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5-th Dolomites Workshop on Constructive Approximation and Applications – Special Issue dedicated to Robert Schaback on the occasion of his 75th birthday

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Abstract

The guest editors discuss the highlights of the *5-th Dolomites Workshop on Constructive Approximation and Applications*, and briefly introduce the papers included in this special issue.

1 Report on the conference

The virtual conference *5-th Dolomites Workshop on Constructive Approximation and Applications DWCAA21* was held on September 6–10 2021. The main scope of the meeting was to provide a forum for researchers to present and discuss ideas, theories, and applications of constructive approximation.

Throughout this five-days event we had a total of 55 speakers, and many topics in different fields of numerical analysis have been discussed. Among them we mention approximation theory, multiresolution techniques and numerical integration.

The complete details of the workshop are available on the website:

https://events.math.unipd.it/dwcaa21/.

The conference featured five invited presentations:

1. M. Buhmann, University of Giessen, Germany:
   Celebrating Robert Schaback's birthday.
2. A. Iske, University of Hamburg, Germany:
   Greedy update strategies for kernel-based approximation algorithms.
3. L. Romani, Alma Mater Studiorum - University of Bologna, Italy:
   Pythagorean-Hodograph curves in exponential polynomial spaces.
4. R. Schaback, Göttingen Georg-August University, Germany:
   My Failures, past and present.
5. H. Wendland, University of Bayreuth, Germany:
   A meshfree method for a PDE-constrained optimisation problem.

Moreover, a poster session with 9 participants, and 5 parallel sessions were organized on the following topics: Approximation theory in imaging science; Meshless methods; Multivariate polynomial approximation; Multiresolution techniques; Numerical integration, integral equations and transforms.

At the end of the conference two awards (both amounting to 500 euros) were assigned to the best talk and the best poster. The winners have been

1. I. Tominec, Uppsala University, Sweden: *An oversampled RBF-FD method for elliptic and hyperbolic PDEs* (best talk).
2. N. Siar: University of Calabria, Italy: *Numerical differentiation on scattered data through multivariate polynomial interpolation* (best poster).

2 Introduction to the special issue

During the workshop the speakers were invited to submit contributions for a special volume of the Dolomites Research Notes on Approximation (DRNA). After peer-revision, we are pleased to announce that the following papers have been included in the volume:

- *Interpolating sequences of 3D-data with C^2 quintic PH B-spline curves*, by G. Albrecht, C. V. Beccari and L. Romani [1]. In this work, PH quintic B-spline curves are employed in solving the C^2 interpolation problem, dealing with sequences of 3D-data. The proposed scheme is efficient and improves the relevant state of the art.

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- *Nonlinear multivariate sampling Kantorovich operators: quantitative estimates in functional spaces*, by N. Çetin, D. Costarelli, M. Natale, and G. Vinti [2]. The work deals with quantitative estimates for the nonlinear sampling Kantorovich operators in the multivariate setting using the modulus of smoothness of $L^p(\mathbb{R}^n)$. The above results have been then extended to the general case of Orlicz spaces $L^\varphi(\mathbb{R}^n)$.
- *Solving surface interpolation problems stochastically and greedily*, by M. Chen, L. Ling, and Y. Su [3]. The task of the paper is to introduce efficient techniques for the optimization of the shape parameter of a Radial Basis Function kernel when solving interpolation problems on a surface. These novel methods are based on (stochastic) Leave One Out Cross Validation, and on a greedy algorithm for the solution of the interpolation linear system.
- *Reconstruction of volatility surfaces: a first computational study*, by S. Cuomo, A. De Rossi, L. Rizzo and F. Sica [4]. In this work, the authors tackle the implied volatility surface interpolation problem by considering a Radial Basis Function technique. By combining both global and a local approach via the partition of unity strategy, this computational study shows the effectiveness of the proposed method both in interpolation and extrapolation tasks.
- *Numerical method for hypersingular integrals of highly oscillatory functions on the positive semiaxis*, by M. C. De Bonis and V. Sagaria [5]. In this paper, the authors analyze and then prove the stability and the convergence of a proposed quadrature rule for the numerical evaluation of hypersingular integrals of highly oscillatory functions on the positive semiaxis.
- *Error bounds and the asymptotic setting in kernel-based approximation*, by T. Karvonen [6]. This work introduces two new computable error bounds for kernel-based interpolation, borrowing ideas from Gaussian process regression. Elements of the theory of the so-called asymptotic setting are furthermore used to argue that these bounds are not conservative. Moreover, interesting aspects of the optimality of existing an new error bounds for kernel interpolation are discussed under a new point of view.
- *Compounded product integration rules on* $(0,+\infty)$, by D. Mezzanotte and D. Occorsio [7]. The paper provides and analyses a new approach for the numerical approximation of certain integral transforms with an exponentially decaying weight and a quite general kernel. A new product rule leveraging a modified Lagrange interpolation technique is introduced, and it is shown that the resulting method is stable and convergent at a certain optimal rate. Moreover, the computational benefits of the new method are demonstrated.
- *Filtered integration rules for finite weighted Hilbert transform II*, by D. Occorsio, M. G. Russo and W. Themistoclakis [8]. In this work, the authors obtain further results concerning a class of quadrature rules for the evaluation of the finite Hilbert transform, which was introduced in a previous paper.
- *Yet another DE-Sinc indefinite integration formula*, by T. Okayama and K. Tanaka [9]. The paper focuses on Sinc approximation combined with the tanh transformation. The authors propose the replacement of the tanh transformation with the double-exponential transformation in Stenger's second formula.
- *Approximation tools for detecting unforeseen sudden events*, by A. Perlo and E. Venturino [10]. In this paper a splines-based algorithm is devised to detect the onset of unforeseen events, such as the opening of windows in a house, causing a sudden drop of the internal temperature.
- *Mean Field limits of trained weights in deep learning*, A. Smirnov, B. Hamzi, and H. Owhadi [11]. The authors study the training of certain Neural Networks from the point of view of the dynamical evolution of the network's weights, establishing the existence of a mean-field limit of this dynamic. This is the first paper published in DRNA addressing the approximation theory of deep Neural Networks, an interesting and emerging topic.

3 Acknowledgements

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