Dynamic Games and Applications
https://doi.org/10.1007/s13235-022-00485-5

RESEARCH

Complexity of Behavioural Strategies and Cooperation in the Optional Public Goods Game

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Accepted: 29 November 2022

Abstract
The problem of collective action where—beside the standard options of cooperating and defecting—there is also the possibility of opting out has been extensively studied through the optional public good game (OPGG). Within this and other social dilemma games, reputation systems, composed of a social norm—assigning reputations to agents—and a set of behavioural strategies using this information to condition their behaviour, are able to sustain cooperation. However, while the relationship between the complexity of social norms and cooperation has been extensively studied, the same cannot be said with respect to behavioural strategies, due to high dimensionality of the strategy spaces involved. We deal with this problem by building an agent-based model where agents adopt simple social norms, play the OPGG and learn behavioural strategies through a genetic algorithm. We show that while social norms which assign different reputations to defectors and to agents opting out achieve the highest levels of cooperation, the social norms that do not distinguish between these actions do improve cooperation levels with respect to the baseline when behavioural strategies are sufficiently complex. Furthermore, we find that cooperation increases when the interaction groups are small enough for agents to accurately distinguish between different behaviours.

Keywords Evolution of cooperation · Reputation · Strategy complexity · Optional public good game

1 Introduction

Cooperative behaviour is pervasive throughout nature and society from the microscopic interactions between cellular organisms [20, 24, 63] to the global matters of trade and collaboration

This article is part of the topical collection "Evolutionary Games and Applications" edited by Christian Hilbe, Maria Kleshnina and Kateřina Staňkovà.

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Published online: 09 January 2023

between governments [56, 80]. A variety of solutions have been proposed [48] by researchers of various disciplines, among others kin selection [25, 26], direct [5, 46], indirect [40, 51, 53, 54, 58, 62] and network [49, 55, 68] reciprocity, punishment [14, 16], policing [15, 18, 19] and herding [71].

A key class of cooperation problems can be ascribed to the issue of identifying the conditions that allow the emergence of collective action, i.e. to find when a group of individuals can work together for the common good. The Public Goods Game is frequently adopted as the game theoretical tool to study this kind of situations. In this type of game, a public good can be built through the contributions of members of a group, but while the common good is maximised when everyone contributes (the social optimum), each group member has the incentive to deviate and withhold their contribution, free-riding on the efforts of others [27]. The public goods game is typically analysed as a choice between contributing (or cooperating) and free-riding (or defecting) [39]. However, there are many situations in which there is also the option not to participate (or to exit) a situation, e.g. due to ostracism [42] or to individual choice [28]. Unlike defectors, who refuse to contribute to the public good while enjoying its proceeds, a so-called "Loner" abstains altogether from interactions, forgoing not only its costs but also its potential benefits, and accepting instead a fixed payoff independent of others' actions. This variant of the Public Goods Game, with the possibility of being a Loner, is the so-called Optional Public Goods game (OPGG).

The complexity and dynamic nature of human interactions frequently involving changes in interacting partners implies a strong role for indirect reciprocity in sustaining cooperation, which emerges when an individual's cooperative behaviour is rewarded not by the recipient of the cooperation, but rather from third parties, who were able to observe [47, 50] or acquired information (e.g through gossip [22, 66]) about the behaviour or the reputation of the individual.

In order for indirect reciprocity to work, there needs to exist a mechanism through which the question "is this person good?" is answered. One method achieves this by assuming that every individual has an intrinsic reputation [50] that acts as a signal of their trustworthiness to help others. The rules that distinguish between actions that are *good* (which should be recognised with a good reputation) and actions that are *bad* (which should be recognised with a bad reputation) are known as social norms. Reputations are assigned by independent third parties who "witness" the interaction and/or spread the information either selectively [62] or publicly [53, 54]. On top of a social norm, indirect reciprocity requires one or more rules to transform reputations of other individuals into actions, answering the question "is this person worth helping?"; these rules are called behavioural strategies.

The literature has explored a wealth of reputational systems, ranging from the simple image scoring [51] (simply prescribing cooperation with collaborative agents), to the more elaborate standing criterion [40, 58] and the Leading Eight [53, 54]. While the complexity of social norms has been carefully assessed [69, 70] showing that relatively simple social norms can achieve high levels of cooperation, the role of complexity in behavioural strategies has been largely unexplored in a systematic fashion. Indeed, until now, approaches to solving the evolution of cooperation through reputation have predominantly been methodical and static searches of pre-determined strategies [52]. While this approach is straightforward when strategies remain relatively simplistic, it quickly becomes computationally intractable as the dimension of the behavioural space increases. In particular, where collective action problems are concerned, social norms help agents to acquire reputational information on their group members, and thus acquiring a rather precise knowledge of the average characteristics of the group. This in turn can serve to condition behavioural strategies of arbitrary complexities, reacting in a (potentially) different way to smaller and smaller changes in average reputation. The relationship between the complexity of a behavioural strategy and the chances of cooperation has not been studied, especially within the context of the OPGG.

While a large part of the literature on reciprocity studies two-player social dilemmas (e.g. the prisoner's dilemma), a number of studies address the emergence and stability of reciprocity in games with many players (i.e. forms of PGG or n-persons prisoner's dilemma). When these games are considered, cooperation is considerably more difficult to achieve [45]; indeed, no finite mix of pure strategies can be evolutionary stable in the absence of errors or a working reputation system [82]. When agents are endowed with memory of past events, reciprocal strategies (both direct and indirect) can also emerge in n-person prisoner's dilemmas [1, 29], leading to sustainable cooperation [37], but only under progressively restrictive assumptions when groups become larger [6, 33].[1]

Specifically concerning indirect reciprocity, unlike two-player games, a simple discriminator strategy, which only cooperates with people who have a good reputation, is evolutionarily stable, even when reputations are assigned simply through image-scoring [75], although the conditions for the evolution of indirect reciprocity are more relaxed when the standing social norm is adopted instead [74]. In more general setups, reciprocal strategies can sustain cooperation through periodic or chaotic oscillations [73]. Also under indirect reciprocity, cooperation again becomes more difficult to sustain in larger groups [74, 76].

Introducing the loner strategy in the OPGG [28], the cooperative dynamics become cyclical: a population dominated by cooperators tends to be invaded by defectors, which are—in turn—vulnerable to invasion by loners. When the latter come to dominate the population, cooperation becomes a viable strategy again starting the cycle anew. Further research showed that while pro-social punishment [30] can break the cycle and increase cooperation, the existence of anti-social punishment [31, 59, 64, 65] re-instates the cyclical dynamics, nullifying the role of pro-social punishment.[2] More recent contributions [61] showed that even a simple reputational system based on social norms assigning a strictly worse reputation to defectors than to loners can increase cooperation with respect to previous findings but with limited success. Furthermore, pro-social punishment and reputation work synergically resulting in no cyclical dynamics and high and persistent levels of cooperation. Crucially, the behavioural strategies studied in [61] are very simple, being effectively limited to playing one of two actions, depending on a single threshold concerning the average reputation of a group. Such strategies can be considered simplistic when human behaviour is concerned. While more complex strategies, based on a more finely grained mapping between group reputation and agents' actions, could in principal be considered, the systematic study though basic evolutionary approaches would rapidly become numerically intractable.

Genetic algorithms (GAs) [34, 38, 44], like many other evolutionary algorithms work through selection, crossover and mutation [23] providing a very natural—and yet little explored—mechanism to study the evolution of cooperation in the presence of complex and numerous behavioural strategies. Nevertheless, while GAs have been shown to be an alternative method of optimisation, widely adopted in areas of environmental modelling [11, 13, 41], operation management and image processing among others [38], they have not yet been fully explored in this area. Their key feature is that they allow behavioural strategies to evolve dynamically starting from a wide and heavily heterogeneous population of tentative strategies, moving towards those that are better suited to a given environment. The GAs allow evolution to focus on the most promising strategies rather efficiently [4], avoiding the limits of systematic studies with fixed strategies. Furthermore, the GAs preserve the ability

[1] This problem can be partially circumvented through punishment [7, 61] or alliances [32]

[2] The emergence of these cycles of punishments have been challenged when loners cannot be punished [21].

of strategies to co-evolve with their cooperation environment. Their main downside is that unlike the systematic exploration of strategies under standard evolutionary approaches, GAs do not ensure that each simulation explores the viability of each single possible strategy. This limitation however can be attenuated through a sufficient number of Monte Carlo simulations.

Despite their benefits, GAs have been little used in studies on indirect reciprocity and cooperation. Some studies [10] investigate repeated dyadic interactions which gives rise to spontaneous and coordinated alternating reciprocity, while others [12] find that the adaptability of individuals in one-shot interactions of the ultimatum game can have a positive impact on cooperation. In the context of reputation-based indirect reciprocity, only a select few works have used a GA to address the explosion of possible strategies and norms. In particular, [78] studies a 2×2 prisoner's dilemma, allowing strategies to evolve through a GA [81]. Relatedly within the context of a giving game, [81] uses a particular application of the GA to study the evolution of social norms, and identifies the key characteristics a social norm requires in order to sustain cooperation.

In this paper, we implement simple image scoring-based social norms as in [50, 61] while manipulating the behavioural strategy complexity to allow agents to respond in a more nuanced manner to their reputation environment. By generalising the choice of actions given the average reputation of some OPGG group, we allow players to learn the situations in which it is optimal to cooperate, to defect or to abstain entirely. It should be noted that the temporal scale explored by this paper is one of social evolution, and not of genetic evolution. GAs are used as a tool to explore a large space of possible strategies efficiently (in line with [4] which used them to study the evolution of preferences in financial markets and [78, 81] where they are used to explore the stability of indirect reciprocity strategies). We find that increasing the complexity of behavioural strategies can improve the chances of cooperation under the social norms that distinguish between being a defector and being a loner. Additionally, we find that increased complexity is most beneficial for smaller OPGG groups, and does not extend to larger groups. In the following, we describe the setup of the model in Sect. 2, state its main results in Sect. 3, and discuss its implications in Sect. 4.

2 Model

Consider a population of N players, associated with a single social norm. Players can have either a good (1), bad (-1) or medium (0) reputation. These are assigned by a social norm on the basis of the last action. We assume that, within a group, cooperation is always assigned the highest reputation (+1). This makes sense considering that cooperation does not involve any intrinsic moral evaluation: within a group cooperating means helping to build the public good of the group. This might be something good for the society as a whole (e.g. engaging in a social activity) but it might also be something socially bad (e.g. engaging in a criminal activity). We then separately consider four social norms that rank the actions of defection and abstaining differently from cooperation. In particular, under the Anti-Defector (AD) social norm, defection is assigned a strictly worse (-1) reputation than the loner action (which is assigned reputation 0). The Anti-Loner (AL) social norm instead assigns the worst reputation to agents who abstain from participation (-1) and a more moderate reputation to defectors (0). The Anti-Neither (AN) and the Anti-Both (AB) norms view both defection and abstaining equally, with the former assigning a reputation of 0, and the latter assigning a reputation of -1. The correspondence between decisions and reputations are summarised in Table 1.

Table 1 Summary of the possible social norm reputation assignments given a particular action

	Cooperating (C)	Defecting (D)	Abstaining (L)
Anti-Defector (AD)	1	−1	0
Anti-Loner (AL)	1	0	−1
Anti-Neither (AN)	1	0	0
Anti-Both (AB)	1	−1	−1

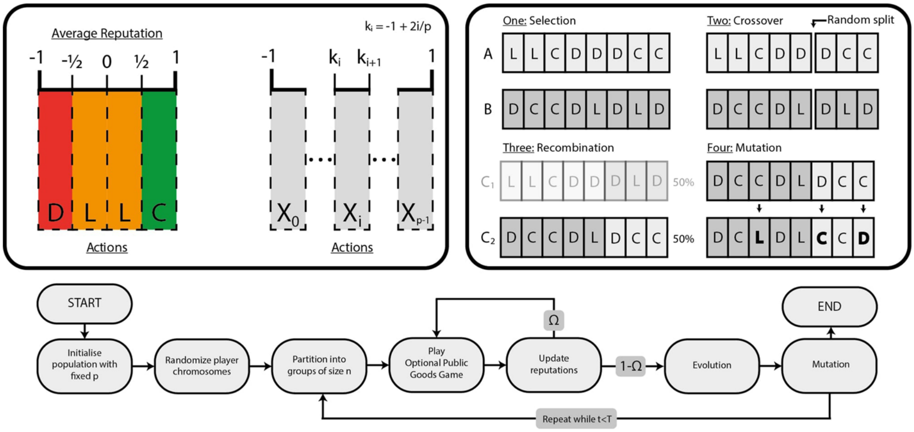

Fig. 1 Model Components. Upper left panel describes an example of a player's behavioural strategy for $p = 4$. Each player has a chromosome of p alleles. The ith allele $i \in [0, p-1]$ encodes an action X $\in$ {C, D, L} in the case that the average group reputation falls in $[k_i, k_{i+1})$ where $k_i = -1+2ip^{-1}$ (thus $k_0 = -1$ and $(k_p = 1)$), except for the $(p-1)_{th}$ allele which encodes for $([k_{p-1}, k_p])$. Upper right panel describes the evolutionary process. Once ζ of the worst performing players are eliminated, we choose two parents proportional to their payoffs, split and swap their chromosomes each at the same randomly chosen allele, choosing one of the two combinations at random for the new child. Once the population is whole again, we choose one player to mutate with probability ϵ_1, randomly choosing each of its chromosome's alleles with probability ϵ_2. Lower panel describes the dynamics of the model

Player j's behavioural strategy is represented as a chromosome vector $s_i \in \{\text{C,D,L}\}^p$ where the ith element (i.e. ith the allele) represents j's action when the average reputation of their OPGG group (excluding themselves) falls between $-1 + 2ip^{-1} \leq \text{rep} < -1 + 2(i+1)p^{-1}$ for $i \in [0, p-2]$ and $-1 + 2ip^{-1} \leq \text{rep} \leq -1 + 2(i+1)p^{-1}$ for $i = p-1$. In other terms, p is the number of possible levels of average reputation that an agent can discern. This allows us to build arbitrarily complex behavioural strategies, able to associate a specific action to slightly different average reputations within their group. Furthermore, this mechanism allows to generalise the simple conditional strategies studied by [61] where $p = 2$ which allowed for only two actions, one if $-1 \leq \text{rep} < 0$, another if $0 \leq \text{rep} \leq 1$.

Every simulation is initialised with a single social norm, a fixed value for p and with the N players in the population each having a random chromosome, with the action associated to each allele being uniformly extracted from the set {C, D, L}, with equal probability. At the start of each period, the population is randomly partitioned into groups of size n to play the OPGG. Each player in the group identifies the average reputation of the group (excluding themselves) and uses their chromosome to decide their action. It should be noted that while it is possible to build strategies of arbitrary complexity p, the actual number of average reputations that a group can assume is limited by the number of agents n forming the group. Indeed for each n, only $2(n-1)+1=2n-1$ possible average levels of average reputation are possible.[3] Once $p > 2n-1$, additional complexity simply adds to the length of the genetic code, but since some intervals of reputation never actually appear (and cannot actually appear) in the population, they are essentially useless. For this reason, we run our baseline simulations with $n = 5$ and we study only larger groups where more complexity can actually be used.

In each interaction, cooperators pay a cost of c to contribute to the public good, defectors participate but pay nothing, and loners do not participate. The total amount contributed to the public good is multiplied by a synergy factor r and the proceeds are equally divided among the total number of participants in the group, regardless of their contribution. Loners are given a payoff of σ where $0 < \sigma < (c-1)r$. Each group plays this game at least once, with subsequent rounds being played with probability Ω, or equivalently the OPGG is played until the period ends with probability $1-\Omega$.

Once the period ends, each player's average payoff is calculated and forms their fitness. The population's players are then sorted in descending order of fitness, and the bottom ζ proportion of the population is eliminated. The eliminated players are then replaced by randomly selecting two parents from the population proportionally to their fitness values. Once selected, a single crossover chiasma point is selected, and the chromosomes are rearranged to form a new chromosome for the resulting child. The parents are unchanged. This is repeated until we return to a full population size. This process tends to create agents whose actions are better fit for their current interaction environment.

At the end of each time step, with probability ϵ_1, a single player is chosen for mutation. Each of the player's alleles have some probability ϵ_2 of being mutated. The new alleles are independently and uniformly selected from the set of possible alleles $\{C, D, L\}$. In baseline simulations, $\epsilon_2 = 1$ meaning that the selected agent's strategy is completely randomised. The mutation process creates the noise required to ensure that the genomes selected by the evolutionary process are robust to the invasion of randomly created new behavioural strategies.

Finally, in line with [53], we consider extensions of our model including two types of errors. The first is an execution error α, which indicates the probability that an agent that would like to play action x ends up playing another—randomly chosen—action $\neg x$.[4] The second is an assignment error β, which indicates the probability that an agent plays an action x but this actions is observed by others as a different action y and thus changes his reputation accordingly. Both α and β are set to zero in baseline simulations and explored in supplementary material. As long as α is not too high ($\alpha < 25\%$ with any β), our key result concerning the relationship between complexity and cooperation is confirmed.

We then repeat the above process of repeated games, evolution, mutation for a very large number of periods T (typically 10^5, see Table S1). We investigate the behaviour of the model through the variables: p (the complexity of a player's chromosome). We further test the model for the robustness of results studying the impact of changing ζ (the proportion of the population that is replaced each period), Ω (the frequency of repeated games within a single period), ϵ_1 (the frequency of mutation) and ϵ_2 (the severity of mutation), α and β (implementation and reputation assignment errors), N (the size of the population), n (the size

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[3] Each individual looks at the average reputation computed across all other agents in the group, excluding herself.

[4] [53] implement the execution errors as a small chance of an agent willing to cooperate being prevented from doing so. We chose to implement it symmetrically due to the addition of the third basic strategy in the OPGG.

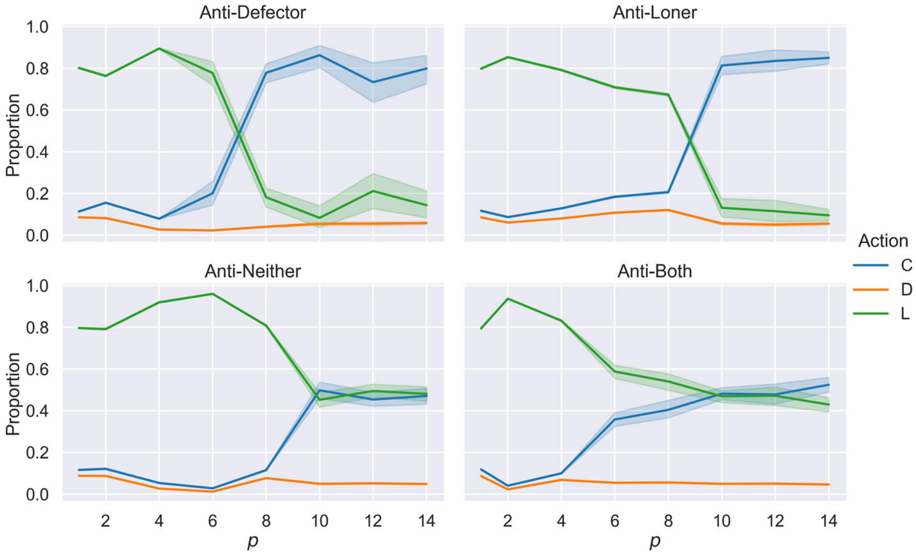

Fig. 2 Cooperation is more successful when social norms clearly distinguish between acts of defection and withdrawal from the game, otherwise cooperation cannot reliably exceed 50% in the population. Defection never truly gains a foothold in the population. The baseline label at $p = 1$ refers to the typical standard OPGG result in the absence of reputation or any other mechanism facilitating cooperation. In all cases, the OPGG is played in groups of five ($n = 5$)

of an Optional Public Good Game), c (the cost of cooperation), r (the group synergy factor), and σ (the loner's payoff). All these robustness checks yield results which are qualitatively similar to those presented in the main text and are thus relegated to supplementary material.

3 Results

In order to assess the relationship between behavioural strategy complexity and cooperation, we report the average proportion of cooperative (blue line), non-cooperative (orange line) and loner (green line) actions with their standard deviations in Fig. 2. For each action and each level of complexity p, the averages are computed first across time (over the second half of a single simulation) and then averaging the results of 20 simulations.

When $p = 1$, the agents play unconditional strategies and do not use the social norms. The leftmost point of each panel in Fig. 2 reports the traditional results of [28] with the majority of the population opting out of most interactions, resulting in very low levels of cooperation and defection (see Figure S1 for the time series of a single simulation for different values of p).

More complex behavioural strategies improve the chances of cooperation in all social norms to varying extents. The Anti-Defector norm is arguably the most effective. When $p \geq 8$, the AD social norm supports consistently high levels of cooperation (around 80% over time and different simulations). A similar outcome is achieved under the Anti-Loner social norm, although achieving similar ($\sim$ 80%) levels of cooperation requires more complex strategies where $p \geq 10$. Below this threshold, the AL norm guarantees only up to $\sim$ 20% of cooperation actions.

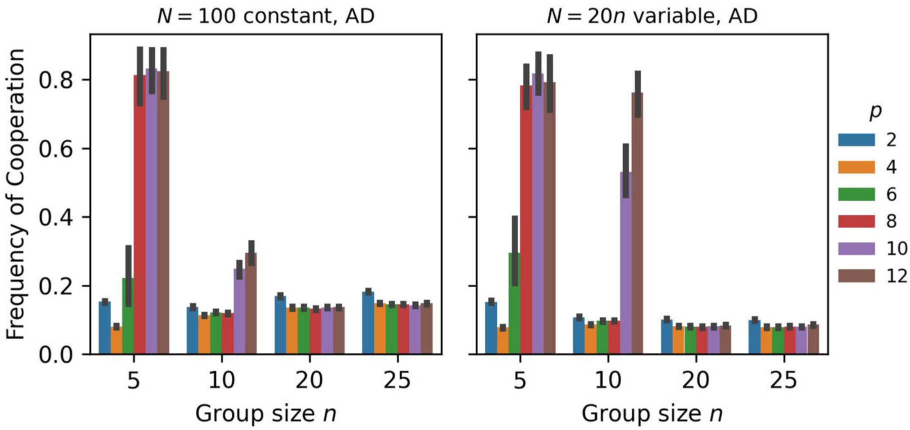

Fig. 3 More complex strategies help, but only in smaller OPGG groups. This is true both when the population size N is constant or proportional to the group size n. Increasing the complexity of behavioural strategies does not help cooperation in larger groups. However, there is a slight improvement of more complex strategies $p \geq 10$ and $n = 10$ when the population grows proportionally to n

Contrastingly, the social norms that do not differentiate between players who abstain instead of directly free-riding result in lower levels of cooperation at all levels of complexity. For both the Anti-Neither and Anti-Both social norms, defection and withdrawal are both seen to be equally worse than cooperation, with the latter social norm taking the harsher view. If the strategies are sufficiently complex, then in either case, at most half of the population will tend to cooperate while the other half abstains. Interestingly, the harsher Anti-Both norm allows a similar level of cooperation to the Anti-Neither, but with less complex strategies (lower p). Indeed, the Anti-Neither norm jumps from almost 0 to 50% cooperation at $p = 9$, while the Anti-Both norm allows some cooperation (30–40%) at $p = 5$ eventually leading to a stable 50% cooperation when $p > 9$ (see Figure S2 for the same graph with odd values of p, yielding qualitatively similar results). The fact that the levels of cooperation reported in Fig. 2 stabilise for $p \geq 10$ shows that—once the strategies are able to suitably represent all possible levels of group reputations—further increases in complexity do not yield any more cooperation.[5]

The relationship between cooperation and the number of individuals involved in the collective action problem has been subject of scientific debate. While some [74, 76] argue that cooperation becomes more difficult as the group size increases, others [36]—through large-scale experimental studies—found that the public good is better sustained with larger groups instead of smaller ones as long as the Marginal Per-Capita Return is fixed [35].

We run two sets of experiments to test (1) whether larger groups increase or decrease cooperation in the OPGG, and (2) how the interaction between strategy complexity and group size reflects on the chances for the emergence of cooperation. We discuss the results of the experiments for the Anti-Defector social norm, which provides the better chances for cooperation at all levels of complexity of the behavioural strategies, while reporting the results for the other social norms in supplementary material. In both experiments, we consider

[5] In Fig. 2, $n = 5$, which means that there are only 9 possible levels of average reputation in a group.

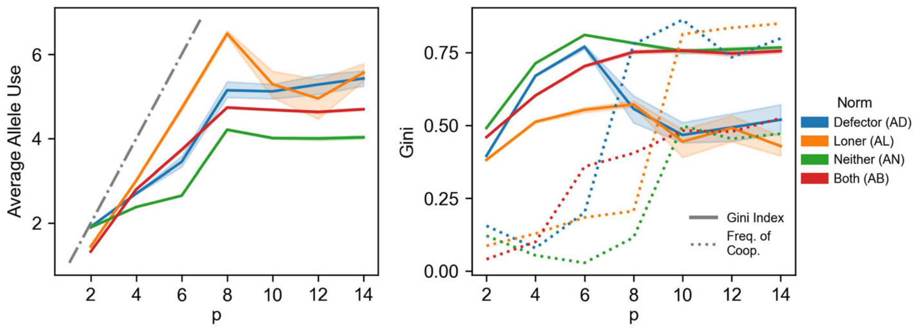

Fig. 4 Actual use of complexity. Left panel: The average number of alleles used as a function of the number of alleles agents have available. The grey line indicates the point at which agents fully utilise their alleles. Right panel: The Gini coefficient is displayed for each social norm, calculated on the alleles that are in use. A high coefficient suggests a large inequality in the alleles that are used suggesting the players regularly face the same OPGG environments, while a low coefficient suggests an equal experience of all kinds of OPGG environments. The dotted lines overlay the level of cooperation exhibited for each social norm alongside the Gini coefficients. In all cases, the OPGG is played in groups of five ($n = 5$)

group sizes larger than $n = 5$, as this value implies that all strategy complexities $p < 10$ yield average reputation values that can all be realised by interacting groups.[6]

In the first experiment, we consider the effect on cooperation when the size of the groups playing the OPGG is increased, while keeping the overall number of groups fixed (Fig. 3 for the AD social norm, and Figure S3 for the other social norms). Naturally, this implies that the population would grow proportionally to the increased group size. Results confirm the observation of the previous literature: larger groups worsens cooperation [74, 76]. Nevertheless, we show that behavioural strategies with higher complexity ($p \geq 8$) can maintain cooperation in larger groups (up to $n = 10$ agents in our experiments), while less complex strategies struggle to sustain cooperative behaviour even for smaller groups.

In the second experiment, we keep instead constant the population size (Fig. 3 for AD and Figure S4 for AL, AN, and AB) and we observe several interesting results. First, while strategies of a higher complexity ($p \geq 6$) are successful in small groups, their success does not translate into larger groups. Secondly, simpler strategies ($p \leq 4$) fare better in larger groups, regardless of the social norm at play (Figure S4). Finally, the relative success of more complex strategies with respect to simpler ones reverses as we increase the group size.

Focusing on Fig. 3, specifically for $p = 10, 12$ for $n = 10$, we see that if we keep the strategy complexity and group size fixed, but allow a larger number of groups and therefore a larger population, there is a greater chance of cooperation implying that populations with larger OPGG groups are better off in proportionally larger populations. This is also seen in the other social norms comparing Figures S3 and S4 for $n > 5$.

Having a larger strategy space does not automatically imply that such space is actually used by the agents. Indeed, this depends on the emerging levels of cooperation (and other strategies) that become dominant in the population, and on their stability over time. In order

[6] Results for $n < 5$ are reported in Figure S5. For $n = 4$ our results are confirmed. When $n = 3$ once the maximum complexity of 5 (since the total number of possible average reputation values in a group of three individuals is $2 * 3 - 1 = 5$) is achieved, cooperation collapses, as increasing p only increase noise. Finally, when $n = 2$, strategies are only focused on the reputation of the single interacting partner, directly discriminating on the opponent's reputation and leading to full cooperation regardless of complexity (there is no uncertainty as the average reputation of the group fully reveals the nature of the opponent allowing perfect discrimination).

to investigate the degree to which players with a more complex strategy space are actually using the alleles available to them, or in other words to understand the spectrum of possible reputations their strategies encode that are actually used, we calculate the average number of times each allele is used as a function of the strategy complexity (or the maximum number of alleles available to them). Figure 4 (left) shows that the average allele use increases for each social norm for $p < 8$[7] at which point further increases in strategy complexity are not utilised. In fact, with the AL norm, we find a drop in alleles used once $p > 8$ which corresponds to the transition between low and high cooperation happening between $p = 8$ and $p = 10$.

Different alleles are used with different intensities, as their use co-evolves with the cooperative environment an agent is facing. For this reason in Fig. 4 (right), we study the the Gini index calculated on the alleles in use within the player strategy chromosomes (solid lines) alongside the average levels of cooperation they exhibit (dotted lines). A high coefficient suggests a large inequality in the alleles that are used suggesting the players regularly face the same OPGG environments, while a low coefficient suggests an equal experience of all kinds of OPGG environments. There are two clear phases of behaviours. Firstly, when $p \leq 8$, we find low cooperation ($\sim$ 10%) and a larger Gini coefficient ($>$ 0.45) presenting high levels of inequality within the use of player strategies. Secondly, when $p > 8$ for the social norms that distinguish between defection and being a loner (AD and AL), we see high levels of cooperation ($\sim$ 80%) with a lower Gini coefficient ($\sim$ 0.5), and for the other social norms (AN and AB), we see a lower level of cooperation ($\sim$ 50%) with a high Gini coefficient ($\sim$ 0.8).

Sensitivity analyses have been run on each of the variables within the model. We present the results of Fig. 2 for all the social norms in Figures S3 and S4. The remainder of the figures each consider two situations, when behavioural complexity is low ($p = 2$) and again when it is high ($p = 8$). All parameters of the OPGG have been systematically explored, with a *ceteris paribus* approach: the cost of cooperation c (Figure S6), the group synergy factor or public good multiplier r (Figure S7), the loner's payoff σ (Figure S8). The speed of evolution of the genetic algorithm is specified by ζ (Figure S9), controlling the proportion of the population that is eliminated each round. The degree of indirect reciprocity within the game is represented by Ω (Figure S10) which controls the likelihood of repeated games following a round of the OPGG. We consider two types of error implemented within our model in line with [53] which mathematically explores execution and assignment error. The former is described by α (Figure S11), and the latter is described by β (Figure S12). We investigate both the effects of mutation frequency ϵ_1 (Figure S13) and the mutation intensity (proportion of the mutant's chromosome that is randomised) ϵ_2 and find that $10^{-2} \leq \epsilon_1 \leq 10^{-1}$ allows the best chances for cooperation while the intensity of mutation has little effect (Figure S14).

In these sensitivity analyses, the results are qualitatively similar and in line with previous literature. However, the baseline results must be qualified after analysing Figure S7, which reports the effect on cooperation when changing the public good multiplier r. For small r, while overall levels of cooperation remain very low, the AD and AN social norms display more cooperation under low ($p = 2$) rather than high ($p = 8$) complexity, which is at odds with our primary result that more complex strategies tend to yield higher cooperation. This result can be explained given the nature of these social norms. Since the AD and AN norms are better in sustaining cooperation, it is natural that they can sustain some level of cooperation even with low multipliers. However, as discussed in [61], their strength follows from the fact that they assign a relatively mild reputational punishment to agents abstaining

[7] If points lie on the grey line, it indicates that players are using the entirety of their strategies.

from the OPGG. More complex strategies require more effort to evolve towards cooperative behaviours (effective actions need to be evolved for a larger set of reputation levels), and given a low enough r, agents find it more convenient to resort to the safer loner action under such setup, even at the cost of a somewhat lesser reputation. This interpretation is confirmed by the fact that the social norms that have the harshest punishments for loners always show higher levels of cooperation when strategies are more complex, regardless of r.

4 Discussion

Understanding how cooperation works, how it can be encouraged and especially how and when it falls apart is important in multiple facets of human society. Studies in this sector can inform policy in prioritising the sustainable governance and longevity of public resources, managing the negative effects of bad players and understanding the situations in which additional external incentives or punishment is required to dissuade deviations from the social norm.

The standard public goods game struggles with maintaining cooperation in experimental setups where participants play repeated games [36, 39] for various reasons [2, 3, 17, 57, 72]. This exemplifies the necessity of further mechanisms to reach cooperation levels implicit in the good working of social systems. Giving individuals the option to withdraw from the game (through the "Loner" strategy) does offer some benefits [28] (as this action cannot be cheated upon by defectors), but creates a cooperator-defector-loner cycle when the population is widely cooperative only for short periods of time. Previous work [61] has shown that the combination of (1) any social norm that values defection to be at least as bad as withdrawal from the game and (2) the threat of punishment, is able to successfully enable cooperation. However, when the punishment mechanism is removed, the reputation mechanism alone is able to only increase the level of cooperation with respect to the baseline in specific circumstances. Indeed, only the social norm that values individuals who withdraw from the public good to be better than those who try to selfishly benefit from it (AD) shows a discernable improvement from the baseline OPGG.

We hypothesised that a social norm assigning moral value to actions in combination with individuals with the ability of acting depending on the relative reputability of their current interaction environment can foster cooperation, if they were able to sufficiently distinguish between good and bad environments. Therefore, building on the work of [61], we developed an agent-based model to test this hypothesis using reputational systems comprising social norms based on an adaptation of image scoring to the OPGG. Using an evolutionary mechanism based on genetic algorithms, we conceptualised that individuals have a behavioural "chromosome" which dictates the individual's behaviour given the state of the immediate interaction environment in the form of the average reputation of the OPGG group. A longer chromosome intuits a more complex behavioural strategy. Therefore, we can explore conditionally cooperative behavioural strategies of—in principle—arbitrary complexity. Our analysis complements the one proposed by [69, 70], which shows that cooperation increases as a function of social norm complexity up to a certain level, with further additions of complexity (chiefly the ability of a social norm to account for behaviour before the last step) not improving the chances of cooperation.

Our first major finding is that, for each given social norm, increasing the complexity of behavioural strategies helps cooperation, but only up to a certain point, after which any additional increase in complexity does not improve the success of collective action (Fig. 4).

Reading this result in conjunction with the one of [70], it can be concluded that complexity in social norms and in the behavioural strategies act as substitutes. Indeed, even with very simple social norms, assimilable to extensions of the image scoring which showed limited ability in sustaining cooperation [8, 9, 40, 43, 58], a sufficiently elaborate level of complexity in the way agents use the reputational information can still sustain high levels of cooperation in the context of collective action problems.

Furthermore, it is interesting to assess our results against those of the literature on the impact of reputational systems in the OPGG [61]. In particular we found that—provided sufficiently complex behavioural strategies are allowed—even the social norms that assign a worse reputation to loners than to defectors can support some limited level of cooperation provided sufficiently complex behavioural strategies are available.

It is important to stress that, while the results that can be compared with those of [61] are indeed very similar, not all results could be replicated quantitatively. In particular, in [61] when behavioural strategies are faced with binary choices in the absence of punishment, roughly 40% cooperation was observed, while the present model can only account for about 20% of cooperation. This follows from the fact that, despite having almost the same parametrisations, these two models have a different selection mechanism, known to be able to cause significant quantitative differences in otherwise equal setups [67]. Relatedly, [79] finds significant differences in quantitative behaviour of models where agents play stern-judging with replicator dynamics versus a genetic algorithm. While the previous model [61] relied on a group-selection mechanism with a limited number of strategies to be studied at the same time, the present work implements selection through genetic recombination of behavioural strategies. The choice depends on the size of the possible strategy space to explore. Indeed, when increasing the complexity of the behavioural strategies, the number of possible strategies to model rapidly becomes intractable. While $p = 2$ affords us $3^2 = 9$ potential strategies, increasing complexity to $p = 10$ implies dealing with $3^{10} = 59049$ different possible strategies. Simulating populations with enough agents to fairly represent the entire strategy space in a timely manner becomes computationally impossible even on large (cluster) computers. Using genetic recombination of strategy chromosomes is more efficient as it allows a more finely tuned and dynamic exploration of the strategy space, allowing the best strategies for each given environment to endogenously emerge from an initial random population and to co-evolve with the environment.

Comparing the different social norms, we find that the social norms that are able to distinguish between the acts of defection and withdrawal are more likely to support higher levels of cooperation in the population. These more distinguishing norms (AD and AL) allow individuals to effectively judge their immediate interaction environment and to associate a reputation to it which in turn is more likely to be a suitable measure of the level of cooperation the public good constituents exhibit. On the other hand, the AB and AN social norms both assign the same numerical reputation to defection and withdrawal. This limits individuals' abilities to decode whether or not their group mostly consists of loners or mostly consists of defectors. Therefore, when a situation arises in which the average reputation is close to either 0 or (-1) (for the AN and AB norms respectively), individuals have to essentially guess their response. This yields a maximum cooperation rate of about 50%.

We show that the AD norm is the one most conducive of cooperation, exhibiting (1) higher levels of cooperation at all levels of complexity and (2) a phase transition to cooperation-dominated populations at lower levels of complexity than the AL social norm. Since the benefits of the AD and the AL social norms are primarily to allow individuals to distinguish between different types of non-cooperative actions, their numerical values are largely irrelevant leading the most popular behavioural strategy under AD and AL to be the same once the numerical values of the Defector and Loner actions are swapped. For example, suppose $p = 3$ and the best strategy with the AD social norm is LDC (be a loner the group's average reputation is less than $-\frac{1}{3}$, defect when it is greater than $-\frac{1}{3}$ but less than $\frac{1}{3}$, and cooperate when it is greater than $\frac{1}{3}$), then the same strategy would be DLC when in the AL norm. We see the same actions, mapping to different reputation ranges as the numerical value associated to defection and to withdrawal are switched.

We have shown that providing agents with the ability to exploit more detailed information about the average cooperativeness of their interaction environment—as proxied through reputational information—fosters cooperation in the OPGG. Given that collective action problems emerge in groups, the natural next question is to assess how complexity and group sizes interact. On this regard, our results show that as the group size and population size increases, the chances for cooperation decrease. When the size of the population is held constant, the simpler strategies are more successful when it comes to larger OPGG groups. These results are in line with our main result, i.e. that the social norm and strategy complexity work in conjunction to identify non-cooperative behaviour in OPGG groups. When larger groups are considered, the behaviour of another individual within the group becomes much less significant as when compared to the group as a whole. In other words, larger groups lead to individual behaviours and reputations being averaged out or lost in the crowd, meaning that more complex strategies are essentially trying to "over-explain" the group behaviour. This increases the computational effort required and harms cooperation by making the evolutionary process less effective (larger chromosomes mean that there is more to learn, but larger groups suggests there is less to learn from as the average reputation is less informative of each single individual's behaviour). By keeping the population size constant, we isolate the effect of the group size and complexity on cooperation removing the overarching disadvantage of larger populations, in line with some debated previous results for the public goods game [35, 60, 77]. However, we find that in certain scenarios, the more complex behavioural strategies in larger groups exhibit a marginal improvement in cooperation when overall size of the population grows proportionally to the size of the OPGG group offering small but specific evidence contradicting previous empirical findings [36].

In summary, we show that a reputation-based social norm can help cooperation in the absence of any further incentive mechanism (e.g without punishment institutions) if individuals are able to identify and condition their actions to a wider range of behaviour/reputation states of OPGG groups. Under these assumptions, the stricter norms show better chances for cooperation and with the lowest complexity requirements. When it comes to smaller groups, more complex strategies are appropriate, but for larger groups, simple strategies are better. Our findings show that behavioural complexity can be beneficial in explaining cooperation when the individuals are capable of a wider range of behaviours in response to any given situation. Further work on this topic might involve the joint study of pro-/anti-social punishment and reputation with the objective of assessing their effect on the level of complexity at which simple social norms can achieve high cooperation and of exploring the interaction of behavioural complexity and punishment and the extent to which it helps cooperation in larger groups. Further work should explore the co-evolution of social norms and behavioural strategies in presence of complexity costs, to assess how the trade off between their relative complexity emerges through evolutionary pressure. Finally, it would be interesting to explore the effects of behavioural strategy complexity in a PGG with compulsory participation.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s13235-022-00485-5.

Author Contributions All authors contributed equally to the development of the research idea, model conception, and simulation design. Shirsendu Podder wrote the code, ran the simulations and generated the graphical outputs. The first draft of the manuscript was written by Shirsendu Podder and Simone Righi provided extensive comments as well as research supervision. All authors read and approved the final manuscript.

Funding Simone Righi gratefully acknowledges funding from "Fondi primo insediamento" of the Ca'Foscari University of Venice. Simulations were run on the Myriad Cluster computer of UCL.

Availability of data and material The data upon which all the figures of this paper are generated are available in the Github folder linked below.

Code Availability All python scripts required to generate the simulation data used in this article, as well as the instruction to run them, are available at the address https://github.com/ShirsenduP/GeneticComplexityInTheOpgg.

Declarations

Conflict of interest Authors declare no conflicts of interests.

Ethics Approval No ethics concerns.

Consent to Participate Not applicable.

Consent to Publish Not applicable.

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