation for the good.

If Bidder 2 does not call a price or calls a price lower than s, Bidder 3 discovers the value of s by observing the price at which Bidder 1 leaves the auction and Bidder 3 stays active up to $s + t_3$. We consider separately the different possible values of t_2.

$t_2 = t_l$. If Bidder 2 calls price $p = 1$, he obtains an expected payoff equal to $s/2$. If he does not call a price or calls a price strictly lower than s, he obtains 0. If he calls a price $p \in [s, 1)$, he cannot obtain more than what he obtains when he calls price $p = 1$. If he calls a price strictly higher than 1, he obtains 0. Hence, there is no profitable deviation.

$t_2 = t_h$. If Bidder 2 calls price $p = 1$, he obtains an expected payoff equal to $(t_h - t_l + s)/2$. If he does not call a price or calls a price strictly lower than s, he obtains $(t_h - t_l)/2$. If he calls a price $p \in [s, 1)$, he cannot obtain more than what he obtains when he calls price $p = 1$. If he calls a price strictly higher than 1, he obtains $\max(0, t_h + s - t_l - 1)/2$. Hence, there is no profitable deviation.

Bidder 3: We first consider deviations that do not involve calling a price and consider separately the different possible values of t_3.

$t_3 = t_l$. If Bidder 2 calls price $p = 1$ (or any price greater or equal than s), he never leaves the auction for a price lower than $s + t_l$. Therefore, conditional on winning the auction, Bidder 3 can only make a negative profit. Leaving the auction at price t_l, Bidder 3 avoids winning and picks a strategy that is not dominated. If no price is called (or a price lower than s is called), Bidder 3 discovers the value of s by observing at which price Bidder 1 leaves the auction. Then, staying active up to $q + t_l$ is a weakly dominant strategy.

$t_3 = t_h$. If Bidder 2 calls price $p = 1$ (or any price greater or equal than s), he never leaves the auction at a price strictly higher than $s + t_h$, which means that Bidder 3 always wins and never makes a loss when winning. Thus, the proposed equilibrium strategy is not dominated. If no price is called (or a price lower than s is called), Bidder 3 discovers the value of s by observing at which price Bidder 1 leaves the auction. Then, staying active up to $q + t_h$ is a weakly dominant strategy.

Now, let us consider deviations that include jump bids.

Suppose that Bidder 3 calls a price lower than 1. This jump bid does not qualify as the highest jump bid in the first stage, so it yields the same outcome as not calling a price at all. Suppose that Bidder 3 calls a price $p \in [1, t_l]$. After the jump bid, Bidder 2 stays active up to v_2. Bidder 3's information is the same as in the case when Bidder 2 is the bidder placing the highest bid in the first stage. Thus, calling a price $p \in [1, t_l]$ cannot be part of a profitable deviation. We can show with the same type of arguments that calling a price $p > t_l$ cannot be part of a profitable deviation either (Bidder 3 does not obtain more information when $t_2 + s \geq p$ and if $t_2 + s \geq p$, the jump bid makes him lose money). □

Proof of Proposition 3

Consider the following strategies:

- Bidder 1. Never calls a price, stays active up to s and leaves the auction if a price higher than s is called.
- Bidder 2. If $t_2 = t_l, t_m$ and if $t_2 = t_h$ and $s > t_h - t_m$, calls price $p = 1$ and stays active up to $s + t_2$ afterwards. If $t_2 = t_h$ and $s < \underline{s}$, does not call a price and stays active up to $t_h + s$. If a bidder calls a price higher than 1, stays active up to $s + t_2$.
- Bidder 3. Never calls a price. If no price is called, Bidder 3 leaves the auction at a price equal to $q + t_3$, q being the price at which Bidder 1 leaves the auction if it is in the interval [0, 1]. If Bidder 1 does not leave the auction at price lower than 1, Bidder 3 stays active up to $1 + t_3$.

  If a price p is called in the first stage:

  (a) If Bidder 2 calls a price p strictly lower than 1, Bidder 3 stays active after the jump bid. Then, if Bidder 1 stays active after the jump bid, Bidder 3 behaves as in the case without jump bid. If Bidder 1 does not stay active after the jump bid, Bidder 3 stays active up to $t_3 + p$. If Bidder 2 calls a price p strictly higher than 1, Bidder 3 stays active after the jump bid up to $1 + t_3$. If Bidder 2 calls a price $p = 1$, Bidder 3 stays active after the jump bid. Then, if Bidder 3's type is t_l, he stays active up to t_l; if $t_3 = t_m$, he stays active up to $2t_h - t_m$; and if $t_3 = t_h$, he stays active up to $t_h + 1$.

  (b) If Bidder 1 calls a price $p < 1$, Bidder 3 stays active and then leaves the auction at a price equal to $q + t_3$, q being defined as before. If Bidder 1 does not leave the auction at a price lower than 1, Bidder 3 stays active up to $1 + t_3$. If Bidder 1 calls a price $p \geq 1$, Bidder 3 stays active up to $1 + t_3$.

If bidders choose these strategies, their behaviors coincide with what we describe in Proposition 3. Now we need to show that these strategies constitute an equilibrium.

Bidder 1: Analogous argument as for the proof of Proposition 2.

Bidder 2: We stress only the parts that differ from the proof of Proposition 2.

$t_2 = t_l$. Same argument as in the proof of 2 except that now the expected payoff in equilibrium is $s/3 + \max(0, t_l + s - 2t_h + t_m)/3$.

$t_2 = t_m$. If Bidder 2 calls price $p = 1$, he obtains an expected payoff equal to $(t_m - t_l + s)/3 + \max(0, 2t_m + s - 2t_h)/3$ and $(t_m - t_l)/3$ if he does not call a price or call a price strictly lower than s. If he calls a price $p \in [s, 1)$, he cannot obtain more than what he obtains when he calls price 1 and if he calls a price strictly higher than 1, he derives $\max(0, t_m + s - t_l - 1)/3$. Hence, there is no profitable deviation when $t_2 = t_m$.

$t_2 = t_h$. If Bidder 2 calls price $p = 1$, he obtains an expected payoff equal to $(t_h - t_l + s)/3 + \max(0, t_m + s - t_h)/3$ and $(t_h - t_l)/3 + (t_h - t_m)/3$ if he does not call a price or call a price strictly lower than s. If he calls a price $p \in [s, 1)$, he cannot obtain more than what he obtains when he calls price 1 and if he calls a price strictly higher than 1, he derives $\max(0, t_h + s - t_l - 1)/3 + \max(0, t_h + s - t_m - 1)/3$. Therefore, Bidder 2's best choice are either calling price 1 or not calling a price. The first alternative gives him $(t_h - t_l + s)/3 + \max(0, t_m + s - t_h)/3$ and the second one $(t_h - t_l)/3 + (t_h - t_m)/3$. When $s \geq \underline{s}$, the first alternative gives him a higher payoff and calling price $p = 1$ is a better response. When $s < \underline{s}$, the second alternative gives him a higher payoff and not calling any price is a better response. Hence, there is no profitable deviation when $t_2 = t_h$.

Bidder 3: We first consider deviations that do not involve calling a price and consider separately the different possible values of t_3.

$t_3 = t_l$. Same argument as in the proof of Proposition 2.

$t_3 = t_m$. If Bidder 2 calls price $p = 1$ and leaves the auction at a price below $t_h + \underline{s}$, the probability that $t_2 = t_h$ is zero. Therefore, $v_2 \leq v_3$ and since Bidder 2 leaves the auction at a price equal to v_2, staying active up to $t_h + \underline{s}$ is not costly and it may be profitable. Therefore, Bidder 3 cannot profitably deviate leaving the auction at a price lower than $t_h + \underline{s}$. Now, suppose that Bidder 3 considers leaving the auction at a price strictly higher than $t_h + \underline{s}$. Since the expected value of v_3 conditional on Bidder 2's leaving the auction at a price p strictly higher than $t_h + \underline{s}$ is strictly lower than p (since $t_h - t_m > t_m - t_l$), such a deviation cannot be profitable either. If no price is called, Bidder 3 discovers the value of s by observing at which price Bidder 1 leaves the auction. Then, staying active up to $q + t_m$ is a weakly dominant strategy.

$t_3 = t_h$. Same argument as in the proof of Proposition 2.

Now, let us consider deviations that include jump bids.

Since Bidder 2 always leaves the auction at a price equal to $s + t_2$, Bidder 3 cannot make any profitable deviation even if it includes a jump bid when $t_3 = t_l$ (he cannot derive any profit) and when $t_3 = t_h$ (he cannot win the auction at a price strictly lower than $t_2 + s$) so that we only need to consider $t_3 = t_m$.

Suppose that Bidder 3 calls a price lower than 1. This jump bid could only affect the auction when $t_2 = t_h$ and $s < \tilde{s}$. However, in that case, Bidder 2 stays active up to $t_h + s$ after the jump bid and Bidder 3 cannot obtain any strictly positive profit.

Suppose that Bidder 3 calls a price $p \in (1, t_l]$. After the jump bid, Bidder 2 stays active up to v_2. Bidder 3's information is the same as in the case when Bidder 2 calls price $p = 1$ except that he can no longer learn the event "$t_2 = t_h$ and $s < \tilde{s}$". Therefore, Bidder 3 is better off leaving the auction at a price equal t_h rather than staying active up to $t_h + \tilde{s}$. Hence, calling price p lowers his expected payoff by $\min(t_l + 1 - t_h, \tilde{s})(t_m - t_l)$. Calling a price $p \in (1, t_l]$ cannot be part of a profitable deviation.

The same type of arguments applies for a jump bid $p > t_l$ so that it cannot be part of a profitable deviation either. □

Proof of Proposition 4

Consider the following strategies:

- Bidder 1. Never calls a price, stays active up to s and leaves the auction if a price higher than s is called.
- Bidder 2. If $s \in [0, 1/2)$ calls price $p = 1/2$ and stays active up to $s+t_2$ afterwards. If $s \in [1/2, 1]$ calls price 1 and stays active up to $s + t_2$ afterwards. If a bidder calls a price higher than the price called by Bidder 2, stays active up to $s + t_2$.
- Bidder 3. Never calls a price. If no price is called, Bidder 3 leaves the auction at a price equal to $q + t_3$, q being the price at which Bidder 1 leaves the auction if it is in the interval $[0, 1]$. If Bidder 1 does not leave the auction at a price lower than 1, Bidder 3 stays active up to $1 + t_3$.

  If a price p is called in the first stage:

  (a) If Bidder 2 calls a price p strictly lower than 1 with $p \neq 1/2$, Bidder 3 stays active after the jump bid. Then, if Bidder 1 stays active after the jump bid, Bidder 3 behaves as in the case without jump bid. If Bidder 1 does not stay active after the jump bid, Bidder 3 stays active up to $t_3 + p$. If Bidder 2 calls a price $1/2$, Bidder 3 stays active after the jump bid. Then, if Bidder 1 stays active after the jump bid, Bidder 3 behaves as in the case without jump bid. If Bidder 1 does not stay active after the jump bid, Bidder 3 stays active up to $t_l + 1/2$ and if $t_3 = t_h$, he stays active up to $t_h + 1$. If Bidder 2 calls a price $p = 1$, Bidder 3 stays active after the jump bid. If Bidder 3 has type $t_3 = t_l$, he stays active up to $t_l + 1/2$; if $t_3 = t_h$, he stays active up to $t_h + 1$. If Bidder 2 calls a price $p = 1$, Bidder 3 stays active after the jump bid. If Bidder 3 has type $t_3 = t_l$, he stays active up to $t_l + 1/2$; if $t_3 = t_h$, he stays active up to $t_h + 1$. If Bidder 2 calls a price p strictly higher than 1, Bidder 3 stays active after the jump up to $1 + t_3$.

  (b) If Bidder 1 calls a price $p < 1$, Bidder 3 stays active and then leaves the auction at a price equal to $q + t_3$, q being defined as before. If Bidder 1 does not leave the auction at a price lower than 1, Bidder 3 stays active up to $1 + t_3$. If Bidder 1 calls a price $p \geq 1$, Bidder 3 stays active up to $1 + t_3$.

If bidders choose these strategies, their behaviors coincide with what we describe in Proposition 4. Now we need to show that these strategies constitute an equilibrium.

Bidder 1: Analogous arguments as in the proof of Proposition 2.

Bidder 2: We stress only the parts that differ from the proof of Proposition 2.

(a) If $s \geq 1/2$
$t_2 = t_l$. If Bidder 2 calls price $p = 1$, he obtains a payoff equal to $(s - 1/2)/2$. If he does not call a price or call a price strictly lower than s, he obtains 0. If he calls a price $p \in [s, 1)$, he does not obtain more than what he obtains when he calls price $p = 1$. If he calls a price strictly higher than 1, he derives obtains 0. Hence, there is no profitable deviation.
$t_2 = t_h$. If Bidder 2 calls price $p = 1$, he obtains a payoff equal to $(t_h - t_l + (s - 1/2))/2$. If he does not call a price or call a price strictly lower than s, he obtains $(t_h - t_l)/2$. If he calls a price $p \in [s, 1)$, he does not obtain more than what he obtains when he calls price $p = 1$. If he calls a price strictly higher than 1, he derives $\max(0, t_h + s - t_l - 1)/2$. Hence, there is no profitable deviation.

(b) If $s < 1/2$
$t_2 = t_l$. If Bidder 2 calls price $p = 1/2$, he obtains a payoff equal to $s/2$. If he does not call a price or call a price strictly lower than s, he obtains 0. If he calls a price $p \in [s, 1]$, he does not obtain more than what he obtains when he calls price $p = 1/2$. If he calls a price strictly higher than 1, he obtains 0. Hence, there is no profitable deviation.
$t_2 = t_h$. If Bidder 2 calls price $p = 1/2$, he obtains a payoff equal to $(t_h - t_l + s))/2$. If he does not call a price or call a price strictly lower than s, he obtains $(t_h - t_l)/2$. If he calls a price $p \in [s, 1]$, he does not obtain more than what he obtains when he calls price $p = 1/2$. if he calls a price strictly higher than 1, he derives $\max(0, t_h + s - t_l - 1)/2$. Hence, there is no profitable deviation.

Bidder 3: $t_3 = t_l$ and $t_3 = t_h$. Same arguments as in the proof of Proposition 2. □

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