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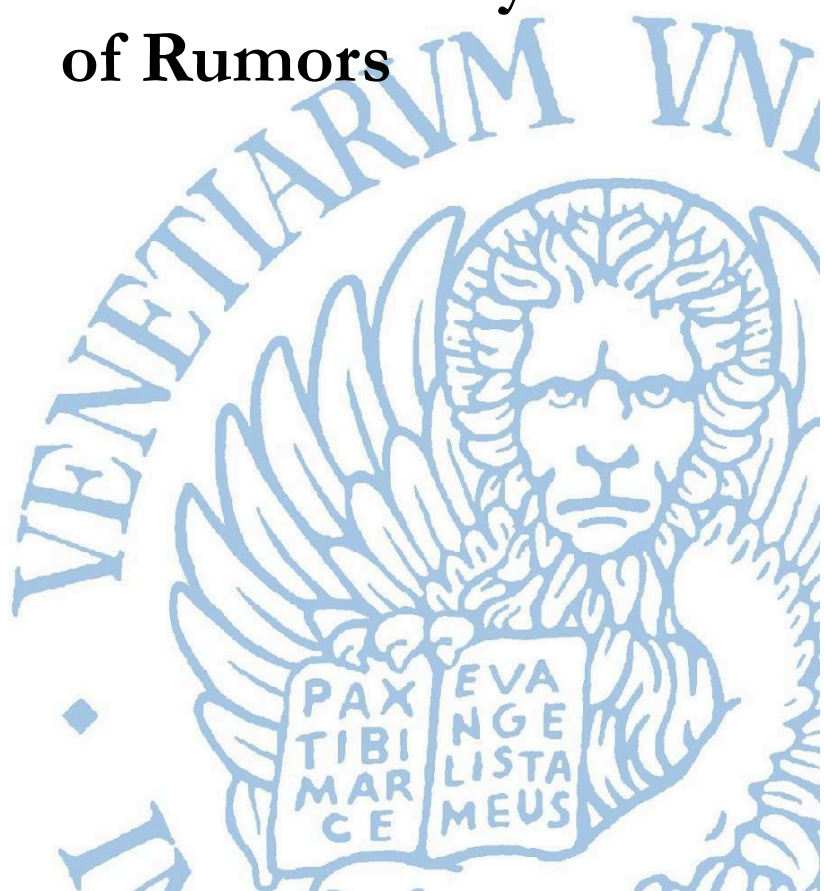
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**On the Profitability
of Rumors**

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Abstract

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Keywords

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1. INTRODUCTION

Since the early days of Internet blogging, there have been several instances in which rumors hyped the price of illiquid penny stocks to the extent that the hypes induced significant price overshooting. In some such instances, individuals spreading rumors made handsome profits by buying the asset before the hypes and selling it at the height of the overshooting. However, the idea that this scheme can be easily implemented is questionable, particularly today, given that large influxes of third-party noise rumors (e.g., unfiltered Internet data) are likely to dilute the overall informativeness of rumors, potentially compromising the overshooting on which the scheme relies.

This article presents a conceptual model for examining the feasibility of the aforementioned scheme. The model distinguishes between two kinds of rumormongers: one with knowledge about the stock's intrinsic value, who provides truthful forecasts through *bearish* or *bullish* rumors, and another who randomly disseminates such rumors and thus generates noise. As suggested in a 2003 article by Jos van Bommel, hereafter referred to as VB, the former may be an amateur analyst spreading rumors through emails, internet blogging, or electronic newsletters. Conversely, as gleanable from Moore et al. (2009), the latter may be a highly skilled or low-skilled hacker pursuing recognition or involved in dubious activities.¹

The model tells us that the amateur may have the opportunity to profit from trading on a price overshooting induced by his own rumors. However, whether or not this opportunity truly exists depends on various factors, such as the probability that a given rumor may be noise rather than truthful, and the shape of the distribution from which the stock's intrinsic value is drawn.

In the model, bearish and bullish rumors are intercepted and followed by a small group of practitioners. Depending on the perceived value of such rumors, this group of rumor followers may trade and spread genuine *sell* or *buy* rumors to linked followers outside the group, triggering an analogous trading and rumor-spreading response among more and more followers that swiftly affect the price through collective trading.

If the initial group intercepts non-conflicting rumors, all followers, both inside and outside the group, spread consistent sell or buy rumors, which is why their collective trading ends up driving the semi-strong efficient price (Fama, 1970) towards one of two "rumor prices," one below and one above the initial price. However, since the source of the intercepted rumors cannot be timely verified,² we have that if they are

¹Hackers do not generally need to write brilliant software to tamper with protected computer systems or alter websites. Instead, they commonly exploit basic vulnerabilities (see Ohm, 2008).

²Even when the source of rumors is observable and trustworthy, it has become increasingly challenging to exclude the possibility

conflicting, the initial group abstains from spreading rumors, thereby nipping any chain of rumor-spreading reactions before it starts. The end result is that, while bearish and bullish rumors occasionally push the price in the right or wrong direction, at other times, they are ultimately ineffective.

Equipped with a unique informational advantage, the amateur has the opportunity to trade twice: once before and once after spreading bearish or bullish rumors. However, as revealed by the model, he will only try to take full advantage of both trading opportunities if he anticipates that, based on his insight into the intrinsic asset value, there is a sufficiently high probability that his rumor will cause the price to significantly overshoot the intrinsic asset value. When this is not the case, depending on the amateur's insight into the asset value, he will either trade solely before his rumor, in the direction of his information, or refrain from trading completely. Nonetheless, when the aforementioned probability is sufficiently high, he will initially trade in the direction of his information, and then, conditional on his rumor actually triggering a significant price overshooting, he will reverse his position at the peak of the overshooting. This is the only instance in which the amateur is able to profit from spreading rumors.

Importantly, there exist three scenarios in which the amateur cannot profit from spreading rumors—regardless of his insight into the intrinsic asset value—and thus, in equilibrium, has no incentive to spread any rumor.

The first scenario arises if the likelihood of noisy rumors is high, while the likelihood of informed rumors is low. Due to semi-strong efficiency, the chain of followers' reactions triggered by the amateur's rumors in such a scenario can only result in a small price shift. Consequently, even if this price shift were to overshoot the intrinsic asset value, the resulting overshooting would not be significant enough for the amateur to profit from a post-rumor trade reversal, once transaction costs are considered.

The second scenario arises if the mass of the asset value distribution is tightly concentrated around the asset's distribution mean. Clearly, because of this concentration, the amateur's rumors can only cause a small

that rumors might indeed have been spread by a hacker. As explained by Joseph J. Cella, former Chief of the Office of Market Surveillance in the Division of Enforcement of the US SEC, and John Reed Stark, formerly in charge of the Division's Internet Programme,

“even if a user knows the identity of the named author of a particular posting [here, the amateur], and trusts that person as a worthy and credible source of information, the author may not actually be that person; the writer may, in fact, be a crook who has used a packet sniffer to steal a legitimate user's identity. Like cat burglars who creep in without ever being seen, stock manipulators [here, the hacker] can steal an identity and make it their own, only the cyber-cat burglars need not climb up the drain pipe for a break-in or even don ski masks to hide their identities” (Cella and Stark, 1997, p. 826).

price shift. As seen with the first scenario, it follows that even if this price shift were to overshoot the asset value, the resulting overshooting would not be significant enough for the amateur to profit from a post-rumor trade reversal.

The third scenario arises if the mass of the asset value distribution is tightly concentrated well above and well below the unconditional mean—which may occur, for instance, in markets for illiquid penny stocks, where there is a substantial gap in the asset’s potential intrinsic values. In this scenario, given that the distribution mass reflects zero possible asset values around the mean, no significant price overshooting is possible, which is why, again, the amateur cannot profit from spreading rumors.

With this paper, we contribute to the important yet relatively small literature started by Roland Benabou and Guy Laroque (1992) on trading and communication in stock markets. Regarding modeling choice, we build on VB, the pioneering theory that studies the profitability of trading based on rumors. Specifically, our paper unifies the communication structures in the VB’s two asset pricing models of informative rumor-spreading, thereby enhancing the accessibility and applicability of the theory. By rigorously assessing the validity of the results suggested by VB’s oft-cited³ theory, our paper clarifies the hidden implications of the assumptions underlying the theoretical framework under scrutiny.

The paper proceeds as follows: Section 2 presents the model’s assumptions, Section 3 derives the model’s predictions, and Section 4 concludes by revisiting the insights from VB using these predictions.

2. ASSUMPTIONS

The model is based on the key assumptions from VB, with some of them being generalized. It considers an asset market in which the asset value, denoted \tilde{v} , is a non-degenerate, independent random variable whose realization, denoted v , is drawn before a sequence of n rounds of activity. The asset value distribution, denoted $f(\cdot)$, is uniform over $[-\beta, -\alpha] \cup [\alpha, \beta]$, where $0 \leq \alpha \leq \beta$.

In the model, there are four kinds of agents: the amateur, noise rumormongers, noise traders, and followers. Let q denote the probability that the amateur is present in the market, and r (or r') denote the probability that a noise rumormonger is present when the amateur is (resp., is not) present. These probabilities are common knowledge and exogenous, with $q(1 - r) \neq 0$. During the first round, the amateur and a noise

³Several papers citing VB have been published over the years, including outstanding works such as Luo (2005), Allen et al. (2006), Aggarwal and Wu (2006), Battalio et al. (2007), Sadka and Scherbina (2007), Goldstein and Guembel (2008), Kyle and Viswanathan (2008), Engelberg and Parsons (2011), García and Sangiorgi (2011), Engelberg et al. (2012), Han and Yang (2013), Lee et al. (2013), Peress (2014), Indjejikian et al. (2014), Ahern and Sosyura (2015), and He and Manela (2016).

rumormonger spread rumors, but only if present in the market. The amateur, who is the sole agent aware of v , spreads “bearish” rumors if $v < 0$ and “bullish” if $v > 0$. In contrast, the noise rumormonger randomizes with equal probability between uninformed “bearish” and “bullish” rumors.

First-round rumors are intercepted by second-round followers, who observe whether the aggregate of these rumors, represented by $A^I + A^U$, is negative, equal to zero, or positive. Specifically, $A^I \in \{-1, 0, 1\}$ equals -1 if the amateur spreads bearish rumors, zero if he spreads no rumor, and 1 if he spreads bullish rumors. Similarly, $A^U \in \{-1, 0, 1\}$ equals -1 if the noise rumormonger spreads bearish rumors, zero if he spreads no rumor, and 1 if he spreads bullish rumors. Consequently, $A^I + A^U$ is negative (or equal to zero, or positive) if second-round followers intercept only bearish rumors (resp., either no or conflicting rumors; only bullish rumors).⁴

In each round $n > 1$, followers trade and spread a rumor $A_{n>1}^F$ that can take on one of three values: -1 implies a sell rumor, 0 implies no rumor, and 1 implies a buy rumor. As in VB, if the expected asset value conditional on round $n - 1$ rumors is negative (or equal to zero, or positive), their rumor equals -1 (resp., 0 ; 1) and their trading demand equals $-x_F m^{(n-1)}$ (resp., zero; $x_F m^{(n-1)}$), where x_F is a positive constant and m is countable and greater than 1 .⁵ In contrast, the demand of noise traders in each round is an independent, normally distributed random variable with mean 0 and variance 1 .

In round n , traders submit orders through a system that publicly reveals the round n aggregate order flow and instantaneously determine the round n price, denoted p_n . In equilibrium, p_n must equal the expected value of the asset conditional upon public information.

The amateur is risk-neutral, and if present in the market, he can trade twice (see VB, beginning of p. 1504): once in round 1 and later on, in round N , when any bearish or bullish rumors, in aggregate, are “*fully incorporated in the price*” (see VB, end of p. 1504). The amateur initially holds no assets, thus having no asset exposure; cannot go longer or shorter than x_L , which is a positive value; and has zero mass. While the amateur’s profits from round n inactivity amount to zero, those from a round n sale and a round n purchase

⁴The implicit assumption that second-round followers neither distinguish the sources of rumors nor whether $|A^I + A^U|$ is equal to 1 or to 2 allows us to generalize VB in a way that increases realism while enabling a direct comparison with VB’s predictions. The rationale behind this implicit assumption is that the number of simultaneous statements or emails that are respectively posted on the amateur’s website or received from the amateur’s email account is inconsequential. The reasonableness of this assumption is supported by the fact that in practical scenarios, as long as simultaneous messages do not conflict with each other, followers are unable to promptly discern whether such messages originate from the same source, and are therefore unable to count them fruitfully.

⁵In what follows, followers’ expectations would remain unchanged if they also conditioned on all past rumors and all publicly available information.

equal, respectively, $p_n - v - c$ and $v - p_n - c$ per asset unit, where c is a positive constant transaction cost per asset unit traded. In equilibrium, the amateur maximizes expected profits.

3. SOLUTION

As a first step to deal with the problem at hand, let $\psi_{q,r,r'}$ denote the probability that, conditional on $A^I + A^U$ being positive, the asset value is also positive. It is straightforward to check that:

$$\psi_{q,r,r'} = \Pr(\tilde{v} > 0 | A^I + A^U > 0) = \frac{\frac{qr}{2} + q(1-r)}{\frac{qr}{2} + q(1-r) + (1-q)r'}. \quad (1)$$

The first lemma follows immediately.

LEMMA 1 *If $A^I + A^U$ is negative (or equal to zero, or positive), the expected value of \tilde{v} given $A^I + A^U$ is equal to $-\psi_{q,r,r'}\xi_f < 0$ (resp., to zero; to $\psi_{q,r,r'}\xi_f > 0$), and so is the expected value of \tilde{v} given $A_{n>1}^F$, where $\xi_f = (\alpha + \beta)/2$.*⁶

PROOF: Available on request. \square

Given the followers' behavior assumed in the previous section, Lemma 1 entails that if $A^I + A^U$ is negative (or zero, or positive), indicating that second-round followers observe only bearish rumors (resp., either no or conflicting rumors; only bullish rumors), each such follower spread rumors to sell (resp., no rumor; rumors to buy), as will followers at any subsequence round.

Once this point is clarified, we can establish the key property of the equilibrium price dynamic. First, recall that in equilibrium, the price must equal to the asset's expected value conditional on the public information available, i.e., on the aggregate flow of orders. Then, note that as a consequence of Lemma 1 and the followers' assumed behavior, if $A^I + A^U$ is negative (or zero, or positive), then round $n > 1$ followers submit a demand equal to $-x_{FM}^{(n-1)}$ (resp., zero; $x_{FM}^{(n-1)}$), indicating that as rounds progress, their demand becomes more and more negative (resp., remains at zero; becomes more and more positive) and is therefore expected to reveal $A^I + A^U$ and, thus, $A_{n>1}^F$ to the public. In fact, as rounds progress, the equilibrium price, i.e., the expected value of \tilde{v} conditional on the aggregate order flow, converges to the expected

⁶For example, if the asset value is uniformly distributed on $[-2, 2]$, then $\alpha = 0$, implying that ξ_f is equal to 1.

value of \tilde{v} given $A^I + A^U$ and, thus, given $A_{n>1}^F$. This unsurprising result is formalized in the following lemma.

LEMMA 2 *As n increases, if $A^I + A^U$ and $A_{n>1}^F$ are negative (or equal to zero, or positive), the equilibrium price p_n converges almost surely to $-\psi_{q,r,r'}\xi_f$ (resp., zero; $\psi_{q,r,r'}\xi_f$).*

PROOF: Available on request. \square

Note in particular that as a consequence of this lemma, if $A^I + A^U$ is negative (or equal to zero, or positive), there exists a finite round N such that when n reaches or exceeds N , the equilibrium price is expected to converge almost surely to $-\psi_{q,r,r'}\xi_f$ (resp., zero; $\psi_{q,r,r'}\xi_f$).

We are now ready to calculate the amateur's equilibrium trading behavior. The calculation leads us to the following important lemma.

LEMMA 3 *In equilibrium, the amateur trades as follows:*

(i) *If he knows that v is such that*

$$|v| < \min[(\psi_{q,r,r'}\xi_f - c), z_{c,f,q,r,r'}], \quad (2)$$

he trades in the first round and with probability $1 - \frac{r}{2}$ in round n . Specifically, he initially sells x_L if $A^I = -1$ or buys x_L if $A^I = 1$, and completely reverses the initial position in round n if $A^I \cdot A_{n>1}^F = 1$, i.e., if his bearish/bullish rumors happen to trigger a chain of sell/buy rumors by followers. His first-round profit is therefore $x_L(|v| - c)$, which may be positive or negative depending on v ; instead, his round n profit is positive and equal to $2x_L(\xi_f\psi_{q,r,r'} - |v| - c)$ if $A^I \cdot A_{n>1}^F = 1$ and is equal to zero otherwise.

(ii) *If he knows that v is such that*

$$\min[(\psi_{q,r,r'}\xi_f - c), z_{c,f,q,r,r'}] \leq |v| \leq \max[(\psi_{q,r,r'}\xi_f - c), c], \quad (3)$$

he does not trade at all and thus earns zero profits.

(iii) *Lastly, if he knows that v is such that*

$$\max[(\psi_{q,r,r'}\xi_f - c), c] < |v|, \quad (4)$$

he trades only in the first round, selling x_L if $A^I = -1$ or buying x_L if $A^I = 1$. This enables the amateur to earn a positive profit equal to $x_L(|v| - c)$. The function $z_{c,f,q,r,r'}$ is explicitly given by

$$z_{c,f,q,r,r'} = \frac{(2-r)\psi_{q,r,r'}\xi_f - c(3-r)}{1-r}. \quad (5)$$

PROOF: Available on request. \square

The above lemma implies the following result:

PROPOSITION 1 *Ex ante, i.e., before v is drawn, the probability that the amateur can profit from spreading rumors in equilibrium is positive if and only if*

$$\Pr \left\{ |v| < \min \left[(\psi_{q,r,r'}\xi_f - c), \frac{(2-r)\psi_{q,r,r'}\xi_f - c(3-r)}{1-r} \right] \right\} > 0. \quad (6)$$

PROOF: Follows directly from Lemma 3. \square

Importantly, note that the satisfaction of Proposition 1's condition crucially depends on q and on r' (respectively, the probability of the amateur being present in the market and the probability of the noise rumormonger being present when the amateur is not). As q decreases to zero and r' increases to one, the function $\psi_{q,r,r'}$ declines to zero. As a result, given that c is fixed and strictly positive, the condition in question turns out to be violated. This leads to our first remark:

REMARK 1 *If q is sufficiently low and r' is sufficiently high, it is never possible for the amateur to profit from rumors.*

As a minor point of clarification, it is worth noting that while the first remark emphasizes the limit values of q and r' for which Proposition 1's condition is inevitably violated, the same first remark refrains from discussing this condition relative to r (the probability of both the amateur and a noise rumormonger being in the market simultaneously). This is because no specific value of r alone leads to the violation of this condition, unless in conjunction with other model parameters that, at certain limit values, independently drive the outcome.

⁷The right-hand side of this equation is a legal fraction because, as assumed, $q(1-r) \neq 0$, ensuring that $1-r \neq 0$.

Next note that the satisfaction of the Proposition 1's condition crucially depends on whether the mass of the asset value distribution is concentrated around the asset's distribution mean. Specifically, if ξ_f is sufficiently small, then the condition in the preceding proposition is inevitably violated, given that c is positive. This leads to our second remark:

REMARK 2 *If the mass of the asset value distribution is sufficiently concentrated around the asset's distribution mean, it is never possible for the amateur to profit from rumors.*

Finally, note that the satisfaction of Proposition 1's condition crucially depends on whether the asset value distribution exhibits any wide discontinuity around its mean. Consider a scenario in which α is equal to zero (indicating a continuous distribution) and the quintuple $\langle q, r, r', c, \xi_f \rangle$ satisfies the condition in question—and in which, therefore, the amateur has a positive ex-ante probability of profiting from spreading rumors. If α becomes positive and, more specifically, its distance from ξ_f becomes sufficiently small (indicating a sufficiently wide distribution discontinuity around zero), then the condition in question no longer holds.⁸ This leads to our third remark:

REMARK 3 *If the asset value distribution exhibits a sufficiently wide discontinuity around its mean, it is never possible for the amateur to profit from rumors.*

Now, turning back to Lemma 3, it is crucial to highlight that the amateur's trading plan depends non-trivially on $\langle q, r, r', c, \xi_f \rangle$. This dependency can be succinctly presented as follows:

LEMMA 4 *In equilibrium, (i) for $\frac{c}{\psi_{q,r,r'}\xi_f} < \frac{1}{2}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is “low”), the amateur always makes a first-round trade in the direction of the information, and reverses completely the initial position if $|v|$ is smaller than $\psi_{q,r,r'}\xi_f - c$. Instead, (ii) for $\frac{1}{2} \leq \frac{c}{\psi_{q,r,r'}\xi_f}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is not “low”), the amateur implements the just-mentioned trading scheme if and only if $|v|$ is either smaller than $z_{c,f,q,r,r'}$ or greater than c , and does not trade in any round otherwise.*

PROOF: Available on request. \square

⁸Firstly note that for this condition to hold, it must be that $\alpha < \min[(\psi_{q,r,r'}\xi_f - c), z_{c,f,q,r,r'}]$. Then, note that as α approaches ξ_f , the right-hand side of this inequality becomes smaller than α . In fact,

$$\lim_{\alpha \rightarrow \xi_f} \min[(\psi_{q,r,r'}\xi_f - c), z_{c,f,q,r,r'}] \leq \lim_{\alpha \rightarrow \xi_f} (\psi_{q,r,r'}\xi_f - c) = \psi_{q,r,r'}\alpha - c < \alpha.$$

Consequently, as α approaches ξ_f , the condition in question is inevitably violated.

Before introducing our next proposition, we shall finally examine another implication of Lemma 3:

LEMMA 5 *In equilibrium, if $f(\cdot)$ is continuous around its mean, then, ex ante, the probability that the amateur can profit from rumors is positive if $0 < z_{c,f,q,r,r'}$, i.e., if $\frac{c}{\psi_{q,r,r'}\xi_f} < \frac{2-r}{3-r}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is not “high”). If this condition is violated, then, regardless of the continuity of $f(\cdot)$, profiting from rumors is impossible.*

PROOF: Available on request. \square

We are now ready to fully characterize the amateur’s equilibrium behavior as a function of $\frac{c}{\psi_{q,r,r'}\xi_f}$.

PROPOSITION 2 *Consider any distribution $f(\cdot)$ and any quintuple $\langle q, r, r', c, \xi_f \rangle$. In equilibrium,*

(i) *in the case in which $\frac{c}{\psi_{q,r,r'}\xi_f} < \frac{1}{2}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is “low”), the amateur always makes a first-round trade in the direction of the information, and reverses completely the initial position if $|v|$ is smaller than $\psi_{q,r,r'}\xi_f - c$;*

(ii) *in the case in which $\frac{1}{2} \leq \frac{c}{\psi_{q,r,r'}\xi_f} < \frac{2-r}{3-r}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is neither “low” nor “high”), the amateur implements the just-mentioned trading scheme if and only if $|v|$ is either smaller than $z_{c,f,q,r,r'}$ or greater than c (where in this second case, $0 < z_{c,f,q,r,r'} < c$), and does not trade in any round otherwise;*

(iii) *in the case in which $\frac{2-r}{3-r} \leq \frac{c}{\psi_{q,r,r'}\xi_f}$ (signifying that $\frac{c}{\psi_{q,r,r'}\xi_f}$ is “high”), the amateur makes a first-round trade in the direction of the information if and only if $|v|$ is greater than c , and finds it impossible to profit from spreading rumors.*

PROOF: Follows directly from Lemmas 3, 4, and 5. \square

While no commentary is needed for the results at Points *i* and *iii* of the above proposition, the one at Point *ii* does require clarification. This result tells us that if $\frac{1}{2} \leq \frac{c}{\psi_{q,r,r'}\xi_f} < \frac{2-r}{3-r}$, then, depending on the exact realization value of v , the amateur may not trade, not even at the first round, which happens for the case in which he observes that v is such that $z_{c,f,q,r,r'} \leq |v| \leq c$. In this case, in fact, since the amateur’s information advantage (measured as $|v|$) is not sufficiently large, he knows that an initial trade can only be loss-making; and since this advantage is not sufficiently close to zero either, he has no expectation that a subsequent trade reversal can offset the initial loss. This is precisely why, for these observed values of v , the amateur refrains from engaging in any trading.

4. CONCLUDING REMARKS

Outlined in the introduction is that our informative rumor-spreading structure unifies the two assumed in VB, where r equals zero, and consequently, $\psi_{q,r,r'}|_{r=0}$ equals $\tilde{\psi}_{q,r'} = \frac{q}{q+(1-q)r'}$. Specifically, in one such VB structure, it is assumed that both r and r' equal zero, implying that the spread rumors can only be informed. In the other, it is assumed that while r equals zero, r' equals one, implying that the spread rumors can either be noisy or informed, but cannot conflict with each other. While these two rumor-spreading structures represent subcases of ours, attainable for specific values of r and r' , it is important to recognize, however, that even for these specific values, the key insights from VB do not align with ours, despite coming out from the same relevant assumptions.⁹

To clarify this matter, it is worth recalling VB’s insights, namely that: (1) regardless of the parameter values, the amateur can profit from spreading rumors; (2) after observing the asset value, the amateur systematically engages in trading; and (3) after a first round trade, the amateur in round N reverses his initial position whenever the round N price overshoots the expected asset value—see VB’s Propositions 2 and 3, respectively assuming a structure without and with noise rumors, in order to track these insights. It is also worth recalling that for the purpose of exemplifying these insights, VB considers a specific asset value distribution (namely, the uniform distribution over $[-2, 2]$), asserting that “[a]ny distribution can be used to demonstrate the intuition of th[e] paper” (see VB’s assumptions section, p. 1502). Armed with the refined predictions from our unified model, it becomes clear that none of these VB insights are actually valid.

In order, consider the insight in Point 1 above. In light of our Proposition 1, it is clear that, in equilibrium,

⁹Three remarks are important: (i) While VB assumes that the cost c must be positive but small, we generalize this assumption by allowing c to take any positive value. This generalization is useful in illustrating that in reality, regardless of the assumed magnitude of c , trading strategy and rumor profitability depend exclusively on the value of v observed by the amateur and on how c relates to q , r , r' , and ξ_f . (ii) While refraining from presenting the amateur’s profit function, VB assumes that in a one-round context, the amateur “buys x_L units whenever she sees $\tilde{v} > c$ and sells x_L when she sees $\tilde{v} < -c$ ” (see VB, p. 1503), as is also evident, for instance, from VB’s Equation A1. This tells us that since the amateur is a “risk-neutral informed trader” (see VB, p. 1502), the amateur’s profits from a round n sale and a round n purchase in the VB’s environment equal, respectively, $p_{n-1} - v - c$ and $v - p_{n-1} - c$ per asset unit, where p_0 is equal to zero. Even though in VB it is additionally assumed that x_L must be small compared to the standard deviation of the noise demand, note that this additional assumption in the VB’s environment does not affect the long-run price (and, in particular, p_{N-1}) at equilibrium, nor does it affect the amateur’s trading problem (which hinges on v , on c , on p_0 , on the amateur’s expectation about p_{N-1} , and on the constraint that he can trade only in rounds 1 and N). Given all this, the amateur’s problem in VB is equivalent to one where the amateur has zero mass, which is in turn equivalent to one where his profits from a round n sale and a round n purchase equal, respectively, $p_n - v - c$ and $v - p_n - c$ per asset unit (as assumed in our paper), resulting in identical solutions for the three problems. (iii) By interpreting the cost c as an implicit charge paid by the amateur to a retail bank for intermediated access to the market, it becomes clear that in the VB’s environment, even though any round n asset unit traded through the bank necessarily executes at the market price p_n , the bank charges the amateur the price p_{n-1} along with the cost c . Adopting this intermediated access interpretation highlights that our model’s environment (where the bank charges our zero-mass amateur the price p_n along with the cost c in round n , therefore) is not only slightly more realistic, but it also ensures that the bank’s round 1 intermediation activity is not loss-making, even for very small values of c .

under VB's rumor-spreading structures, the amateur can profit from spreading rumors if and only if the absolute value of the observed v falls below a certain threshold (i.e., $\min[(\tilde{\psi}_{q,r'}\xi_f - c), (2\tilde{\psi}_{q,r'}\xi_f - 3c)]$). However, VB overlooks this condition, leading to the flaw in the insight presented in Point 1. Specifically, as highlighted by our Remark 1, if r' is high and q is small (as possible in VB's rumor-spreading structure with noise rumors), the mentioned condition is inevitably violated, irrespective of the exact transaction cost c and the shape of the asset value distribution (and consequently, even in the example presented in VB's Proposition 3). Similarly, as highlighted by our Remarks 2 and 3, if the mass of the asset value distribution is concentrated around the distribution's mean (formally, if ξ_f is small), or if the distribution exhibits a wide discontinuity around the mean, then the aforementioned condition is violated, regardless of which of the two VB's rumor-spreading structures we consider.

Next, to underscore the flaw in the insight presented in Point 2 above, let us narrow the focus to the case where the asset value distribution is continuous. Note that in this case, our Proposition 2 implies that under VB's two rumor-spreading structures, even when restricting attention to those triples $\tilde{\psi}_{q,r'}, \xi_f, c$ such that $\frac{c}{\tilde{\psi}_{q,r'}\xi_f}$ is smaller than $\frac{2}{3}$ (i.e., even when, ex ante, the amateur can profit from rumors), it is only for $\frac{c}{\tilde{\psi}_{q,r'}\xi_f}$ smaller than $\frac{1}{2}$ that the amateur engages systematically in trading—contrary to what VB claims. Note in particular that in VB's rumor-spreading structure with noise, even if $\frac{c}{\tilde{\psi}_{q,r'}\xi_f}$ is initially smaller than $\frac{1}{2}$, decreasing q towards zero inevitably leads to $\frac{c}{\tilde{\psi}_{q,r'}\xi_f}$ becoming greater than $\frac{1}{2}$. Thus, under the structure with noise, the insight in Point 2 is not valid, for any considered values of c and ξ_f (and consequently, even for those in the example presented in VB's Proposition 3).

Finally, to underscore the flaw in the insight presented in Point 3, note how our Proposition 2 indicates that even under VB's two assumed rumor-spreading structures, the amateur reverses his position only if the round N price overshoots the expected asset value by more than c —contradicting the assertions made in VB's Propositions 2 and 3, from which the third insight above arises.

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