

Shuttle Diplomacy^{*}

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Abstract

In practice mediation operates through shuttle diplomacy: the mediator goes back and forth between parties, meeting them in private. We model shuttle diplomacy as a dynamic procedure. The mediator helps each party to gradually discover (privately) her value from settlement and re-assess her bargaining position, while also proposing the terms of the deal. We show that shuttle diplomacy always allows parties to achieve an ex-post efficient final settlement. In contrast, this is not possible with a static mediation procedure. In addition, if parties have symmetric prior value distributions, shuttle diplomacy guarantees a fair split of the social value from settlement.

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1 Introduction

In April of 2019, U.S. District Court Judge Vince Chhabria appointed Ken Feinberg to facilitate a settlement between Bayer and over 13,000 plaintiffs who had alleged that the weed killer Roundup causes non-Hodgkin’s lymphoma due to its active ingredient, glyphosate. Feinberg is an expert mediator who had previously mediated many disputes, including settlement of the 9/11 victims fund and BP Deepwater Horizon disaster. In seeking to bring about a mutually-agreed upon resolution, mediators provide two services. First, in communicating with parties they acquire information that is privately known to one party and selectively transmit it to the other party. Second, the mediator (usually a retired judge or well-experienced lawyer) brings her own expertise to bear on the question of case value. The mediation literature often labels these two roles as facilitative versus evaluative mediation styles.¹ In lawsuit mediation the mediator’s expertise and role as an evaluator is especially important if the case involves issues of first impression, like the Roundup litigation; that is, new legal issues or interpretations brought before a court that have not been addressed before by that court or that court’s jurisdiction. A mediator who has been involved in other mediations, many of which are not in the public domain, can help parties to better assess costs and benefits of the case, for example with regard to the likely outcomes of the litigation process.

As pointed out by Carbone (2019): “The best mediators will use an approach that draws upon both styles as the needs of the case require.” As an evaluator, the mediator helps parties to assess the merits of the case and provides them with reality checks; she engages in “shuttle diplomacy,” meeting with each side in a private caucus. Shuttle diplomacy also allows the mediator to act as a facilitator, collecting information from a party and then leaking some of it back to the other side. It is important to stress that, unlike in arbitration, mediators are not allowed to impose a judgment on the parties. The mediator suggests and seeks to bring about a mutually-agreed upon resolution. The dynamic process of mediation has been described as follows:

“Mediations are in the main assisted negotiations. The mediator goes back

¹ See for example Smartsettle (2017): “The mediator who evaluates assumes that the participants want and need her to provide some guidance as to the appropriate grounds for settlement based on law, industry practice or technology – and that she is qualified to give such guidance by virtue of her training, experience, and objectivity the facilitative mediator assumes that her principal mission is to clarify and to enhance communication between the parties in order to help them decide what to do.”

and forth with demands and offers (and counter demands and counter offers), while, at the same time, the mediator asks questions and makes comments about the dispute, so that each side can more objectively and realistically consider the facts and think about what might happen if the case were to go to a verdict.”²

There are many other forms of mediation, besides lawsuit mediation. Informal mediation, for example, has a long tradition across different cultures; in China it goes back to Confucianism and recently “top political-legal authorities of the Chinese Communist Party have been promoting mediation as the key to resolving all disputes” (Pissler, 2013). Disputes which formal or informal mediation is commonly used to resolve include: conflict between members of a family or business relationship, conflict in the workplace, conflict arising out of a commercial transaction including cross border trade, real estate matters and personal property. In many cases transfer payments are commonly used.

A good mediator should be fair. According to the Model Standards of Conduct for Mediators (2005), prepared by the American Arbitration Association, the American Bar Association’s Section of Dispute Resolution, and the Association for Conflict Resolution, Impartiality is Standard II: “A. A mediator shall decline a mediation if the mediator cannot conduct it in an impartial manner. Impartiality means freedom from favoritism, bias or prejudice. B. A mediator shall conduct a mediation in an impartial manner and avoid conduct that gives the appearance of partiality.”

In this paper we examine the mediator’s role in helping two parties to reach an agreement about an ongoing dispute regarding an economic transaction. The parties use a mediator to help them discover their costs and benefits from settlement and in suggesting a fair resolution, inclusive of compensation from one party to the other. In short, our mediator is both a mechanism designer and an information designer. At the start, parties do not fully know their values for the ways of settling the dispute. A novelty of our approach is that the mediator acts as an expert evaluator, who can transmit to each party information about the case that she does not have, thus providing her with reality checks. We maintain the assumption that the mediator can only provide each party with information about her own value and that whatever a party learns about it remains her private information. As in the Bayesian persuasion literature (Kamenica and Gentzkow,

²National Arbitration and Mediation (2019).

2011), we give extended leeway to the mediator about how much the parties discover about their values; we allow revelation of any information that is Bayes consistent with the prior. A task of the mediator is then to set up an information discovery policy: what to say about the case, to whom in a private session and at what point in the negotiations? But, given the parties' private information, the mediator also plays a facilitative role, by designing procedures to extract information from parties regarding what they learnt and proposing them possible trading deals.

We show that ex-post efficiency, and hence maximal ex-ante social value, can always be achieved with a simple, dynamic, shuttle diplomacy procedure that resembles the mediation procedures we often observe in practice. In addition, if parties have symmetric prior value distributions, shuttle diplomacy guarantees a fair split of the total gains from a settlement. The key features that allow shuttle diplomacy to attain ex-post efficiency and (under symmetry) fairness are that the procedure limits the parties to small information discoveries at each point in time, thus reducing their opportunities to misreport their information and slackening their incentive constraints. At the same time, it ensures that parties have enough information to assess whether or not carrying out the economic transaction is ex-post efficient. Also, the symmetry of the information provided to the parties help to sustain the fairness of the outcome. Furthermore, we show the mechanism we consider, based on the shuttle diplomacy procedure, has some robustness features, which allow to extend the validity of our results to the case where parties have some private information regarding their value at the start of the procedure.

Our findings provide a clear, strong, rationale for shuttle diplomacy, because they demonstrate that focusing on static mechanisms, in which the mediator simultaneously chooses the distribution of the signals received by the parties and there is then only round where a transaction can occur is with loss of generality; ex-post efficiency cannot be achieved.

The paper is organized as follows. Section 2 introduces the setting. Section 3 presents the shuttle diplomacy procedure, first in an example and then in full generality, and shows that it always allows to implement an ex-post efficient outcome. Section 4 contains some extensions, showing that a fair outcome is implementable under symmetry of the parties prior value distributions, the robustness of the result to the presence of initial private information, and more. Section 5 considers some alternative sequential mediation

procedures that we show also allow to implement an ex post efficient outcome and discusses their properties. Section 6 examines static mediation procedures and the impossibility to attain efficiency in that case. Section 7 discusses the related literature and Section 8 concludes. All the proofs are in Appendix A. Additional material is in Appendix B.

2 The Setting

We consider two parties seeking to reach an agreement about an ongoing economic dispute regarding an outstanding transaction. For concreteness, we will refer to two applications. The first is a dispute regarding the provision of a service from one party, the seller, to the other party, the buyer. The second is a dispute regarding the transfer of ownership of an asset from the seller to the buyer. The settlement options are that the transaction occurs, in exchange of some compensation to the seller, or that it does not occur. The parties use a mediator to help them discover the costs and benefits of providing the service, or of holding the asset, (i.e., of the alternative ways of settling the dispute) and to suggest the compensation, or price, the buyer must pay if the service is provided, or trade takes place.

Let $v_B \in [0, 1]$ be the buyer's value for the service, or for acquiring the asset. Let $v_S \in [0, 1]$ be the seller's cost of providing the service, or her value for retaining ownership of the asset. They are independently and privately drawn from distributions $F_B(v) = \Pr(v_B \leq v)$ and $F_S(v) = \Pr(v_S \leq v)$. We assume that F_B and F_S have no atoms and admit densities $f_B(v) \geq 0$ and $f_S(v) \geq 0$ almost everywhere; the expectations according to these distributions are $\mathbb{E}_{F_B}[v_B] = \int_0^1 v dF_B(v)$ and $\mathbb{E}_{F_S}[v_S] = \int_0^1 v dF_S(v)$. We will also use the notation $\mathbb{E}_{F_B}^I[v_B]$, $\mathbb{E}_{F_S}^I[v_S]$ to denote the expectation conditional on the value being in an interval $I \subseteq [0, 1]$, and $\mathbb{E}_{F_B, F_S}^I[v_B | v_B > v_S]$, $\mathbb{E}_{F_B, F_S}^I[v_B | v_B < v_S]$, $\mathbb{E}_{F_B, F_S}^I[v_S | v_S > v_B]$, $\mathbb{E}_{F_B, F_S}^I[v_S | v_S < v_B]$ for the expectation taken over both values conditional on the values being in I and one value being larger than the other.

The mediator is benevolent and her goal is to maximize social surplus, or ex-post efficiency; that is, ideally the mediator would want the service to be provided, or control of the asset transferred to the buyer, if $v_B > v_S$ and the service not be provided, or the asset to remain in the seller's control, if $v_B < v_S$. To this end, the mediator sets up experiments (or discovery policies) that inform each party independently about features

of the environment. The privately observed outcomes of these experiments allow a party to learn about her value from settlement. To stack the cards against the possibility of implementing an efficient outcome, we assume the party’s discoveries and updated value remain private and are not observed by the other party, or the mediator.³ To understand this, we can think of a party as learning something about the different features of providing the service, or of owning the asset, from the experiments set up by the mediator, but how much she values these characteristics is her private information. As in a cryptographic protocol, each party knows private keys that allows her to discover her value after the mediator discloses public keys. Sole knowledge of the party’s public keys reveals no information about the party’s value to either the mediator or the other party. An alternative interpretation is that each party directly acquires information about her value, but the mediator monitors and enforces the parties’ commitment to discover the prescribed – limited – amount of information.

Apart from this privacy constraint, we allow the mediator to use the most general signaling technology; that is, as in the literature on Bayesian persuasion (e.g., see Kamenica and Gentzkow, 2011, and Bergemann and Morris, 2019) the mediator is free to choose any discovery policy that is consistent with the prior distribution. Prior to the experiments designed by the mediator, neither the buyer, the seller or the mediator have any information regarding the draws from F_B and F_S , but we will later investigate the robustness of our results to the presence of some private information from the outset.⁴

3 Shuttle Diplomacy

As our interest is in understanding why mediators engage in *shuttle diplomacy*, we focus on a sequential procedure that exhibits features of what mediators do in practice. The mediator begins by posting the price p that the buyer would have to pay to the seller if the service is provided, or the asset traded. The mediator then approaches the parties repeatedly over time, making them discover the outcome of suitably designed private experiments. It is convenient in what follows to model the discovery process as taking

³If the parties’ values became known to the mediator, she could then just propose a mutually acceptable price at which the transaction should occur if it is efficient. Similarly efficiency would be easier to attain if parties could learn each other’s values.

⁴ In Section 4.3, we will also argue that our results extend to the case where values are correlated.

place in continuous time.

An Illustrative Example

To facilitate understanding of the procedure and the role played by each of its ingredients, we begin with an example. Suppose the values of the buyer B and the seller S are both uniformly distributed in $[0, 1]$. The mediator then posts the price $p = 0.5$. At time $t \in [0, 0.5]$ each party discovers whether her value is equal to t , to $1 - t$, or whether it belongs to the interval $(t, 1 - t)$. The information disclosure process is thus perfectly symmetric: at any date t , prior to the discoveries occurring in that period, the interval of remaining uncertainty is $[t, 1 - t]$ and the expected value is equal to 0.5 for both parties.

After the time t discovery, each party has the option to stop the discovery phase of the mediation procedure. If none stops, then the discovery phase continues. If the buyer stops, she then decides whether or not the service should be provided (equivalently, the asset traded) at price p . If the seller stops, the seller decides whether or not to provide the service (trade the asset) at price p . If both stop, one party is selected according to the outcome of a fair coin toss and this party then decides whether or not the service is provided (the asset traded) at price p . The decision of the party that stopped the discovery phase (or the randomly chosen one if both stopped) is then implemented and the procedure terminates.

Consider then the following *stop-at-value strategies*. For the buyer, the strategy is:

- Stop at t if and only if $v_B \in \{t, 1 - t\}$, i.e., the value has been discovered;
- If $v_B = t$, then choose No Service (equivalently, do not buy the asset);
- If $v_B = 1 - t$, then choose Service (equivalently, buy the asset) at price p .

Similarly, for the seller, the stop-at-value strategy is:

- Stop at t if and only if $v_S \in \{t, 1 - t\}$, i.e., the value has been discovered;
- If $v_S = t$, then choose Service (equivalently, sell the asset) at price p ;
- If $v_S = 1 - t$, then choose No Service (equivalently, do not sell the asset).

It is immediate to see that if both parties follow a stop-at-value strategy, then the service is provided if and only if it's efficient (i.e., $v_B \geq v_S$). We now argue that it is a perfect Bayesian equilibrium for both parties to follow such strategies.

If at t a party discovers her value, it is clearly optimal to follow the stop-at-value strategy, as it guarantees her the highest possible continuation payoff, independently of the other party's strategy. We then only need to consider the case in which a party does not discover her value at t . Note first that if the other party follows a stop-at-value strategy, by also following a stop-at-value strategy the party guarantees the service is provided if and only if it's efficient, as argued above. Consider then the buyer. At t , her expected value conditional on her value being higher than the seller's value is $\frac{1}{3}t + \frac{2}{3}(1-t) = \frac{2-t}{3}$. Hence, by not stopping and thus following the stop-at-value strategy the buyer obtains a positive payoff, as with a positive probability her value is higher than the seller's value, in that event her expected value is $\frac{2-t}{3}$, which is greater than 0.5 and the service is provided at a price $p = 0.5$. By stopping, on the other hand, she would get a zero payoff no matter whether the service is provided or not, as her expected value at t is $\frac{1}{2}t + \frac{1}{2}(1-t) = 0.5 = p$. Hence, it is a best response for the buyer to follow the stop-at-value strategy at t .

Now consider the seller. At t , her expected value conditional on it being higher than the buyer's value is also $\frac{2-t}{3}$. If she follows the stop at value strategy, she will provide the service at $p = 0.5$ whenever her value is lower than the buyer's, an event with probability 0.5. Thus by following the stop-at-value strategy the seller obtains a payoff equal to $\frac{1}{2} \cdot \frac{2-t}{3} + \frac{1}{2} \cdot 0.5$. This is greater than her payoff if she stops: if she stops she can either provide the service at $p = 0.5$, or not provide it and obtain an expected value (or cost saving) of 0.5. Hence, it is also a best response for the seller to follow the stop-at-value strategy at t .

Thus, in this example, the shuttle diplomacy procedure implements the ex-post efficient outcome. Also, at any date t , prior to the information discovery the two parties have the same information and their expected payoff from the procedure is also the same.

The Procedure

We now describe the shuttle diplomacy mediation procedure in full generality for the case of general value distributions. The first idea behind the construction of the procedure

is that at each point in time t parties should make symmetric discoveries, that reveal whether their values are or not at the boundaries of the current interval of remaining uncertainty $[\alpha_t, \beta_t]$. With symmetric discoveries the interval of uncertainty evolves in the same way for the two parties,⁵ as long as they make no discoveries. Still, with asymmetric value distributions, the distribution of values over this interval, the expected values and hence the extent of learning typically differ across the two parties. The second idea is that at each date t the interval of remaining uncertainty includes the price p posted at the outset by the mediator. As t approaches the terminal date T the size of this interval progressively shrinks and approaches zero. The third idea is that the posted price and the path of the intervals of remaining uncertainty needs to be chosen so that each party is incentivized to stop the discovery phase of mediation and make a service provision (or asset trading) decision if and only if she has discovered her value. Combined, we will show these three ingredients allow the implementation of the ex-post efficient outcome. Formally, the procedure is as follows:

PRELIMINARY STAGE . The mediator selects:

- A posted price $p \in (0, 1)$.
- The length of the discovery phase, T , and a collection of intervals of remaining uncertainty: (α_t, β_t) , $t \in [0, T]$, with: $\alpha_0 = 0$, $\beta_0 = 1$; $\alpha_T = \beta_T = p$; $\alpha_t \geq \alpha_{t'}$ and $\beta_t \leq \beta_{t'}$ if $t > t'$.

ROUND $t \in [0, T]$, DISCOVERY PHASE.

- First each party privately discovers whether her value is α_t , β_t , or belongs to the interval of remaining uncertainty (α_t, β_t) .
- Then each party decides whether to stop the discovery phase of mediation. If $t = T$ or at least one party decided to stop, then the discovery phase terminates and the procedure moves to the allocation phase.

ROUND $t \in [0, T]$, ALLOCATION PHASE.

⁵As the densities f_B, f_S could be zero in some interval, the distributions F_B and F_S may have different supports. Hence the interval of remaining uncertainty should be understood as including all - but possibly not only - the values which a party views as having positive density, given the available information.

- If only one party stopped, she becomes the deciding party. If both stopped, or $t = T$, then a fair coin is tossed to randomly select one of the two parties as the deciding party.
- The deciding party chooses whether or not the service should be provided (equivalently, whether or not the ownership of the asset should be transferred to the buyer). If the service is provided, then the buyer pays the posted price p to the seller.

The symmetry of the discovery process helps to ensure the efficiency of the outcome as well as, when value distributions are the same, its fairness. Note also that setting a constant price at which the service is provided, whenever the discovery phase is terminated, adds a robustness feature to the mechanism,⁶ since the price does not depend on the exact beliefs entertained by parties along the process. Furthermore, having a constant price simplifies the mechanism, lowering the computational burden of parties.

Stop-at-value Strategies

In the shuttle diplomacy procedure we have described, each party's strategy specifies whether or not to stop the discovery process at any date t , given the discoveries made at that date, and the decision after stopping the process. Under a stop-at-value strategy a party stops the procedure only if she just discovered her value and chooses then the allocation that gives her the highest payoff:

- A party stops the discovery phase of mediation at $t < T$ if and only if she has just discovered her value, which is then either $\alpha_t < p$ or $\beta_t > p$;
- If B is the deciding party at $t < T$, then she decides that the service is provided if her value is high, $v_B = \beta_t$, and that is not provided if it is low, $v_B = \alpha_t$;
- If S is the deciding party at $t < T$, then she decides that the service is provided if her value is low, $v_S = \alpha_t$, and that it is not provided if it is high, $v_S = \beta_t$;
- At $t = T$ the deciding party chooses that the service is not provided.⁷

⁶On robust mechanism design, see Bergemann and Morris (2005).

⁷If the discovery phase stops at T , then both parties have discovered that their value is equal to the posted price p and hence are indifferent about which decision is made. For completeness, we have selected that the choice is that the service is not provided.

If both parties follow the stop-at-value strategy the discovery process terminates each time a party discovers her value. If none of them has stopped the mechanism, then before the discovery at t they both know that their values lie in the interval $[\alpha_t, \beta_t]$; the interval $[\alpha_t, \beta_t]$ converges to $[\alpha_T, \beta_T] = [p, p]$.

Efficiency under stop-at-value strategies

If both parties use the stop-at-value strategy, then the outcome of the shuttle diplomacy mechanism is ex-post efficient. To see this, note first that if the discovery phase stops at $t = T$, the two parties have the same value $v_i = p$ and any decision is then efficient. Second, if party $i \in \{B, S\}$ stops at $t < T$ after having discovered that her value is α_t and she is the deciding party, then she will select to provide the service if $i = S$ and not to have it provided if $i = B$. Since the other party's value is in the interval $[\alpha_t, \beta_t]$, this decision is efficient. Third, if i stops at $t < T$ after having discovered that her value is β_t and she is the deciding party, then she will select to have the service provided if $i = B$ and not to provide it if $i = S$. Again, the other party's value is in the interval $[\alpha_t, \beta_t]$ and hence the decision is efficient.

Incentive compatibility under stop-at-value strategies

Note that, as there are no atoms in the value distribution, the probability that the two parties stop at the same time, when they both follow stop-at-value strategies, is zero.⁸ Hence a party who decides to stop expects she will decide the allocation with probability one.

We show first that when a party discovers her value at t , stopping the discovery process is a weakly dominant behavioural strategy. Consider the buyer; if she discovers that her value is $v_B = \alpha_t$, stopping the process and deciding the service is not provided gives her a payoff of zero. Any other choice could only lower her payoff, since it may happen the service is provided, in which case she has to pay a price p which is above her value. If the buyer discovers that her value is $v_B = \beta_t$, then stopping the discovery phase and deciding to have the service provided yields her a payoff of $\beta_t - p > 0$. Any other choice could only lower her payoff, since the price is constant and she may end up with the service not being provided in which case she gets a payoff of zero. Now consider the seller; if she discovers that her value is $v_S = \alpha_t$, then stopping the discovery phase and deciding to provide the

⁸Note that in that event the decision made by whichever party is randomly selected by the coin toss is efficient.

service to the buyer gives her a payoff of $p > \alpha_t$. Any other choice could only lower her payoff, since she might end up not providing a service that she values less than p . If the seller discovers that her value is $v_S = \beta_t$, stopping the discovery phase and deciding not to provide the service yields her a payoff of $\beta_t > p$. Any other choice could only lower her payoff, since if she ends up providing the service she gets a lower payoff of p .

Thus, to prove that both parties adopting the stop-at-value strategy constitutes a (perfect Bayesian) equilibrium of the mediation procedure, it only remains to show that for no party there exists a round $t < T$ at which she prefers to stop and end the procedure even though she has not discovered her value, assuming the other party follows the stop-at-value strategy.

Consider first the buyer. Suppose she has not yet discovered her value. If she deviates from the stop-at-value strategy and stops, then her continuation payoff is zero if she decides that the service is not provided by the seller, and it is $\mathbb{E}_{F_B}^{(\alpha_t, \beta_t)}[v_B] - p$ if she decides to have the service provided. If instead she follows the stop-at-value strategy, the buyer's continuation payoff is equal to the expected benefit of ultimately having the service provided at price p whenever provision is ex-post efficient. Letting $\Pr_{F_B, F_S}^{(\alpha_t, \beta_t)}(v_B > v_S)$ be the probability that the buyer's value is greater than the seller's value, conditional on both values being in the interval (α_t, β_t) , this continuation value is equal to:

$$\left(\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B > v_S] - p \right) \cdot \Pr_{F_B, F_S}^{(\alpha_t, \beta_t)}(v_B > v_S) \quad (1)$$

It thus follows that for the buyer to want to follow the stop-at-value strategy at t it is necessary and sufficient that the following incentive constraint holds:

$$\left(\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B > v_S] - p \right) \cdot \Pr_{F_B, F_S}^{(\alpha_t, \beta_t)}(v_B > v_S) \geq \max \left\{ 0, \mathbb{E}_{F_B}^{(\alpha_t, \beta_t)}[v_B] - p \right\} \quad (2)$$

Observe that⁹

$$\begin{aligned} \mathbb{E}_{F_B}^{(\alpha_t, \beta_t)}[v_B] - p &= \left(\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B > v_S] - p \right) \cdot \Pr_{F_B, F_S}^{(\alpha_t, \beta_t)}(v_B > v_S) \\ &\quad + \left(\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B < v_S] - p \right) \cdot \Pr_{F_B, F_S}^{(\alpha_t, \beta_t)}(v_B < v_S). \end{aligned}$$

⁹Note that the independence of the value distributions implies $\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B] = \mathbb{E}_{F_B}^{(\alpha_t, \beta_t)}[v_B]$.

Using this condition to rewrite (2), we obtain the following necessary and sufficient condition for the buyer to be willing to follow the stop-at-value strategy, for all $t \in [0, T]$:

$$\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_B | v_B < v_S] \leq p \leq \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_B | v_B > v_S]. \quad (3)$$

Now consider the seller. Suppose she has not discovered her value at t . If she deviates from the stop-at-value strategy and stops, her payoff is p if she decides to provide the service to the buyer, while if she decides not to provide the service her payoff is

$$\begin{aligned} \mathbb{E}_{F_S}^{(\alpha_t, \beta_t)} [v_S] &= \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S > v_B] \cdot \Pr(v_S > v_B) \\ &+ \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S < v_B] \cdot \Pr(v_S < v_B) \end{aligned}$$

If instead she follows the stop-at-value strategy, then her continuation payoff is:

$$\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S > v_B] \cdot \Pr(v_S > v_B) + p \cdot \Pr(v_S < v_B) \quad (4)$$

Hence, it is optimal for the seller to follow the stop-at-value strategy at t if and only if the following condition holds:

$$\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S > v_B] \cdot \Pr(v_S > v_B) + p \cdot \Pr(v_S < v_B) \geq \max \left\{ p, \mathbb{E}_{F_S}^{(\alpha_t, \beta_t)} [v_S] \right\}$$

or, equivalently

$$\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S < v_B] \leq p \leq \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_S > v_B] \quad (5)$$

We can thus conclude that if the incentive compatibility conditions (3) and (5) hold for all $t \in [0, T]$, then the shuttle diplomacy procedure implements an ex-post efficient allocation.

The Uniform Example Revisited

If F_B and F_S are uniform, then (3) and (5) reduce to the following, single condition:

$$\int_{\alpha_t}^{\beta_t} \frac{v(\beta_t - v)}{\int_{\alpha_t}^{\beta_t} (\beta_t - v) dv} dv \leq p \leq \int_{\alpha_t}^{\beta_t} \frac{v(v - \alpha_t)}{\int_{\alpha_t}^{\beta_t} (v - \alpha_t) dv} dv, \quad (6)$$

for all $t \in [0, T]$.

Proposition 1 below establishes that equal sized discovery intervals on each side of the price p , as in the earlier discussion of this example, satisfy the incentive constraints (6) for an interval of possible values of the price p . This interval includes the value $p = \frac{1}{2}$ considered earlier.

Proposition 1 *Suppose the prior value distributions F_B and F_S are uniform. Set $\alpha_t = \frac{t}{T}p$ and $\beta_t = 1 - \frac{t}{T}(1 - p)$. Then, the buyer and seller following a stop-at-value strategy is a perfect Bayesian equilibrium of the shuttle diplomacy mechanism (implementing an ex-post efficient outcome) if and only if the posted price p satisfies:*

$$\frac{1}{3} \leq p \leq \frac{2}{3}.$$

General Prior Distributions

In the case of general distributions F_B, F_S , we begin by establishing some preliminary results about the conditional expectations that appear in the incentive constraints (3) and (5). It is convenient here to use $i, j \in \{B, S\}$, with $j \neq i$, to denote the two parties.

The first claim is that, conditional on $v_j < v_i$ and both values belonging to the same interval (α, β) , the expectation of v_i is greater than the expectation of v_j .

Claim 1 *For all $(\alpha, \beta) \subseteq [0, 1]$ and $i, j \in \{B, S\}$, $i \neq j$,*

$$\mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_j \mid v_j < v_i] < \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_i \mid v_j < v_i]$$

The second claim states that the expectation of v_j conditional on $v_j < v_i$ is smaller than the expectation of v_j conditional on $v_j > v_i$.

Claim 2 *For all $(\alpha, \beta) \subseteq [0, 1]$ and $i, j \in \{B, S\}$, $i \neq j$,*

$$\mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_j \mid v_j < v_i] < \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_j \mid v_j > v_i]$$

The next claim is an important step towards the proof that the parameters of the shuttle diplomacy mechanism – intervals of uncertainty and posted price – can be selected so that the perfect Bayesian equilibrium outcome of the mechanism is ex-post efficient. The claim states that, for any given interval (α, β) , we can find some prices (which depend on the interval) which satisfy the incentive constraints (3) and (5) for $(\alpha_t, \beta_t) = (\alpha, \beta)$. Claim

3 follows directly from Claims 1 and 2, as they imply that each expectation appearing on the left hand side of the inequality in Claim 3 is smaller than each expectation on the right hand side.

Claim 3 For all $(\alpha, \beta) \subseteq [0, 1]$,

$$\max \left\{ \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_B | v_B < v_S], \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_S | v_S < v_B] \right\} < \min \left\{ \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_B | v_B > v_S], \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_S | v_S > v_B] \right\}$$

We are now ready to show that there exist a posted price under which the shuttle diplomacy mechanism satisfies the incentive compatibility constraints in all rounds for appropriately chosen intervals of uncertainty (α_t, β_t) , thus ensuring that the equilibrium outcome is ex-post efficient. By Claim 3, to satisfy incentive compatibility at the beginning of the shuttle diplomacy mechanism, when the interval of uncertainty (after the initial discoveries) is $(0, 1)$, the posted price p must satisfy the following condition:

$$\max \left\{ \mathbb{E}_{F_B, F_S}^{(0, 1)} [v_B | v_B < v_S], \mathbb{E}_{F_B, F_S}^{(0, 1)} [v_S | v_S < v_B] \right\} \leq p \leq \min \left\{ \mathbb{E}_{F_B, F_S}^{(0, 1)} [v_B | v_B > v_S], \mathbb{E}_{F_B, F_S}^{(0, 1)} [v_S | v_S > v_B] \right\} \quad (7)$$

Proposition 2 below shows that once the mediator has selected one such a posted price, she can always find intervals of remaining uncertainty (α_t, β_t) , of decreasing size, with $\alpha_0 = 0$, $\beta_0 = 1$, $\alpha_T = \beta_T = p$ (i.e., a discovery policy), such that the posted price p satisfies the incentive compatibility inequalities (3) and (5) for all intervals of uncertainty.

Proposition 2 Consider any prior value distributions F_B and F_S . For all posted prices satisfying (7) there exist an increasing function α_t , with $\alpha_0 = 0$ and $\alpha_T = p$, and a decreasing function β_t , with $\beta_0 = 1$ and $\beta_T = p$, defining the intervals of remaining uncertainty (α_t, β_t) , such that buyer and seller following the stop-at-value strategy is a perfect Bayesian equilibrium of the shuttle diplomacy procedure which implements an ex-post efficient outcome.

Note the result requires no condition on the distributions of values for seller and buyer, besides the assumed property that there are no atoms. Hence they may have different supports. The existence of an interval of prices satisfying (7) for which efficiency can be attained guarantees that the shuttle diplomacy procedure is robust to the presence of some initial, private information of parties about their values, as we will show in Section

4.2.

We now provide some further intuitive explanation for why a shuttle diplomacy procedure implements an efficient outcome. We may interpret stopping the discovery phase and deciding about service provision by a party as reporting her value. The stop-at-value strategy can thus be seen as truthful reporting and the procedure as a dynamic direct mechanism with slow information discovery. Limiting the parties to small information discoveries at each point in time and stopping the discovery of a party when the other party stops the procedure after having discovered her value reduces their misreporting opportunities, by limiting the lies available. The payoff from terminating the mediation process, as determined by the design of the allocation phase and the constant price, also contributes to slacken their incentive constraints. At the same time, the shuttle diplomacy procedure guarantees that parties have enough information to assess whether $v_B > v_S$ or $v_B < v_S$ and hence whether the provision of the service is or is not ex-post efficient. This is because the party making the discovery learns her value is one of the extremes of the interval of remaining uncertainty while the other party only knows her value lies in that interval.

Intuitively speaking, participating in shuttle diplomacy requires little strategic sophistication from buyer and seller. Parties can just follow the stop-at value strategy and stop the discovery phase when they discover their value. Greater sophistication is required from the mediator, who must compute the “right” posted price, but note that if parties naively follow the stop-at-value strategy, any choice of a posted price will induce an efficient outcome. Of course, sophisticated parties would deviate from the stop-at-value strategy if the price is not in the right interval.

We should point out one other feature of the shuttle diplomacy procedure as described. When a party stops the discovery process, she decides the allocation, that is whether or not the service is provided at the price p . The other party must comply with this. The fact that she chose not to stop the procedure at earlier dates means her expected payoff from its continuation was positive. When the process is stopped however her payoff, evaluated on the basis of the information available to her at that moment, could end up being negative if the decision is that the service is provided. Consider the case where the buyer discovers her value is β_t and chooses that the service is provided. In that case the seller’s payoff, conditional on her available information, is $p - \mathbb{E}_{F_S}^{(\alpha_t, \beta_t)}[v_S]$. A similar situation

may arise for the buyer, in the symmetric case where the seller discovers her value is α_t and the service is provided, in exchange for the payment p by the buyer. When value distributions are symmetric the price p can always be set at a value such that not only the incentive constraints are satisfied, (7), but in addition these payoffs are non negative.¹⁰ This cannot be ensured when value distributions are asymmetric. Hence in such cases the procedure described requires some commitment to the mechanism from the parties. In the Section 5.1 we present an alternative specification of a shuttle diplomacy procedure where parties along the process trade back and forth the asset (or claims to the service being provided). This procedure also allows to attain (approximate) ex post efficiency and the expected payoff from such trades, at any point in time, including the one where the process terminates, is non negative.

4 Extensions

4.1 Fair Price

Define the fair price at each t as follows:

$$p_t^F = \frac{1}{2} \cdot \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_B | v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)} [v_S | v_B > v_S], \quad (8)$$

At this price the expected payoffs of the two parties from the shuttle diplomacy procedure at time t , conditionally on the information available to them at that date (prior to discoveries occurring in that period), are equal. This price ensures that the expected surplus generated by providing the service, or trading the asset, whenever that is ex post efficient, is equally split among the two parties. The question is, can the shuttle diplomacy procedure be designed so that the posted price p is equal to the fair price p_t^F specified in (8) in each round t ? For general value distributions, the answer is No. There are two reasons.

First, at the beginning of the procedure fairness requires the posted price to equal

$$p_1^F = \frac{1}{2} \cdot \mathbb{E}_{F_B, F_S}^{(0,1)} [v_B | v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F_B, F_S}^{(0,1)} [v_S | v_S < v_B],$$

¹⁰This is an immediate implication of the fairness property of the procedure we establish in the next Section 4.1.

but for some distributions F_B, F_S this value of p_1^F lies outside the interval defined by (7) and thus does not satisfy the incentive constraints for the stop-at-value strategies to be an equilibrium. To illustrate this possibility, consider the case where F_B has almost all mass concentrated in a small interval around $v_B = \frac{1}{2}$ and F_S has almost all mass equally concentrated in two small intervals around $v_S = 0$ and $v_S = 1$. Then:

$$\mathbb{E}_{F_B, F_S}^{(0,1)}[v_B | v_B < v_S] \approx \mathbb{E}_{F_B, F_S}^{(0,1)}[v_B | v_B > v_S] \approx \frac{1}{2};$$

$$\mathbb{E}_{F_B, F_S}^{(0,1)}[v_S | v_S < v_B] \approx 0 \quad \text{and} \quad \mathbb{E}_{F_B, F_S}^{(0,1)}[v_S | v_S > v_B] \approx 1.$$

The fair price at $t = 1$ is then $p_1^F \approx \frac{1}{4}$ and it violates condition (7), which in this case requires that $p \approx \frac{1}{2}$.

Second, if p_1^F belongs to the interval defined by (7), by Proposition 2 we can set $p = p_1^F$ and construct a path of intervals of remaining uncertainty such that p_1^F satisfies the incentive compatibility conditions at all dates, inducing buyer and seller to follow the stop-at-value strategy. However, we cannot be sure that the intervals of uncertainty are such that $p_1^F = p_t^F$ at all rounds; fairness may then be violated at some $t > 0$.

An important special case where we can set the posted price p equal to the initial fair price p_1^F and make sure that it is also a fair price at all $t \in [0, T]$ is the symmetric environment, where the prior distributions of valuations are the same for the two parties; that is, when $F_B = F_S = F$. This is because with identical distributions the two expectations on the left hand side and the two on the right hand side of inequality (7) coincide; that is, $\mathbb{E}_{F, F}^{(0,1)}[v_S | v_S < v_B] = \mathbb{E}_{F, F}^{(0,1)}[v_B | v_B < v_S]$ and $\mathbb{E}_{F, F}^{(0,1)}[v_S | v_S > v_B] = \mathbb{E}_{F, F}^{(0,1)}[v_B | v_B > v_S]$. Furthermore, we have:

$$\begin{aligned} p_1^F &= \frac{1}{2} \cdot \mathbb{E}_{F, F}^{(0,1)}[v_B | v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F, F}^{(0,1)}[v_S | v_B > v_S] \\ &= \mathbb{E}_{F, F}^{(0,1)}[v_B | v_B > v_S] \cdot \Pr\{v_B > v_S\} + \mathbb{E}_{F, F}^{(0,1)}[v_S | v_S > v_B] \cdot \Pr\{v_S > v_B\} \\ &= \mathbb{E}_F^{(0,1)}[v]. \end{aligned}$$

Since: $\mathbb{E}_{F, F}^{(0,1)}[v_B | v_B < v_S] < \mathbb{E}_F^{(0,1)}[v_B] < \mathbb{E}_{F, F}^{(0,1)}[v_B | v_B > v_S]$, by setting $p = p_1^F$ condition (7) is satisfied. Moreover, the intervals of uncertainty (α_t, β_t) can be chosen so that, for all rounds t , we have $\mathbb{E}_F^{(\alpha_t, \beta_t)}[v] = \mathbb{E}_F[v]$. In particular, if F is uniform, then the fair price is $p_1^F = \frac{1}{2}$.

Generally speaking, what makes it impossible to set the posted price equal to the fair

price is the presence of large asymmetries in the prior distributions of buyer and seller. We formalize the result for symmetric prior uncertainty in the following proposition.

Proposition 3 *With symmetric prior uncertainty, i.e., $F_B = F_S = F$, there always exist intervals of remaining uncertainty defined by an increasing function α_t , with $\alpha_0 = 0$, $\alpha_T = p_1^F$, and a decreasing function β_t , with $\beta_0 = 1$, $\beta_T = p_1^F$, such that: (i) the posted price $p_1^F = \mathbb{E}_F^{(0,1)}[v]$ is fair, and (ii) it is a perfect Bayesian equilibrium of the shuttle diplomacy mechanism for buyer and seller to follow the stop-at-value strategy.*

4.2 Private Information before Mediation

We have assumed so far that the two parties have no private information before the start of the mediation procedure, but in some practical disputes they are likely to have some partial information at the outset. It has been well known since Myerson and Satterthwaite (1983) that if parties have full private information about their values, then no mechanism can implement the ex-post efficient outcome. An important question is thus whether the shuttle diplomacy procedure we considered is robust to the presence of some amount of private information. The answer is yes.

Suppose that, before the start of the procedure, the buyer privately observes a signal $\theta_B \in [0, 1]$ about her own value and the seller a signal $\theta_S \in [0, 1]$ about her value. Then let $F_B(\cdot | \theta_B)$ be the distribution of the buyer's value v_B conditional on the buyer having received private signal θ_B . Similarly, let $F_S(\cdot | \theta_S)$ be the distribution of the seller's value v_S conditional on the seller having received private signal θ_S . A special case is the situation studied so far, where the distributions of values are independent from the signals θ_B, θ_S . At the other extreme is the case where signals are fully informative, with the distributions putting all mass on v_i being equal to θ_i , $i \in \{B, S\}$.

Following the same approach as the one followed above for the case of no private information, we can derive the incentive constraints that guarantee that the stop-at-value strategies are an equilibrium when parties have private information. They are analogous to constraints (3) and (5) obtained for the case of no private information. Let $\mathbb{E}_{F_B(\cdot | \theta_B), F_S}^{(\alpha, \beta)} [v_B | v_B < v_S; \theta_B]$ be the expectation of v_B , conditional on v_B, v_S being in the interval (α, β) , on $v_B < v_S$, and also on the signal realization of the buyer being θ_B . Similarly specify $\mathbb{E}_{F_B, F_S(\cdot | \theta_S)}^{(\alpha, \beta)} [v_S | v_S > v_B; \theta_S]$ and the analogous expectations conditional

on $v_B > v_S$. The incentive constraints can then be rewritten as:

$$\mathbb{E}_{F_B(\cdot|\theta_B), F_S}^{(\alpha_t, \beta_t)} [v_B | v_B < v_S; \theta_B] < p < \mathbb{E}_{F_B(\cdot|\theta_B), F_S}^{(\alpha_t, \beta_t)} [v_B | v_B > v_S; \theta_B] \quad (9)$$

$$\mathbb{E}_{F_B, F_S(\cdot|\theta_S)}^{(\alpha_t, \beta_t)} [v_S | v_S < v_B; \theta_S] < p < \mathbb{E}_{F_B, F_S(\cdot|\theta_S)}^{(\alpha_t, \beta_t)} [v_S | v_S > v_B; \theta_S] \quad (10)$$

and must hold for all $t \in [0, T]$ and all $\theta_B, \theta_S \in [0, 1]$.

It is immediate to see that Claim 2 generalizes to this environment in the sense that, for all $\theta_j, i, j \in \{B, S\}$, it is the case that:

$$\mathbb{E}_{F_j(\cdot|\theta_j), F_i}^{(\alpha, \beta)} [v_j | v_j < v_i; \theta_j] < \mathbb{E}_{F_j(\cdot|\theta_j), F_i}^{(\alpha, \beta)} [v_j | v_j > v_i; \theta_j].$$

Claim 1 (and hence Claim 3), on the other hand, need not generalize; that is, it needs not be the case that for all $\theta_j, \theta_i, i, j \in \{B, S\}$:

$$\mathbb{E}_{F_j(\cdot|\theta_j), F_i}^{(\alpha, \beta)} [v_j | v_j < v_i; \theta_j] < \mathbb{E}_{F_j, F_i(\cdot|\theta_i)}^{(\alpha, \beta)} [v_i | v_j < v_i; \theta_i].$$

The reason is that the conditional distributions used to evaluate the expectation on the right and the left hand side of the above inequality are different. In general, Claim 1 fails to generalize if the influence of private information is “strong”. For example, in the extreme case where the signals received by each party are fully informative about their values (i.e., $v_i = \theta_i, i \in \{B, S\}$), the inequality above becomes $\theta_j < \theta_i$, which cannot hold for all $\theta_i, \theta_j \in [0, 1]$. More generally, when the conditional distributions are quite sensitive to the signal realizations the incentive constraints cannot be satisfied by some price p for all θ_B, θ_S . On the contrary, if the influence of private information is “not too strong”, then the conditional expectations do not vary much with θ_B and θ_S and we can find a value of p such that the incentive constraints are satisfied for all θ_B and θ_S . In such a case the result in Proposition 2 extends to this environment with private information; it is possible to find a discovery policy (i.e., intervals of uncertainty) under which the shuttle diplomacy mechanism implements an efficient outcome.

Consider for example the case in which $v_i = (1 - \lambda)\omega_i + \lambda\theta_i$, with $\lambda \in [0, 1]$, ω_i and θ_i uniformly distributed in $[0, 1]$, with ω_i initially unknown and θ_i known by party $i \in \{B, S\}$. Letting U_i be the (uniform) distribution of ω_i and F_i be the distribution of $v_i = (1 - \lambda)\omega_i + \lambda\theta_i$, the incentive constraints (9) and (10) reduce to the following condition, which must hold for any $i \neq j \in \{B, S\}$ and for all θ_i, θ_j :

$$\lambda\theta_i+(1-\lambda)\mathbb{E}_{U_i,F_j}^{(\alpha_t,\beta_t)}[\omega_i|\lambda\theta_i+(1-\lambda)\omega_i < v_j] < p < \lambda\theta_j+(1-\lambda)\mathbb{E}_{U_j,F_i}^{(\alpha_t,\beta_t)}[\omega_j|\lambda\theta_j+(1-\lambda)\omega_j > v_i]$$

Taking $\theta_i = 1$ and $\theta_j = 0$, the condition reduces to

$$\lambda + (1 - \lambda)\mathbb{E}_{U_i,F_j}^{(\alpha_t,\beta_t)}[\omega_i|\lambda + (1 - \lambda)\omega_i < v_j] < p < (1 - \lambda)\mathbb{E}_{U_j,F_i}^{(\alpha_t,\beta_t)}[\omega_j|(1 - \lambda)\omega_j > v_i],$$

which can be satisfied if and only if λ is below some upper bound λ^* .

4.3 Correlation of Values

We have assumed that the buyer and seller's values are independently drawn, but in several applications it seems reasonable to allow them to be correlated. For example, when applying the model to litigation, part of the plaintiff's cost of selling a legal claim (i.e., settling) and part of the defendant's value of buying the legal claim could be correlated, as both the defendant's and the plaintiff's benefit of reaching an agreement relative to going to court depend on the settlement decision that the court will impose if mediation fails. The court's decision in turn depends on characteristics of the case which are relevant for both parties and over which an expert mediator may shed light.

One could then assume that the values of buyer and seller are drawn from an atomless joint distribution $F(v_B, v_S)$ on $[0, 1]^2$ with atomless conditional distributions $F(v_B|v_S = v)$ and $F(v_S|v_B = v)$ with support $[0, 1]$ and non negative conditional densities, for all $v \in [0, 1]$. It is then possible to show that Claims 1–3 still hold for this joint distribution $F(v_B, v_S)$ and then Proposition 2 holds as well. Thus, our results extend to the case of correlated values and, in particular, the ex-post efficient outcome can be implemented without resorting to the side-betting mechanisms of Crémer and McLean (1988).

4.4 Outside Options

If the parties refuse to enter the mediation process, we have assumed that there is no service provision (equivalently, there is no trade and the seller keeps the asset). We find this assumption natural in the case of service provision. It also has the advantage of stacking the deck against shuttle diplomacy; it is well known in the mechanism design literature that asymmetric ownership claims of an asset make it more difficult to achieve ex post efficiency (e.g., see Cramton et al., 1987, and Makowsky and Mezzetti, 1993,

1994). In many instances, of course, the alternative to mediation is to go to trial and a trial judge may impose a different outcome than the seller keeping the asset, or the service not being provided. While considering alternative outside options might be interesting, we argue that they will not be an obstacle to shuttle diplomacy being able to achieve an ex-post efficiency outcome. Suppose for example that if the parties reject mediation and go to trial, then the court sets a price and leaves the parties to decide whether they want to trade or have the service provided. It is immediate to see that in such a case, as long as the mediator posts a price equal to the price chosen by the court, the parties would prefer to continue using the shuttle diplomacy procedure run by the mediator, as that procedure guarantees efficient trade and makes both parties better off.

5 Other Mechanisms Achieving Efficiency

5.1 Back-and-forth Diplomacy

In the shuttle diplomacy mechanism parties make discoveries and trading decisions in continuous time. We show next that a discrete-time, back-and-forth version of the procedure also implements, albeit only “approximately”, the efficient outcome. The key differences of the discrete-time version of shuttle diplomacy are that the mediator approaches the parties sequentially and that in each round, as long as the procedure is not terminated, the asset, or the right to decide provision of the service, is transferred back and forth between the two parties,¹¹ always at the posted price p . The mediator interacts with one party at a time, who makes a discovery. She then alternates between the two parties, in a sequence of T rounds. Each round $t \in \{1, \dots, T\}$ has two steps that are designed so that discoveries in each round, though asynchronous, are symmetric: at the beginning of round t the interval of remaining uncertainty is $[\alpha_{t-1}, \beta_{t-1}]$ for both parties, with $\alpha_0 = 0$, $\beta_0 = 1$ $\alpha_T = \beta_T = p$, α_t increasing and β_t decreasing, as in the continuous time procedure.

In the first step of each round t , B learns whether her value is in the lower end of the interval of remaining uncertainty, that is in $I_t^L = [\alpha_{t-1}, \alpha_t)$, and decides then whether or not to stop the procedure. If she stops the process ends and she walks away without the

¹¹Since trading back and forth ownership claims or decisions rights appear more natural in the case of the dispute regarding the ownership of the asset, we chose to phrase the presentation of the mechanism here with regard to this second application.

asset. If B doesn't stop, it is then S 's turn to learn whether her value is in the upper end of the interval of remaining uncertainty, $I_t^R = (\beta_t, \beta_{t-1})$, and to decide whether to stop the procedure, in which case she retains the asset. If S also decides not to stop in step 1, then the ownership claim to the asset is transferred to B in exchange of the payment of the price p and the procedure moves to step 2 of round t . In step 2, S learns whether her value is in I_t^L and decides whether she wants to stop the procedure. If S doesn't stop, then B learns whether her value is in I_t^R and decides whether to stop mediation. If any of them stops, the ownership of the asset remains with B . If instead none stops in step 2, then the ownership claim to the asset is transferred back to S for a payment of the price p and the procedure moves to round $t + 1$. When one party stops, the resulting allocation is final.

The procedure is described formally in Appendix B. There we show that if the parties adopt the stop-at-value strategy whereby mediation is stopped only when they discover their value, as the number of rounds T increases and the discovery intervals $I_t^L = [\alpha_{t-1}, \alpha_t)$ and $I_t^R = (\beta_t, \beta_{t-1})$ shrink, the outcome implemented is “approximately” efficient. This is because an inefficient allocation may only result when a party stops mediation after discovering that her value belongs to I_t^L or I_t^R , and the other party's value happens to be in the very same interval. Such inefficiency loss is bounded by the size of the discovery intervals I_t^L and I_t^R and converges to zero as T grows large and I_t^L and I_t^R shrink. We also show that we can always find a value of p , a sufficiently large T and a sequence of small discovery intervals I_t^L and I_t^R such that the two parties following stop-at-value strategies is a perfect Bayesian equilibrium of the diplomacy procedure. Formally:

Proposition 4 *For all posted prices satisfying (7)¹² there exist a T^* such that for all $T > T^*$ there are an increasing sequence $\{\alpha_t\}_{t=0}^T$, with $\alpha_0 = 0$ and $\alpha_T = p$, and a decreasing sequence $\{\beta_t\}_{t=0}^T$, with $\beta_0 = 1$ and $\beta_T = p$, such that buyer and seller following the stop-at-value strategy is a perfect Bayesian equilibrium of the back-and-forth diplomacy procedure which implements an approximately ex-post efficient outcome.*

Trading back and forth the claims to the asset and the asynchronous discoveries within a round are needed to incentivize parties to stop mediation when they discover their value. Note that if both parties follow the stop-at-value strategy, then back and forth trade only

¹²More precisely, the inequality in (7) must hold strictly.

stops when one party has approximately discovered her value. It is important to highlight that in this procedure, when a party stops the discovery process she decides that the allocation resulting from the previous sequences of transactions is final, no further exchanges occur. Hence in contrast from the shuttle diplomacy procedure considered in Section 3 the other party does not have to carry out actions, as transferring the asset or providing the service, which may give her a negative expected payoff, given her information.

Consider the case where the original buyer stops the procedure when owning the asset, and keeps it. Recall this happens when the original buyer discovers that her value is above the posted price p . As argued at the end of Section 3, in this case it is possible, if value distributions are asymmetric, that the price p is lower than the current expected value of the original seller. The original seller would not want to sell the asset at this price if she still owned it. However trade of a claim on the asset took place at the end of the previous step, when no party had discovered her value and the seller decided to sell based on the expectation that she would buy the asset back if her value were higher than the original buyer's value. Given the allocation of the claim to the asset as determined by past trades, the payoff of the original seller when the buyer stops is thus equal to zero, individual rationality is so satisfied. The situation in the case where the original seller stops the procedure after having relinquished the asset and hence the original buyer keeps it is perfectly symmetric.

Without a sequence of back and forth trades of the asset, the same ultimate transfer of the asset to the party with the higher value could only be achieved if parties committed in advance to the trading outcome of the procedure, irrespective of whether, once they obtain the information along the process, it is individually rational for them to do so. As we noticed this is the case for the continuous time shuttle diplomacy procedure we considered before. We can argue that such commitment (and hence the continuous-time procedure) is more natural when the dispute is about service provision. Back-and-forth trading of ownership claims and hence back-and-forth diplomacy appears more natural when the dispute is about asset ownership.

5.2 Single-Round Diplomacy

We now show that a single round of diplomacy is sufficient to guarantee the achievement of an ex-post efficient outcome. This however comes at the cost of giving all the surplus

generated by the allocation to a single party and it is not robust to the introduction of a small amount of private information from the outset.

In *single-round diplomacy* parties make a single discovery and do so in sequence. Differently from shuttle diplomacy, the discovery process is thus not gradual, but the mediator asks the first party who makes a discovery to send her a (private) report about what she learnt. Then the mediator, on the basis of the report received, determines the discovery the second party makes and sets the price p at which the two parties can trade at the end of the discovery phase.

More precisely, the mediator designs an experiment which fully informs the first party about her value. In what follows, we consider the case in which the party informed first is the seller; the analysis when the buyer is first informed is analogous. Let \hat{v}_S be the value reported by the seller. The mediator then lets the buyer only observe whether her value is above or below \hat{v}_S and sets the posted price $p(\hat{v}_S)$. After the buyer also has made her discovery, both parties are asked if they wish to trade the asset (or have the service provided) at $p(\hat{v}_S)$. Trade takes place if and only if both buyer and seller accept to trade at the posted price. The procedure then ends in any case.

It is clear that if the seller sends a truthful report about her value and the price satisfies the inequalities $\hat{v}_S \leq p(\hat{v}_S) \leq \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq \hat{v}_S]$, both parties find it optimal to trade if and only if the buyer's value is above the seller's reported value. In that case this procedure also implements an ex-post efficient outcome.

The next proposition shows that there exists a unique price function that satisfies the above inequalities and incentivizes the seller to report truthfully. The price function gives all the surplus from trade to the seller, the party that is fully informed. Intuitively, if the seller's report is truthful it is clearly a strictly dominant continuation strategy for her to accept to trade at the posted price. The buyer is also willing to trade at this price when her value is above the seller's, as in that case she obtains the same payoff (equal to zero) both when trade occurs and when it does not occur. Thus the seller appropriates all the gains from trade and we show it is always optimal for her to report sincerely.

Proposition 5 *Under single-round diplomacy, with the seller discovering her value first and the buyer discovering whether her value is above or below the value \hat{v}_S reported by the seller, $p_S(\hat{v}_S) = \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq \hat{v}_S]$ is the unique price function for which it is a perfect Bayesian equilibrium for the seller to report sincerely her value and for buyer and seller*

to accept to trade at the posted price if and only if it is ex-post efficient to trade.

Two features of single-round diplomacy allow the implementation of an ex-post efficient outcome. *The first is that it limits the information that is made available to parties, and by so doing it reduces their misreporting opportunities, slackening the incentive constraints.* This is similar to what we already saw in the case of shuttle diplomacy. However the non gradual nature of the discovery process implies that the party who ends up not discovering her true value now learns less, in fact she only obtains the minimal information that is needed to assess whether trading is efficient. *The second feature is that the provision of this minimal information is implemented by conditioning the discovery made by the second party on the value reported by the first one.* This conditioning introduces correlation between the information available to the two parties, which is exploited by the shuttle diplomacy procedure to achieve first best efficiency. This correlation is endogenously determined by the first party's report. The situation is then quite different from the one arising with exogenous correlation among the different buyers' values exploited by Crémer and McLean (1988) to construct side-betting, Bayesian, trading mechanisms in which the seller extracts the full surplus.

The outcome of single-round shuttle diplomacy, while allowing to achieve surplus from the allocation, is extremely lopsided in its distribution. One simple adjustment of the procedure to ensure a more equitable outcome is to flip a coin to determine which of the two parties is the one to discover her value fully, and thus to extract all social value. While this procedure would guarantee an equal division of the surplus ex-ante, the extreme lack of fairness ex-post remains. It also exposes parties to maximum uncertainty about the trading price and their payoffs.

The single-round diplomacy procedure has another important drawback: the result that it implements an efficient outcome is not robust to the introduction of a small amount of private information that parties may have about their own value before the start of the mediation procedure. As we saw above, the need to induce the seller to report her true value v_S after she has discovered it uniquely pins down the trading price. This must be set so that the seller extracts all expected surplus from trade: that is, it must be that $p(v_S) = \mathbb{E}_{F_B}^{[0,1]} [v_B | v_B > v_S]$. But a buyer who receives an even very noisy private signal at the outset, whenever the signal realization θ_B is such that $\mathbb{E}_{F_B(\cdot|\theta_B)}^{[0,1]} [v_B | v_B > v_S; \theta_B] < \mathbb{E}_{F_B}^{[0,1]} [v_B | v_B > v_S]$ will not want to trade at $p(v_S)$; hence the ex-post efficient outcome can

no longer be implemented by a single-round procedure. Contrary to the case of shuttle diplomacy, the implementation of an efficient outcome under single-round diplomacy is not robust to the introduction of even a small amount of private information prior to the start of the mediation process.

5.3 Partnership Formation and Dissolution

When the dispute is over the provision of a service, it is not natural to think of the two parties as possible partners with joint ownership rights. On the other hand, when the dispute is over asset ownership, forming and dissolving a partnership might be an available option. It is known from Cramton, Gibbons and Klemperer (1987) (henceforth CGK) that even with fully informed parties ex-post efficient trade might be possible if the two parties are partners that initially jointly own the asset.

In this section, we consider a *partnership-formation-and-dissolution diplomacy* procedure that could be adopted by the mediator if joint ownership of a disputed asset were feasible. As CGK, we consider the symmetric case in which the prior distributions of B and S are the same, $F_B = F_S = F$.¹³ According to such procedure, parties first form a partnership by transferring a share $s = 1/2$ of the asset from the seller to the buyer in exchange for a payment p . Then the mediator privately discloses the true values to each party and the parties participate in the Bayesian mechanism with transfers described in CGK to dissolve the partnership; that is, to allocate the asset to the party with the highest value.

The Bayesian mechanism with transfers that implements an ex-post efficient dissolution of the partnership when parties are fully informed and have equal shares of the asset is a bidding game in which the parties bid for the share $s = 1/2$ of the other party. The highest bidder obtains the asset and pays a transfer to the other party equal to the difference between the highest and the lowest bid. CGK show that the Bayesian equilibrium bid β of a party with value v_i is $\beta(v_i) = \int_0^{v_i} u dF(u)$.

It remains to show that it is possible to find a price p at which B and S are willing to form an equal share partnership when they do not have any information about the disputed asset's value. If they form the partnership, given the symmetry of the value distributions and ownership shares, the expected payoff of each party is equal to half the

¹³ See however footnote 14.

maximal value of the asset across the parties, $\mathbb{E}_{F,F}[\max\{v_B, v_S\}]/2$. Thus for B to be willing to pay a price p to acquire a share of $1/2$ in the partnership, it must be that:

$$\frac{1}{2} \cdot \mathbb{E}_{F,F}[\max\{v_B, v_S\}] - p \geq 0.$$

By not selling a share $s = 1/2$ and keeping full ownership of the asset (i.e., not participating in the partnership-formation-and-dissolution diplomacy procedure), S obtains a payoff of $\mathbb{E}_F[v_S]$. Hence, for S to be willing to form a partnership with equal ownership shares, the price p she receives must satisfy:

$$p + \frac{1}{2} \cdot \mathbb{E}_{F,F}[\max\{v_B, v_S\}] \geq \mathbb{E}_F[v_S].$$

Since $\mathbb{E}_{F,F}[\max\{v_B, v_S\}] > \mathbb{E}_F[v_S]$, it follows that such a price can be found and the partnership-formation-and-dissolution diplomacy procedure implements an ex-post efficient outcome.

We remark that the equilibrium in the partnership-formation-and-dissolution procedure requires strategic sophistications from buyer and seller, as they must compute the correct bidding function of the other party (and their own). In contrast, as we argued before, little strategic sophistication is required from parties in the shuttle diplomacy and back-and-forth diplomacy procedures.¹⁴

¹⁴ Figueroa and Skreta (2012) have shown that there are ownership structures that guarantee that ex-post efficiency is achievable when parties are fully informed about their values even if the prior value distributions of B and S are different. However, such ownership structures depend subtly on the value distributions and might be extremely unequal. In addition, the mechanism implementing ex-post efficiency needs not be a simple bidding game as in the case of symmetric distributions. Let r_i be the ownership share of party $i \in \{B, S\}$ that guarantees that ex-post efficiency is possible when parties are fully informed, and let $V_i(r_i)$ be the ex-ante expected value of party i in the mechanism implementing ex-post efficiency. Let p be the price that B pays to acquire ownership share r_B from S at the ex-ante stage, before knowing her value. It must be $p \leq V_B(r_B)$ for the buyer to want to buy the share r_B and $\mathbb{E}[v_S] \leq p + V_S(1 - r_B)$ for the seller to want to sell it before the mediator has fully informed the parties about their values. Since $\mathbb{E}_{F_B, F_S}[\max\{v_B, v_S\}] = V_B(r_B) + V_S(1 - r_B)$, this shows that by selecting a price p such that $V_B(r_B) - (\mathbb{E}_{F_B, F_S}[\max\{v_B, v_S\}] - \mathbb{E}[v_S]) \leq p \leq V_B(r_B)$, the mediator may use a generalized version of the partnership-formation-and-dissolution procedure to implement the ex-post efficient outcome, even when value distributions are asymmetric. Such procedure however requires even more sophistication from parties than in the symmetric case.

6 Impossibility of Efficiency with Static Mechanisms

We now examine the case in which the information discovery is static; that is, parties make only a single discovery and do so simultaneously. Ex-post efficiency cannot be attained in this case. This is true even if no restriction is imposed on the allocation mechanism, that is, if we allow for arbitrary Bayesian mechanisms where the posted price depends on the parties reports of the information they independently discovered. The reason is that, when the parties' signal distributions are chosen simultaneously, to ensure that the transaction is carried out if and only if doing so is efficient the parties must essentially be fully informed, in which case, as is well known (Myerson and Satterthwaite, 1983), ex-post efficiency is not attainable.

Nevertheless, it is still interesting to see what we can obtain with a static procedure. Characterizing the optimal static Bayesian procedure, where the price p is set after the parties' discoveries, on the basis of the reports sent by them, is a difficult task. Schottmüller (2023) provides a closed form solution for the case in which the prior type distribution is binary. For the case of continuous distribution studied in this paper, he shows that the optimal information structure can be approximated arbitrarily closely by a finite monotone partition of the type space and a deterministic mechanism.

To facilitate comparison with shuttle diplomacy, here we limit ourselves to consider static mechanisms in which the mediator posts a price before the parties make their discoveries. The mediator simultaneously chooses both the distribution of the signals independently received by buyer and seller and the posted price p . Buyer and seller, after observing the realization of their own signal, decide whether or not they are willing to trade the asset (or have the service provided) at p . Trade only takes place if both are willing. From the point of view of practical application in mediation, an advantage of such static mechanisms is that the choice of whether to trade by a party is not subject to strategic uncertainty. It is a dominant strategy for a party to want to trade if and only if doing so would increase her payoff.

To find the optimal static mechanism we must consider all possible (static) discovery policies. They can be summarized by the induced posterior probability distributions of the expected value of the asset for each party. From the point of view of the mediator (and the other party), party i not discovering any information corresponds to the posterior

distribution of i 's expected value having a unit mass atom on $\mathbb{E}_{F_i}[v_i]$, while i having discovered her value corresponds to the posterior distribution of the expected value being F_i , the true distribution from which the value is drawn. Intermediate discovery policies must lead to distributions \tilde{F} that are mean preserving spreads of the distribution with unit mass on $\mathbb{E}_{F_i}[v_i]$ and such that F_i is a mean preserving spread of \tilde{F} . Thus, for party $i \in \{B, S\}$, the family of signal distributions over her expected value that can be feasibly induced by the mediator is:¹⁵

$$\mathcal{F}_i = \left\{ \tilde{F} : \int_0^1 v d\tilde{F}(v) = \mathbb{E}_{F_i}^{[0,1]}[v_i] \text{ and } \int_0^z \tilde{F}(v) dv \leq \int_0^z F_i(v) dv \text{ for all } z \in [0, 1] \right\}.$$

We can now state formally the mediator's problem in the present environment. The mediator chooses a *static discovery and trading mechanism* $\langle \tilde{F}_B, \tilde{F}_S, p \rangle$; that is, signal distributions $\tilde{F}_B \in \mathcal{F}_B$ and $\tilde{F}_S \in \mathcal{F}_S$ for B and S , together with a trading price $p \in \mathbb{R}_+$. After they receive their signals from \tilde{F}_B and \tilde{F}_S , respectively, each party decides whether or not to trade at p . It is immediate to verify that it is a dominant strategy for the buyer to accept to trade if and only if the signal is an expected value $v_B \geq p$ and for the seller to do so if and only if the signal received is $v_S \leq p$. Thus, since the mediator's goal is to maximize the social value, the optimal static discovery and trading mechanism is obtained as a solution of the following problem:

$$\max_{p \in [0,1], \tilde{F}_B \in \mathcal{F}_B, \tilde{F}_S \in \mathcal{F}_S} \int_p^1 \int_0^p (v_B - v_S) d\tilde{F}_S(v_S) d\tilde{F}_B(v_B)$$

or, equivalently:

$$\max_{p \in [0,1], \tilde{F}_B \in \mathcal{F}_B, \tilde{F}_S \in \mathcal{F}_S} \left(\mathbb{E}_{\tilde{F}_B}^{[0,1]} [v_B | v_B \geq p] - \mathbb{E}_{\tilde{F}_S}^{[0,1]} [v_S | v_S \leq p] \right) R_{\tilde{F}_B}(p) \tilde{F}_S(p) \quad (11)$$

Lemma 1 in Appendix A shows that it is sufficient for the mediator to select a discovery policy that only has two realizations, a high and a low signal. This is because each party faces a binary decision, to accept or reject trade at the price posted by the mediator. Building on this, we can show that the solution of the mediator's problem has a simple, binary and monotone, partition structure.¹⁶ A threshold is chosen for each trader, who

¹⁵ As the signal distributions could be continuous or discrete, all the integrals in the paper should be understood as Stieltjes integrals.

¹⁶ This is reminiscent of the result in Bergemann and Pesendorfer (2007), that monotone partitions are the optimal static information structures for a revenue maximizing seller in an auction setting.

then discovers whether her value is above or below the threshold. Buyer types above the buyer's threshold and seller types below the seller's threshold will want to trade, while the other types will refuse to trade. We thus see that, like the shuttle diplomacy procedure, the optimal static mechanism limits the amount of information discovered by the parties, but to a greater extent.

The optimal binary partitions that constitute the signals for seller and buyer are characterized in the next proposition.

Proposition 6 *In the static information discovery and trading mechanism that maximizes social value, the buyer observes whether her value is strictly below some threshold x and the seller observes whether her value is strictly above some other threshold y , with x, y satisfying:*

$$\mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq y] = x \quad \text{and} \quad \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x] = y. \quad (12)$$

The trading price can be any $p \in [\mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq y], \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x]] = [x, y]$.

It is immediate to verify that system (12) always admits a solution for x, y and that $x < y$.¹⁷ Trade occurs when the buyer's valuation is above x and the seller's valuation below y . Thus, since $x < y$, trade may occur when the buyer's value is below the seller's value. This is an important difference relative to the case where parties are fully informed and also to the optimal shuttle diplomacy mechanism we studied in Section 3. Under the latter, trade is ex-post efficient. Under full information, only buyers with a value above the posted price p and sellers with a value below p trade.

The reason why a full information discovery policy is not optimal is that it does not generate enough trade: any efficient trade with either (i) $p > v_B > v_S$, or (ii) $v_B > v_S > p$ is lost. By limiting the information available to parties, the optimal static information discovery and trading mechanism guarantees completion of a higher volume of trades. Some of the most valuable trades which are lost under full information – those with (i)

¹⁷ Since F_B, F_S have no atoms, $\mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq y] < y$ for all $y > 0$ and is continuous and strictly increasing in y , while $\mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x] > x$ for all $x < 1$ and is continuous and strictly increasing in x . Hence if a solution exists, we have $x < y$. To show existence define the following function of x with domain and range $[0, 1]$: $\mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x]]$. Since it is the composite function of two continuous functions, it is continuous. By Brouwer's fixed point theorem it has a fixed point x^* and thus x^* and $y^* = \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x^*]$ is a solution of (12). Note that multiple solutions may exist, in which case one of them is the optimum.

$v_B = p - \varepsilon_B$ and $v_S = \varepsilon_S$ and those with (ii) $v_B = 1 - \varepsilon_B$ and $v_S = p + \varepsilon_S$ (for $\varepsilon_B, \varepsilon_S$ small) – take place under the optimal mechanism.¹⁸ Inducing completion of valuable trades in the optimal mechanism comes at a cost: as we noticed, some inefficient trades are also completed (this never happens when parties are fully informed), but those are the ones that have smaller losses; trades with $v_S = y - \varepsilon_S > v_B = x + \varepsilon_B$. Moreover, there are also some less valuable, but still efficient, trades that are not completed in the optimal mechanism (e.g., those for which $x > v_B > v_S$ or $v_B > v_S > y$).

We should also add that in the optimal static information discovery and exchange mechanism we characterized, trade occurs whenever its expected benefit exceeds the cost; as long as $x < p < y$ incentive constraints do not bind.

The freedom the mediator has in the choice of the price allows her to pursue the additional goal of a fair division of social value. A 50-50 split of the interim expected surplus between buyer and seller can always be obtained by posting the fair price $p = \frac{x+y}{2}$. Recall that, on the contrary, with shuttle diplomacy only for some (e.g., under symmetry, $F_B = F_S$), but not all, distributions of values F_B and F_S the mediator can ensure that the posted price at which trade occurs is equal to the fair price. This is because, with shuttle diplomacy and asymmetric distributions, the fair price may be different at different points in time.

When F_B and F_S are uniform, the solution of the optimal discovery policy we obtain from (12) is $x = 1/3$ and $y = 2/3$; the buyer observes whether her value is above or below $1/3$, while the seller observes whether her cost is above or below $2/3$. Any price $p \in [1/3, 2/3]$ is an optimal trading price. The expected total surplus generated by the allocation attained at the optimal static mechanism is $(\frac{2}{3} - \frac{1}{3}) \frac{2}{3} \frac{2}{3} = \frac{4}{27}$ or 89% of the ex-post efficient level achieved by the shuttle diplomacy mechanism, which is $\frac{1}{6}$. As an additional comparison, when parties are fully informed about their own values, the optimal posted price is $p = \frac{1}{2}$, which yields an expected surplus of $(\frac{3}{4} - \frac{1}{4}) \frac{1}{2} \frac{1}{2} = \frac{1}{8}$, or 75% of the ex-post efficient level. The increase in surplus at the optimal static mechanism relative to full information is due to an increase in the volume of trade: trade occurs whenever the buyer has a value greater than $1/3$ and the seller a value smaller than $2/3$.¹⁹

¹⁸ This can be seen most clearly when $x < p < y$.

¹⁹ The expected level of total surplus with F_B and F_S uniform is also higher in our optimal static mechanism than in the optimal Bayesian mechanism with fully informed traders where, as shown by Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983), trade takes place whenever

7 Related Literature

In the law and economics models that study the role of mediation (e.g., Brown and Ayres, 1994, Doornik, 2014, and Goltsman et al., 2009), each party is assumed to have full information about her own value at the outset of the process. The mediator thus only plays a facilitative role, collecting information from both parties and conveying it strategically via a proposed trading deal, which allows parties to update their view of their bargaining position. Brown and Ayres (1994) argue that mediators reduce adverse selection by committing parties to simple mechanisms; e.g., “(1) by committing parties to break off negotiations when private representations to a mediator indicate that there are no gains from their interaction; (2) by committing parties to equally divide the social surplus; and (3) by committing to send noisy translations of information disclosed during private caucuses.” Doornik (2014) argues that the role of a mediator is to avoid a costly trial by verifying the private information of an informed party and communicating it to the other party, without disclosing confidential details that would disadvantage the informed party. Goltsman et al. (2009) study mediation in a cheap talk framework and show that by adding noise a mediator may relax the incentive compatibility constraint of the informed party and thus facilitate information transmission. While insightful, these models cannot explain either the evaluative role of the mediator, or the benefits of the mediator engaging in shuttle diplomacy.

Hörner et al. (2015) and two recent papers by Fanning (2021, 2023) also focus on the facilitative role of mediation. They study situations not involving transfer payments. Hörner et al. (2015) analyze the role of mediation in disputes regarding situations of conflict in international relations. Parties are fully informed about their own type, and they model shuttle diplomacy as a procedure whereby the mediator asks parties to privately report their type and the mediator then makes private recommendations. They argue that the private nature of these recommendations allows to limit what parties learn about their counterparts and hence to relax their incentives to report truthfully. Besides this mediated communication stage, the mechanism is a static one. In this paper, we show that if the mediator also plays an evaluative role, then incentives can be further enhanced by limiting the information parties acquire about their own type, and a fully sequential

$v_B \geq v_S + 1/4$ and the expected surplus is $9/64$, or 84% of the ex-post efficient level.

procedure allows to further improve welfare. Fanning (2021, 2023) focuses on the facilitative role of mediation in the framework of the dynamic, reputational bargaining model of Abreu and Gul (2000). In this model each of the two parties is privately informed about whether she is rational, or a commitment type who demands a fixed share of the surplus and does not accept less. Without a mediator, Abreu and Gul (2000) show that there is a unique equilibrium, similar to the equilibrium in the war of attrition. Fanning (2021) then shows that an uninformed mediator may help parties to achieve a better outcome by collecting private messages from them at the beginning of the game, and revealing with positive but less than one probability whether parties have communicated their willingness to compromise. Fanning (2023) extends this result by taking a mechanism design approach and looking at the optimal mechanism with an uninformed mediator. she shows that noisy communication remains an important feature of the optimal mechanism.

The Bayesian persuasion literature, initiated by Kamenica and Gentzkov (2011) (see also Rayo and Segal, 2010, Kolotilin et al., 2017 and Li and Norman, 2021) and recently reviewed by Bergemann and Morris (2019), studies how the release of appropriately chosen information may allow a principal to incentivize an agent to behave in the desired way. Information design in dynamic settings with a sequence of information releases has then been studied, among others, in Ely et al. (2015), Ely (2017), Ely and Szydlowski (2020), Ball (2023), Orlov et al. (2020) and Zhao, Mezzetti, Renou and Tomala (2024). We should also mention Khantadze, Kremer and Skrzypacz (2022). They compare simultaneous and sequential disclosure procedures in a setting with multiple agents taking multiple actions and provide conditions under which the optimal sequential procedure leads to a higher payoff to the principal.

As the information designer in this literature, our mediator is able to commit to an information structure that maps states of the world (parties' values) into stochastic signals privately disclosed (to each party) and not observed by the mediator. In addition, and also in common with this literature, our mediator knows the prior value distributions and may use all feasible information discovery policies. But in contrast to the information designer of the persuasion literature, who cannot affect the outcomes available to the players, and like the designer in the classic mechanism design literature (e.g., see Myerson and Satterthwaite, 1983), our mediator may also affect outcomes, by setting the price at which parties may trade. Thus, our mediator plays both the role of an information

designer in the persuasion literature, and the classical designer in the mechanism design literature (see Mezzetti, 2019, for a brief discussion on this dual role of a designer).

This feature is shared by our paper with the literature on information discovery and surplus extraction by the seller of a single item, where the seller chooses both an information and a sale policy (Bergemann and Pesendorfer, 2007, Esö and Szentes, 2007, Li and Shi, 2017 and Krähmer, 2020).²⁰ There are several differences, however, apart from the objective of our mediator being to maximize welfare. First, in our paper there is private information on both sides of the market and an information structure must be chosen for both. In contrast, in the papers cited above there is no uncertainty regarding the seller's value but there could be several buyers and the focus is then on the information of each of them about their own value.²¹ Second, we focus on simple price posting mechanisms,²² and do not allow the mediator to add privately observed (but payoff irrelevant) signals to the information structure that is provided to the buyer and the seller.²³ Third, and most important, as opposed to the static discovery policies of most of the surplus extraction literature, our main contribution is to study a sequential, shuttle diplomacy procedure and show that a sequence of simple discoveries allows to significantly increase the set of outcomes the mediator may achieve; in particular, it allows implementation of the first best outcome.

We are only aware of two important exceptions to static discovery policies in the surplus extraction literature, Bergemann and Wambach (2015) and Heumann (2020).

²⁰The combination of information and mechanism design is also present in Roesler and Szentes (2017) and Condorelli and Szentes (2020), but in those papers it is the buyer that selects her own information structure so as to protect herself from the seller's choice of a sale mechanism aiming to maximize her surplus.

²¹These papers impose, like us, a privacy constraint on the information disclosed to each buyer. In contrast, Bergemann et al. (2015) examine how the division of the gains from trade associated with the optimal pricing policy of a monopolist seller varies with the amount of information the seller has about buyers' private valuations.

²²It is useful to point out that when parties are fully informed about their values, the only dominant strategy mechanisms that balance the budget at all signal realizations (i.e., such that the law of one price holds) and satisfy parties' participation constraints ex-post are price posting mechanisms in which the mediator posts a single price at which trade may take place (see Hagerty and Rogerson, 1987, Čopič and Ponsati, 2016, and Čopič, 2017; for the robustness of dominant strategy mechanisms, see Bergemann and Morris, 2005).

²³In contrast, Krähmer (2020) allows the seller to: (i) randomize secretly among alternative information structures all of which fully inform the buyer about her own valuation, and (ii) condition the terms of trade on the buyer's report of the signal obtained from the realized information structure. This guarantees that a lie is detected with positive probability and can be severely punished. Hence the seller can extract all the surplus from the buyer; that is, the good is allocated efficiently and the buyer can be charged her valuation.

Bergemann and Wambach (2015) must be credited for being the first to show the benefits of slow information release. They introduce an auction with sequential discovery of information in which each bidder learns a progressively higher lower bound v on her value for the item for sale. Each bidder may elect to continue to stay or to stop and drop out of the auction. Once all but a single bidder i have dropped out, bidder i gets the item and is charged her expected value conditional on her value being above the value v at which the last bidder dropped out. Thus, clearly, this auction implements an efficient outcome and extracts all surplus. Our single-ride mechanism also gives all the surplus to one of the parties, but it is different as it does so by revealing all information to one party, and linking what is revealed to the other party to the report of the informed one. Our shuttle diplomacy mechanism also reveals information gradually and only allows parties to choose if they want to stop or continue, but there are many differences with Bergemann and Wambach (2015). Contrary to their model, with shuttle diplomacy: *(i)* stopping might imply that the seller keeps the asset; *(ii)* agents progressively discover whether their values are high or low; *(iii)* the main goal is efficiency, which cannot be achieved if the agents are fully informed, contrary to the auction setting; *(iv)* a fair outcome can be achieved with symmetric agents, and finally and importantly *(v)* agents trade the claim on the asset back and forth along the mediation procedure.

Heumann (2020) studies a single-buyer, single-seller, model with variable quantity, in which the type θ of the buyer affects both her valuation for the quantity q of the good supplied by the seller and the seller's cost of supplying it. As in Bergemann and Wambach (2015), the seller wants to maximize profit. To do so, the seller can choose the information content of a continuous-time signal process observed by the buyer, the quantity to be sold, and the price to be charged, as functions of the entire history of reports by the buyer about the information she has obtained. Heumann (2020) provides a lower bound on the rent that the buyer can attain and demonstrates the optimality of an upward disclosure policy as in Bergemann and Wambach (2015), where the buyer learns a progressively higher lower bound θ , which corresponds to a higher lower bound on her value for any quantity q of the good. As in our paper, slow information release relaxes the incentive constraints; however, in our paper uncertainty and private information concerns both the buyer and seller, the goals are efficiency and fairness, and it is not sufficient only to provide information about a lower boundary on value. Downward disclosures (i.e., the

slow release of information on the upper bound of value), must go hand in hand with upward disclosures.

8 Conclusions

Psychologists have argued that mediators help parties to overcome psychological barriers to conflict resolution. We have argued that a mediator will also help when parties are rational, strategic negotiators. We consider a shuttle diplomacy procedure in which private information is progressively and symmetrically revealed to the parties involved in a dispute, and each time new information arrives parties decide whether to settle the dispute. We have shown that shuttle diplomacy allows the mediator to achieve an efficient outcome by reducing the opportunities for misrepresentation of value. With symmetric prior value uncertainty, the mediator is also able to set a fair price at which the parties share equally the social value.

A first insight from our paper is that shuttle diplomacy requires an expert mediator, but little strategic sophistication from the parties to a dispute. They only need to wait until they have discovered their value to decide on settlement. Another important insight is that to facilitate efficient settlement, mediators should make sure that parties discover one small bit of information at a time, and that information about extreme values is discovered first. Finally, while the mediator could suggest different settlement terms along the way (the price could vary), depending on the feedback she receives along the mediation procedure, another insight of this paper is that efficient settlement can be obtained even when the terms are determined at the start. This is a feature that simplifies the mediation procedure and increase its robustness, by making it less dependent on the details of the parties' beliefs and removing strategic uncertainty.

The approach adopted in this paper may be described as an information discovery and allocation mechanism design approach. Both the private information disclosed to parties and the terms of settlement are chosen by a designer, in our case the mediator. The approach blends the classical mechanism design approach with the information design, or Bayesian persuasion, approach. In classical mechanism design, agents have full private information and the designer only selects a procedure to determine the allocation as a function of the information reported by the agents. In information design, the allocation

mechanism is exogenously fixed and the designer may only select the information discovery policy. We believe that the approach considered in this paper could be fruitfully applied to other settings, beyond the case of the a dispute between two parties that we have considered.

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Appendix A

In this appendix we present results and proofs omitted from the main body of the paper.

Proof of Proposition 1 Condition (6) can be written as

$$\int_{\alpha_t}^{\beta_t} \frac{\beta_t v - v^2}{\frac{1}{2}(\beta_t - \alpha_t)^2} dv \leq p \leq \int_{\alpha_t}^{\beta_t} \frac{v^2 - \alpha_t v}{\frac{1}{2}(\beta_t - \alpha_t)^2} dv, \quad \text{or}$$

$$\frac{1}{2}\beta_t(\beta_t^2 - \alpha_t^2) - \frac{1}{3}(\beta_t^3 - \alpha_t^3) \leq \frac{1}{2}(\beta_t - \alpha_t)^2 p \leq \frac{1}{3}(\beta_t^3 - \alpha_t^3) - \frac{1}{2}\alpha_t(\beta_t^2 - \alpha_t^2), \quad \text{or}$$

$$\frac{\beta_t}{3}(\beta_t^2 - \alpha_t^2) - \frac{2\alpha_t^2}{3}(\beta_t - \alpha_t) \leq (\beta_t - \alpha_t)^2 p \leq \frac{2\beta_t^2}{3}(\beta_t - \alpha_t) - \frac{\alpha_t}{3}(\beta_t^2 - \alpha_t^2), \quad \text{or}$$

$$\frac{\beta_t}{3}(\beta_t + \alpha_t) - \frac{2\alpha_t^2}{3} \leq (\beta_t - \alpha_t)p \leq \frac{2\beta_t^2}{3} - \frac{\alpha_t}{3}(\beta_t + \alpha_t), \quad \text{or}$$

$$\frac{\beta_t^2 - \alpha_t^2}{3} + \frac{\alpha_t(\beta_t - \alpha_t)}{3} \leq (\beta_t - \alpha_t)p \leq \frac{2(\beta_t^2 - \alpha_t^2)}{3} - \frac{\alpha_t}{3}(\beta_t - \alpha_t), \quad \text{or}$$

$$\frac{\beta_t^2 - \alpha_t^2}{3} + \frac{\alpha_t(\beta_t - \alpha_t)}{3} \leq (\beta_t - \alpha_t)p \leq \frac{2(\beta_t^2 - \alpha_t^2)}{3} - \frac{\alpha_t}{3}(\beta_t - \alpha_t), \quad \text{or}$$

$$\frac{1}{3}\beta_t + \frac{2}{3}\alpha_t \leq p \leq \frac{2}{3}\beta_t + \frac{1}{3}\alpha_t$$

Since the above inequality must hold at the beginning of the discovery procedure, $t = 0$, when $\alpha_0 = 0$ and $\beta_0 = 1$, it must be $\frac{1}{3} \leq p \leq \frac{2}{3}$. To see that this is the only required condition, let $\alpha_t = \frac{t}{T}p$ and $\beta_t = 1 - \frac{t}{T}(1 - p)$, so that all discovery intervals on the same side of p are of the same size. Then the above condition, for all t , reduces to:

$$\frac{1}{3} \left(1 - \frac{t}{T}(1 - p) \right) + \frac{2}{3} \frac{t}{T} p \leq p \leq \frac{2}{3} \left(1 - \frac{t}{T}(1 - p) \right) + \frac{1}{3} \frac{t}{T} p, \quad \text{or}$$

$$\frac{1}{3} \left(1 - \frac{t}{T} \right) + \frac{t}{T} p \leq p \leq \frac{2}{3} \left(1 - \frac{t}{T} \right) + \frac{t}{T} p, \quad \text{or}$$

$$\frac{1}{3} \leq p \leq \frac{2}{3}.$$

□

Proof of Claim 1 It follows from the fact that:

$$\mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_j | v_j < v_i] - \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_i | v_j < v_i] = \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)} [v_j - v_i | v_j < v_i] < 0.$$

□

Proof of Claim 2 For fixed v_i we have :

$$\mathbb{E}_{F_j}^{(\alpha, \beta)}[v_j | v_j < v_i] < v_i < \mathbb{E}_{F_j}^{(\alpha, \beta)}[v_j | v_j > v_i].$$

Integrating over v_i yields the Claim. □

Proof of Claim 3 Let $j = \operatorname{argmax}_{B,S} \left\{ \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_B | v_B < v_S], \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_S | v_S < v_B] \right\}$.

By Claim 1:

$$\mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_j | v_j < v_i] < \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_i | v_i > v_j].$$

By Claim 2:

$$\mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_j | v_j < v_i] < \mathbb{E}_{F_B, F_S}^{(\alpha, \beta)}[v_j | v_j > v_i].$$

The inequality in Claim 3 then follows. □

Proof of Proposition 2 For $\lambda \in (0, 1)$, select the posted price

$$p = \lambda \max \{ \mathbb{E}_{F_B, F_S}[v_B | v_B < v_S], \mathbb{E}_{F_B, F_S}[v_S | v_S < v_B] \} + (1 - \lambda) \min \{ \mathbb{E}_{F_B, F_S}[v_B | v_B > v_S], \mathbb{E}_{F_B, F_S}[v_S | v_S > v_B] \}. \quad (13)$$

Define i_t, j_t as follows:²⁴

$$i_t = \operatorname{arg max}_{B,S} \left\{ \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B < v_S], \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_S | v_S < v_B] \right\}$$

$$j_t = \operatorname{arg min}_{B,S} \left\{ \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_B | v_B > v_S], \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_S | v_S > v_B] \right\}$$

Letting $h_t, k_t \in \{B, S\}$, $h_t \neq i_t$, $k_t \neq j_t$, to prove the proposition we will show that, for all t , there is a choice of α_t and β_t such that the price p defined in (13) satisfies the following condition for all $t \in \{0, \dots, \infty\}$:

$$p = \lambda \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{i_t} | v_{i_t} < v_{h_t}] + (1 - \lambda) \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{j_t} | v_{j_t} > v_{k_t}] \quad (14)$$

$$= \lambda \int_{\alpha_t}^{\beta_t} \frac{v_{i_t} [F_{h_t}(\beta_t) - F_{h_t}(v_{i_t})]}{\int_{\alpha_t}^{\beta_t} [F_{h_t}(\beta_t) - F_{h_t}(v)] dF_{i_t}(v)} dF_{i_t}(v_{i_t}) + (1 - \lambda) \int_{\alpha_t}^{\beta_t} \frac{v_{j_t} [F_{k_t}(v_{k_t}) - F_{k_t}(\alpha_t)]}{\int_{\alpha_t}^{\beta_t} [F_{k_t}(v) - F_{k_t}(\alpha_t)] dF_{j_t}(v)} dF_{j_t}(v_{j_t})$$

²⁴ Note it could be $i_t = j_t$ or $i_t \neq j_t$.

Take an increasing function α_t with $\alpha_0 = 0$, $\alpha_T = p$ and use (14) to implicitly define an associated function β_t . It is clear that it must be $\beta_0 = 1$ and $\beta_T = p$. Thus, it only remains to show that the function β_t is decreasing. To establish that this is the case, we totally differentiate (14) with respect to α_t and β_t to obtain:

$$\begin{aligned}
0 = & \left(\lambda \left[\int_{\alpha_t}^{\beta_t} \frac{v_{i_t}}{F_{i_t}(\beta_t) - F_{i_t}(\alpha_t)} dF_{i_t}(v_{i_t}) - \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{i_t} | v_{i_t} < v_{h_t}] \right] \frac{[F_{i_t}(\beta_t) - F_{i_t}(\alpha_t)]f_{h_t}(\beta_t)}{\int_{\alpha_t}^{\beta_t} [F_{h_t}(\beta_t) - F_{h_t}(v)]dF_{i_t}(v)} \right. \\
& + (1 - \lambda) \left[\beta_t - \mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{j_t} | v_{j_t} > v_{k_t}] \right] \frac{[F_{k_t}(\beta_t) - F_{k_t}(\alpha_t)]f_{j_t}(\beta_t)}{\int_{\alpha_t}^{\beta_t} [F_{k_t}(\beta_t) - F_{k_t}(v)]dF_{j_t}(v)} \left. \right) d\beta_t \\
& + \lambda \left(\left[\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{i_t} | v_{i_t} < v_{h_t}] - \alpha_t \right] \frac{[F_{h_t}(\beta_t) - F_{h_t}(\alpha_t)]f_{i_t}(\alpha_t)}{\int_{\alpha_t}^{\beta_t} [F_{h_t}(\beta_t) - F_{h_t}(v)]dF_{i_t}(v)} \right. \\
& + (1 - \lambda) \left[\mathbb{E}_{F_B, F_S}^{(\alpha_t, \beta_t)}[v_{j_t} | v_{j_t} > v_{k_t}] - \int_{\alpha_t}^{\beta_t} \frac{v_{j_t}}{F_{j_t}(\beta_t) - F_{j_t}(\alpha_t)} dF_{j_t}(v_{j_t}) \right] \frac{[F_{j_t}(\beta_t) - F_{j_t}(\alpha_t)]f_{k_t}(\alpha_t)}{\int_{\alpha_t}^{\beta_t} [F_{k_t}(\beta_t) - F_{k_t}(v)]dF_{j_t}(v)} \left. \right) d\alpha_t
\end{aligned}$$

Both the term multiplying $d\beta_t$ and the one multiplying $d\alpha_t$ are positive. Hence, if α_t is increasing (i.e., $d\alpha_t > 0$, then β_t implicitly defined by (14) is decreasing (i.e., $d\beta_t < 0$). This concludes the proof. \square

Proof of Proposition 5 The seller's payoff when she has value v_S and reports \widehat{v}_S is:

$$u_S(\widehat{v}_S; v_S) = \max \{ [p_S(\widehat{v}_S) - v_S] [1 - F_B(\widehat{v}_S)], 0 \}$$

At an interior solution, the first order condition of the seller's problem of choosing \widehat{v}_S to maximize $u_S(\widehat{v}_S; v_S)$ is:

$$\frac{dp_S(\widehat{v}_S)}{d\widehat{v}_S} [1 - F_B(\widehat{v}_S)] - f_B(\widehat{v}_S) [p_S(\widehat{v}_S) - v_S] = 0. \quad (15)$$

Denote by $u_S(v_S) = u_S(v_S; v_S)$ the seller's indirect utility when reporting sincerely (i.e., $\widehat{v}^S = v_S$) is optimal. By the envelope theorem, treating the true value v_S as a parameter, we have:

$$\frac{du_S(v_S)}{dv_S} = -[1 - F_B(v_S)].$$

Integrating both sides from v_S to 1, using the boundary condition $u_S(1) = 0$, we obtain:

$$[p_S(v_S) - v_S][1 - F_B(v_S)] = u_S(v_S) = \int_{v_S}^1 [1 - F_B(\tilde{v}_S)] d\tilde{v}_S.$$

Integrating by parts and rearranging we obtain:

$$p_S(v_S) = \int_{v_S}^1 \frac{\tilde{v}_S}{1 - F_B(v_S)} dF_B(\tilde{v}_S) = \mathbb{E}_{F_B}^{[0,1]} [v_B | v_B \geq v_S]. \quad (16)$$

This shows that $p_S(\hat{v}_S) = \mathbb{E}_{F_B}^{[0,1]} [v_B | v_B \geq \hat{v}_S]$ is the only price function that satisfies the first order condition for truthful reporting of her value by the seller to be incentive compatible. Replacing (16) into (15), the first order condition can be written as:

$$-\hat{v}_S f_B(\hat{v}_S) + p_S(\hat{v}_S) f_B(\hat{v}_S) - f_B(\hat{v}_S) [p_S(\hat{v}_S) - v_S] = f_B(\hat{v}_S) [v_S - \hat{v}_S] = 0.$$

Differentiating it with respect to the report \hat{v}_S and evaluating it at $\hat{v}_S = v_S$, shows that the second order condition is satisfied:

$$\left(\frac{df_B(\hat{v}_S)}{d\hat{v}_S} [v_S - \hat{v}_S] - f_B(\hat{v}_S) \right) \Big|_{\hat{v}_S=v_S} = -f_B(v_S) \leq 0.$$

□

Lemma 1 *Given any solution to the mediator's maximization problem (11), there is a payoff equivalent solution in which the mediator chooses a two-point signal distribution both for the buyer and the seller.*

Proof Denote by $R_{\tilde{F}_B}(v) = \Pr(v_B \geq v) = 1 - \tilde{F}_B(v) + \mathbf{1}_{\tilde{F}_B}(v)$ as the reliability function associated with the distribution $\tilde{F}_B(v)$, where $\mathbf{1}_{\tilde{F}_B}(v)$ is the probability mass on $v_B = v$.

Suppose $p, \tilde{F}_B, \tilde{F}_S$ are maximizers of (11). We show in what follows that p and the following two point distributions are also maximizers of (11). For buyers, take the distribution that puts mass $1 - R_{\tilde{F}_B}(p)$ on $\mathbb{E}_{\tilde{F}_B}[v_B | v_B < p]$ and mass $R_{\tilde{F}_B}(p)$ on $\mathbb{E}_{\tilde{F}_B}[v_B | v_B \geq p]$. For sellers, take the distribution that puts mass $\tilde{F}_S(p)$ on $\mathbb{E}_{\tilde{F}_S}[v_S | v_S \leq p]$ and mass $1 - \tilde{F}_S(p)$ on $\mathbb{E}_{\tilde{F}_S}[v_S | v_S > p]$. It is then immediate to see that the expression in (11), evaluated at the solution $p, \tilde{F}_B, \tilde{F}_S$, is equal to:

$$\left(\mathbb{E}_{\tilde{F}_B}^{[0,1]} [v_B | v_B \geq p] - \mathbb{E}_{\tilde{F}_S}^{[0,1]} [v_S | v_S \leq p] \right) R_{\tilde{F}_B}(p) \tilde{F}_S(p),$$

that is, to the value of (11) at the two-point distributions described above. Note that if $\tilde{F}_B \in \mathcal{F}_B$, then the described two-point distribution for the buyer also belongs to \mathcal{F}_B , and similarly for the seller. This concludes the proof that the mediator may restrict attention to two-point discrete distributions. \square

Proof of Proposition 6 To establish the result, we characterize first the classes of feasible two-point signal distributions. For buyers, let $\{v_B^L, v_B^H\}$ be the set of possible signals with associated probabilities $\tilde{f}_B^L, \tilde{f}_B^H = 1 - \tilde{f}_B^L$. The following constraints must hold to guarantee that the true distribution F_B is a mean preserving spread of the two-point signal distribution:

$$v_B^L \tilde{f}_B^L + v_B^H (1 - \tilde{f}_B^L) = \mathbb{E}_{F_B}^{[0,1]}[v_B] \quad (17)$$

$$(x - v_B^L) \tilde{f}_B^L \leq \int_0^x F_B(v) dv \quad \text{for } x \in [v_B^L, v_B^H] \quad (18)$$

$$(v_B^H - v_B^L) \tilde{f}_B^L + (x - v_B^H) \leq \int_0^x F_B(v) dv \quad \text{for } x \in [v_B^H, 1] \quad (19)$$

We can then solve (17) for v_B^L and replace the solution into the other two constraints:

$$(x - v_B^H) \tilde{f}_B^L - \mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - \int_0^x F_B(v) dv \leq 0 \quad \text{for } x \in [v_B^L, v_B^H] \quad (20)$$

$$x - \mathbb{E}_{F_B}^{[0,1]}[v_B] - \int_0^x F_B(v) dv \leq 0 \quad \text{for } x \in [v_B^H, 1] \quad (21)$$

It is immediate to see that (21) holds since, integrating by parts,

$$\begin{aligned} x - \mathbb{E}_{F_B}^{[0,1]}[v_B] - \int_0^x F_B(v) dv &= x[1 - F_B(x)] - \mathbb{E}_{F_B}^{[0,1]}[v_B] + \int_0^x v dF_B(v) \\ &= xR_{F_B}(x) - \int_x^1 v dF_B(v) \leq 0. \end{aligned}$$

It is also immediate to see that the left hand side of (20) is a concave function of x . Define x_B as the solution to $\tilde{f}_B^L = F_B(x)$. There are three possible cases.

Case 1: $x_B \in [v_B^L, v_B^H]$. Then the left hand side of (20) is maximized at x_B and condition

(20) holds if it is satisfied for $x = x_B$. Hence we can rewrite this condition as:

$$\begin{aligned} (x - v_B^H)F_B(x) - \mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - xF_B(x) + \int_0^x v dF_B(v) \leq 0 \quad \text{or,} \\ v_B^H R_{F_B}(x) - \int_x^1 v dF_B(v) \leq 0 \end{aligned} \quad (22)$$

Condition (22) is quite intuitive: it says that the value of the buyer when she receives the high signal, v_B^H , cannot exceed the expected value of the buyer, conditional on this value lying above some threshold x , evaluated according to the posterior distribution F_B .

Case 2: $x_B < v_B^L$, that is the two point signal distribution is such that $F(v_B^L) > \tilde{f}_B^L$. In this case the left hand side of (20) is maximized at $x = v_B^L$, hence condition (20) is always satisfied as it can be rewritten as:

$$- \int_0^{v_B^L} F_B(v) dv \leq 0.$$

Case 3: $x_B > v_B^H$, the two point signal distribution is such that $\tilde{f}_B^L > F(v_B^H)$. The expression on the left hand side of (20) is now maximized at $x = v_B^H$ and we can rewrite (20) as:

$$-\mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - \int_0^{v_B^H} F_B(v) dv \leq 0, \quad (23)$$

which we can also show always to hold.²⁵

We may repeat the same argument for the seller, letting $\{v_S^L, v_S^H\}$ be the set of possible signals with probabilities $\tilde{f}_S^L, \tilde{f}_S^H$, with $v_S^L \tilde{f}_S^L + v_S^H \tilde{f}_S^H = \mathbb{E}_{F_S}^{[0,1]}[v_S]$ and the counterparts of (20) and (21). Using y instead of x , we see that the left hand side of the counterpart of (20) is a concave function of y . The expression on the left hand side of the constraint reaches the highest value either at y equal to v_S^L or v_S^H , in which case the constraint is

²⁵ To see that (23) holds, we can rewrite it as

$$\begin{aligned} -\mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H [1 - F_B(v_B^H)] + \int_0^{v_B^H} v dF_0(v) \leq 0 \quad \text{or,} \\ v_B^H [1 - F_B(v_B^H)] - \int_{v_B^H}^1 v dF_B(v) \leq 0. \end{aligned}$$

satisfied, or at y such that $\tilde{f}_S^L = F_S(y)$, in which case the constraint can be rewritten as:

$$\begin{aligned} v_S^H R_{F_S}(y) - \int_y^1 v dF_S(v) &\leq 0 \quad \text{or,} \\ \int_0^y v dF_S(v) - v_S^L F_S(y) &\leq 0 \end{aligned} \quad (24)$$

We can then use Lemma 1 and the characterization we obtained of two-point signal distributions to rewrite the mediator's problem, (11). As we said, a mediator aiming to maximize the gains from trade will choose distributions and a price such that the buyer is willing to trade when she gets the high signal and the seller with the low signal, that is: $v_B^H \geq p \geq v_S^L$. Also, we can show that the situation described in Cases 2 and 3 above never arises at a solution of the mediator's problem,²⁶ hence neither constraint (22) nor (24) can be ignored, and we can replace the choice variables \tilde{f}_B^L and \tilde{f}_S^L with $F_B(x)$ and $F_S(y)$. The mediator problem (11) can then be rewritten as follows:

$$\begin{aligned} \max_{v_B^H, v_S^L, x, y} & (v_B^H - v_S^L) F_S(y) R_{F_B}(x) \quad \text{s.t.} \\ & v_B^H R_{F_B}(x) + v_B^L F_B(x) = \mathbb{E}_{F_B}^{[0,1]}[v_B] \\ & v_B^H R_{F_B}(x) - \int_x^1 v dF_B(v) \leq 0 \\ & v_S^H R_{F_S}(y) + v_S^L F_S(y) = \mathbb{E}_{F_S}^{[0,1]}[v_S] \\ & \int_0^y v dF_S(v) - v_S^L F_S(y) \leq 0 \end{aligned} \quad (25)$$

Furthermore, it is immediate to see that both inequality constraints must bind, otherwise the mediator would profit from raising v_B^H or lowering v_S^L . Hence, substituting the constraints into the objective function, the problem reduces to:

$$\max_{x, y} \left(\mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x] - \mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq y] \right) F_S(y) R_{F_B}(x) \quad (26)$$

²⁶ Consider the information provided to the buyer. Suppose first the solution of the mediator's problem falls in Case 2. Then $F(v_B^L) > \tilde{f}_B^L = F(x_B)$ and it is possible to raise the social value by keeping \tilde{f}_B^L constant, reducing v_B^L and increasing v_B^H while satisfying the only binding constraint $v_B^L \tilde{f}_B^L + v_B^H (1 - \tilde{f}_B^L) = \mathbb{E}_{F_B}^{[0,1]}[v_B]$, a contradiction. Second, suppose the solution falls in Case 3. We have so $F(v_B^H) < \tilde{f}_B^L = F(x_B)$. If $x_B < 1$ it is again possible to raise the social value by keeping \tilde{f}_B^L constant, reducing v_B^L and increasing v_B^H while satisfying $v_B^L \tilde{f}_B^L + v_B^H (1 - \tilde{f}_B^L) = \mathbb{E}_{F_B}^{[0,1]}[v_B]$, a contradiction. If instead $x_B = 1$, then $f^L = 1$ and with probability 1 there is no trade; this is also a contradiction as full discovery and any interior posted price $p \in (0, 1)$ would generate positive social value.

The interpretation of (26) is as follows. The fact that the two constraints in (25) hold as equality means that the mediator lets the buyer observe exactly whether her value is greater than or equal to x and lets the seller observe whether her value is smaller than or equal to y . Trade takes place when both events realize, as ensured by posting any price $p \in \left[\mathbb{E}_{F_S}^{[0,1]}[v_S | v_S \leq y], \mathbb{E}_{F_B}^{[0,1]}[v_B | v_B \geq x] \right]$. The values of x and y are then optimally chosen to maximize the expected social value.

The final step of the proof is then to characterize the solutions of program (26). Since the domain of (x, y) is compact (the unit square) and the objective function and constraints are continuous in x and y , program (26) has a solution. Setting $x = 1$ or $y = 0$ cannot be optimal, as it yields a zero payoff to the mediator, which is less than the payoff that could be achieved by setting $0 < x = y < 1$. Thus the only possible boundary solutions have $x = 0$ and/or $y = 1$.

Since F_B and F_S have no atoms, the first order conditions of program (26), taking into account the constraints $x \geq 0$ and $1 - y \geq 0$, are:

$$\begin{aligned} -x f_B(x) F_S(y) + \int_0^y v dF_S(v) f_B(x) &\leq 0 \\ \left(-x f_B(x) F_S(y) + \int_0^y v dF_S(v) f_B(x) \right) x &= 0 \\ \int_x^1 v dF_B(v) f_S(y) - y f_S(y) R_{F_B}(x) &\geq 0 \\ \left(\int_x^1 v dF_B(v) f_S(y) - y f_S(y) R_{F_B}(x) \right) (1 - y) &= 0 \end{aligned}$$

Note that if $x = 0$, the first inequality is violated, as the term on the left hand side is strictly positive. Similarly, if $y = 1$ the second inequality is violated, as the expression on the left hand side is strictly negative. Thus there are no boundary solutions, the solution is interior and satisfies the conditions:

$$\begin{aligned} -x f_B(x) F_S(y) + \int_0^y v dF_S(v) f_B(x) &= 0 \quad \text{and} \\ \int_x^1 v dF_B(v) f_S(y) - y f_S(y) R_{F_B}(x) &= 0 \end{aligned}$$

which can be written as

$$\mathbb{E}_{F_S}^{[0,1]}[v \mid v \leq y] = x \quad \text{and}$$
$$\mathbb{E}_{F_B}^{[0,1]}[v \mid v \geq x] = y.$$

This concludes the proof of the proposition. □

Appendix B

The back-and-forth diplomacy procedure is composed by Round 0, when the mediator selects the mechanism parameters, and then a number of rounds of discovery and asset claim trading.

ROUND 0. The mediator selects:

- The maximum number T of rounds of value discovery and asset claim trading of the procedure.
- A posted price p at which trading may take place in each round t , $1 \leq t \leq T$.
- A collection of lower value discovery intervals: $\{[\alpha_{t-1}, \alpha_t)\}_{t=1}^T$ with $\alpha_0 = 0$, $\alpha_t > \alpha_{t-1}$ and $\alpha_T = p$.
- A collection of upper value discovery intervals: $\{(\beta_t, \beta_{t-1}]\}_{t=1}^T$ with $\beta_0 = 1$, $\beta_t < \beta_{t-1}$, and $\beta_T = p$.

ROUND $t \leq T$, STEP 1. At the beginning of this step the seller has a claim on the asset.

- The buyer discovers whether her value is in the interval $[\alpha_{t-1}, \alpha_t)$ and decides next whether or not she is willing to acquire the claim on the asset at price p .
- If the buyer decides not to acquire the claim, then the procedure stops and the seller retains the asset.
- If the buyer is willing to acquire the claim, then the seller discovers whether her value is in the interval $(\beta_t, \beta_{t-1}]$ and subsequently decides whether or not she is also willing to transfer the claim on the asset to the buyer at price p .
- If the seller decides not to transfer the claim, then the procedure stops and the seller retains the asset.
- If the seller is willing to transfer the claim on the asset, then the transfer takes place at price p .

ROUND $t < T$, STEP 2. At the beginning of this step the buyer has a claim on the asset.

- The seller discovers whether her value is in the interval $[\alpha_{t-1}, \alpha_t)$ and subsequently decides whether or not she is willing to buy back the claim on the asset at price p .
- If the seller decides not to buy back the claim, then the procedure stops and the transfer of the asset to the buyer is finalized.
- If the seller wishes to acquire the claim, the buyer discovers whether her value lies in the interval $(\beta_t, \beta_{t-1}]$ and decides next whether she is willing to transfer the claim on the asset to the seller at price p .
- If the buyer decides not to transfer back the claim, the procedure stops and the buyer ends up with the asset.
- If the buyer is willing to transfer back the claim on the asset, then the transfer takes place at price p and the procedure moves to round $t + 1$.

In this discrete time setting, the stop-at-value strategy requires that a party stops the procedure if and only if she approximately discover her value. We now argue that if both parties use the stop-at-value strategy, and the size of the information discovery intervals at round t , $d\alpha_t = \alpha_t - \alpha_{t-1} > 0$ and $d\beta_t = \beta_{t-1} - \beta_t > 0$, is “small”, then the outcome of the back-and-forth diplomacy mechanism is approximately ex-post efficient. An inefficient outcome can only arise in two circumstances, both occurring in step 1 of a round t . The first is if the buyer stops the procedure after discovering that her value is in the interval $[\alpha_{t-1}, \alpha_t)$; the seller retains the asset, but her value could also be in that interval and be lower. The maximal size of this efficiency loss is equal to the interval size $d\alpha_t$, and, given that F_B and F_S are atomless, its probability is $[F_B(\alpha_{t-1} + d\alpha_t) - F_B(\alpha_{t-1})][F_S(\alpha_{t-1} + d\alpha_t) - F_S(\alpha_{t-1})]$, which for small $d\alpha_t$ is approximately equal to $f_B(\alpha_{t-1})f_S(\alpha_{t-1})(d\alpha_t)^2$. The second circumstance is if the seller stops the procedure and retains the asset after discovering that her value is in the interval $(\beta_t, \beta_{t-1}]$, but the yet undiscovered buyer’s value is also in that interval and is higher. The maximal size of the efficiency loss in this case is again given by the size of the interval, $d\beta_t$, and its probability by $[F_S(\beta_{t-1}) - F_S(\beta_{t-1} - d\beta_t)][F_B(\beta_{t-1}) - F_B(\beta_{t-1} - d\beta_t)]$, which for small $d\beta_t > 0$ is approximately equal to $f_B(\beta_{t-1})f_S(\beta_{t-1})(d\beta_t)^2$. It is then immediate that the efficiency loss when parties use stop-at-value strategies converges to zero as $d\alpha_t$ and $d\beta_t$ converge to zero for all t .

Next we argue that the incentive constraints in back-and-forth diplomacy are the same as for the continuous-time shuttle diplomacy. The only additional argument that is needed is to show that if a party follows the stop-at-value strategy, then, as established in Lemma 2 below, for sufficiently large T and small discovery intervals $\{d\alpha_t, d\beta_t\}_{t=1}^T$ her continuation payoff is approximately equal to the expected benefit of ultimately acquiring the asset at price p whenever trade is ex-post efficient. Given this, and Claims 1-3, it follows that the equivalent of Proposition 2 holds for back-and-forth diplomacy.

Lemma 2 *If buyer and seller follow the stop-at-value strategy in the back-and-forth diplomacy procedure, then their payoffs in step 1 of round t converge to the expressions in (1) and (4) as $T \rightarrow \infty$ and $d\alpha_t$ and $d\beta_t$ converge to zero.*

Proof To compute the buyer's and seller's continuation payoffs from step 1 of round t when they follow the stop-at-value strategy, observe that if they do, then the outcome of the back-and-forth diplomacy procedure is that the buyer ends up owning the asset when it is efficient – that is, her value v_B is higher than the seller's value v_S – with the exception of two events.

The first event is when in step 1 of some round $\tau \geq t$ the seller stops the mechanism, but the buyer's value is higher. The buyer loses and the seller gains relative to efficient trading as they both value the asset more than the posted price and the asset would end up in the buyer's hands under efficient trading. The probability of this event can be approximated by $f_S(\beta_\tau)d\beta_\tau f_B(\beta_\tau)d\beta_\tau$. An upper bound on the loss to the buyer and on the gain to the seller relative to the payoff under efficient trading is $\beta_{\tau-1} - p$. Thus, relative to the payoff from trading at price p when it is efficient, the loss in the expected payoff of the buyer and the expected gain for the seller due to this event occurring at $\tau \geq t$ is (approximately) bounded above by:

$$(\beta_{\tau-1} - p) \frac{f_S(\beta_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]} \frac{f_B(\beta_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]} (d\beta_\tau)^2.$$

Adding up over all $\tau \geq t$ yields the following upper bound on the expected loss in payoff for the buyer and gain for seller with respect to the payoff from efficient trading:

$$\sum_{\tau=t}^T (\beta_{\tau-1} - p) \frac{f_S(\beta_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]} \frac{f_B(\beta_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]} (d\beta_\tau)^2,$$

which converges to zero as $T \rightarrow \infty$ and $d\beta_\tau \rightarrow 0$.

The second event is when in step 1 of some round $\tau \geq t$ the buyer stops, but her value is higher than the seller's value and hence it would be efficient for the buyer to obtain the asset. In this case, the departure from efficiency is beneficial to the buyer and costly for the seller, as they both value the asset less than the posted price. The probability of this event happening is equal to the probability that v_B and v_S are both in $[\alpha_{\tau-1}, \alpha_\tau)$, which is approximately equal to $f_S(\alpha_\tau)d\alpha_\tau f_B(\alpha_\tau)d\alpha_\tau$. An upper bound on the buyer's payoff gain and seller's loss relative to efficient trading is $p - \alpha_{\tau-1}$. Thus, relative to the payoff from trading at price p , the buyer's expected gain and seller's expected loss from this event happening at τ is (approximately) bounded above by

$$(p - \alpha_{\tau-1}) \frac{f_S(\alpha_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]} \frac{f_B(\alpha_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]} (d\alpha_\tau)^2.$$

It then follows that, as for the first event, adding up over all $\tau > t$ yields again an upper bound on the expected gain of the buyer and loss of the seller, and this upper bound converges to zero as $T \rightarrow \infty$ and $d\alpha_\tau \rightarrow 0$.

Hence, as $T \rightarrow \infty$ and $d\alpha_t$ and $d\beta_t$ converge to zero for all rounds $\tau \geq t$, the buyer and seller's expected payoffs from the stop-at-value strategies, evaluated before step 1 of round t , converge, respectively, to:

$$\left(\mathbb{E}_{F_B, F_S}^{[\alpha_{t-1}, \beta_{t-1}]} [v_B | v_B > v_S] - p \right) \cdot \Pr(v_B > v_S),$$

which is equal to (1), and

$$\mathbb{E}_{F_B, F_S}^{[\alpha_{t-1}, \beta_{t-1}]} [v_S | v_S > v_B] \cdot \Pr(v_S > v_B) + p \cdot [1 - \Pr(v_S > v_B)],$$

which is equal to (4). □