No Country for Young People?

The Rise of Anti-Immigration Politics in Ageing Societies*

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Abstract

We investigate the effects of (1) population ageing and (2) rising income inequality on immigration policies using a citizen-candidate model of elections. In each period, young people work and pay taxes while old people receive social security payments. Immigrants are all young, meaning they contribute significantly to financing the cost of public services and social security. Among natives, the elderly and the poor benefit the most from public spending. However, because these two types of voters do not internalise the positive fiscal effects of immigration, they have a common interest in supporting candidates who seek to curb immigration and increase the tax burden on high-income individuals. Population ageing and rising income inequality increase the size and, in turn, the political power of such sociodemographic groups, resulting in more restrictive immigration policies, a larger public sector, higher tax rates, and lower societal well-being. Calibrating the model to UK data suggests that the magnitude of these effects is large. The implications of this model are shown to be consistent with patterns observed in UK attitudinal data.

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What are the effects of population ageing and rising income inequality on immigration policy? Why are anti-immigration politicians and political parties increasingly successful in rapidly ageing countries, which arguably need more legal immigration to mitigate the impact of population ageing on public finances? Should we expect increasing restrictions on immigrant workers inflow in these countries?

This paper aims to answer these questions using a theoretical model and providing suggestive empirical evidence.

This study is motivated by three key findings from the empirical literature on migration:

1. Aversion to immigration (Dustmann & Preston, 2007; Facchini & Mayda, 2007; Card et al., 2011) and support for anti-immigration political parties (Becker & Fetzer, 2017; Van der Brug et al., 2000) tend to be strong among the elderly (Fig. 1A) and the low-income native citizens relative to people in other sociodemographic groups (Fig. 1B).

2. Economic hostility towards immigration is primarily motivated by concerns about its effects on public finances, specifically those related to public spending policies (Dustmann & Preston, 2006, 2007; Boeri, 2010). This view suggests a perception among natives of competition with immigrants over welfare benefits and the use of crowded-out public services.

3. Immigrants are, on average, net fiscal contributors. The empirical evidence indicates that this is true both in the UK (Dustmann et al., 2010; Dustmann & Frattini, 2014), and the United States (Lee & Miller, 2000; Orrenius, 2017), implying that, at least in the long run, immigrants do not directly draw fiscal resources from the natives.

For instance, in 2017 61% of the British citizens over 60 wanted less immigration, while just 45.3% of those under 35 years felt the same way (BSA 2017). In the US, the corresponding values for 2016 are 27.8% and 44.1% (General Social Survey, 2016).

The evidence regarding other European countries is heterogeneous (Boeri, 2010). For an extensive survey on the
These three empirical regularities lead to a two-fold puzzle. First, why is hostility towards immigration motivated by concerns about its fiscal effects in countries where these effects are generally positive in aggregate? Second, why are the elderly and the poor — who benefit the most from the fiscal surplus from immigration — more averse to immigration-friendly policies?

We propose a channel that can explain this puzzle. Throughout this paper, we provide theoretical and empirical arguments to argue that it is a key channel.

We study a dynamic economy in which the resident population in each period consists of two age groups: young and old, and two legal groups: citizens and immigrants. Individuals, both citizens and immigrants, live for two periods at most, differ in their productivity levels (low, medium or high), and derive utility from the consumption of private goods and government services. The citizens also have an exogenous common taste for immigration which is meant to capture any non-economic factors affecting their immigration policy preferences, but relative policy preferences are entirely driven by economic factors. All new immigrants are young, possess the same average productivity as the natives and cannot vote. However, they acquire citizenship (and voting rights) after one period of residency in the host country.

In each period, society chooses a two-dimensional policy consisting of an immigration quota and governmental service provisions. The elderly (both native and naturalised immigrants) receive a public pension. For simplicity, we assume that pension transfers are exogenously determined and financed solely by tax revenues, but both these assumptions can be easily relaxed (see section 6.1). The government collects revenue through a linear tax on labour income and use it to finance public spending, resulting in a redistributive welfare system. There is no public debt and the budget is assumed to be balanced in each period. Thus, the policy choice endogenously determines the income tax rate.

In this setup, voters can choose both the immigration policy and how society divides the net fiscal benefits from immigration. This novel feature of the model generates the following key tradeoffs. First, immigration widens the tax base of the receiving country generating a fiscal surplus, which can be employed to (1a) increase public spending and/or (1b) reduce taxes. Second, an increase in public spending can be financed through (2a) higher tax rates and/or (2b) further immigration. The elderly and the low-income citizens are less affected by income tax changes than are the young and rich citizens. Thus, choices (1a) and (2a) mostly benefit the former sociodemographic groups, whereas

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3These factors include, among others, the effects of immigration on compositional amenities documented in Card et al. (2012). We discuss the role played by such factors in section 6.2.
choices (1b) and (2b) favour the latter.

These tradeoffs generate the key intuition underpinning our results: the elderly and the low-income citizens not only (a) support higher public spending than the young and rich, but also (b) prefer to finance this spending through higher income tax rates rather than through further immigration. Channels (a) and (b) imply that politicians seeking to represent the interests of the old and the poor citizens propose relatively restrictive immigration policies, high public spending, and high taxes (anti-immigration candidates).

Conversely, young and rich citizens primarily seek to ease their tax burden. Because both increased immigration and cuts to public spending contribute to reducing the tax rate, politicians aiming to represent the interests of those types of citizens offer less restrictive immigration policies, a smaller government, and lower taxes in their electoral platforms (pro-immigration candidates). As a consequence, open immigration policies are always endogenously bundled with a relatively small government in the platforms of such candidates.

Note that the model generates no actual competition between immigrants and natives over welfare benefits because the fiscal gains from immigration always outweigh its crowding-out effect on public services (with a fixed tax rate). Nevertheless, the political process induces perceived competition. The mechanism is the following. Pro-immigration candidates propose more immigration, less public spending, and lower taxes than anti-immigration candidates. Because the elderly and the poor are almost unaffected by a fall in the tax rate but they are strongly harmed by cuts on public spending, the policy platform of a pro-immigration candidate – if implemented – produces a negative short-term fiscal effect on those types of citizens relative to that of an anti-immigration candidate. This prompts elderly and poor voters to behave as though they are competing with immigrants over public benefits.\footnote{This mechanism applies even if the elderly and the poor benefit from public spending financed through immigration as much as or more than the rich.}

That is, they support relatively anti-immigration candidates in the elections on the grounds of the negative fiscal effects of pro-immigration policy platforms, in line with the stylised facts.

Demographic shocks tilt the relative power of the two opposing political factions; population ageing and increasing income inequality result in a larger share of elderly and poor voters. Thus, they gain in both size and political power, which fuels the support for anti-immigration candidates. This yields an equilibrium policy of low immigration and high public spending. This channel underpins the main analytical results of this paper, which are as follows:

1. A rise in longevity and/or a fall in the birth rate increases the share of the elderly, while a rise
in income inequality increase the number of poor voters. These factors determine the electoral success of a relatively anti-immigration politician.

2. The election of an anti-immigration politician leads to a tightening in immigration policy, an increase in public spending, and a sharp increase in the tax rate. Hence, the political process tends to exacerbate the effects of population ageing on public finances.

3. The tightening of immigration policy generates a welfare loss for the entire society, though it mostly harms middle- and upper-class workers as well as future generations.

Moreover, we provide two sets of quantitative results:

1. We calibrate a parametric infinite-horizon version of the model to UK data. This exercise reveals that the magnitude of the analytical effects described above may be rather large. For instance, 5 more years of life expectancy at 65 yields a policy allowing for 11.3% fewer working-age immigrants. A 10% decrease in income inequality (measured with the Gini coefficient) yields 11.9% more immigrants.

2. We show that the patterns of aversion towards immigration observed in British Social Attitudes Survey data from the 1995–2017 period are consistent with the fiscal channel proposed in this paper. Specifically, age is positively correlated with aversion to immigration, even after controlling for cohort effects and non-economic factors such as education qualification. Similarly, household income is negatively correlated with the same attitudinal measure.

These results are consistent with the correlation patterns between population ageing, income inequality, and restrictions to immigration observed in the UK during the post-WWII period (Fig. 2). Moreover, they provide a rationale to explain why ageing countries, which would arguably benefit from more immigration, tend to limit it. Ageing societies tend to disregard the wellbeing of young people – natives and immigrants alike – and future generations. Our analysis suggests that this dynamic, which has widespread economic, demographic, and political consequences, is unlikely to change.

Lastly, the anti-immigration rhetoric is deemed to be one of the key features of the wider and multi-faceted phenomenon of right-wing populism in Western democracies (Mudde, 2007; Guiso et al., 2019). In the discussion section, we argue that our analysis may also help in identifying one of the mechanisms underpinning the proliferation and electoral success of right-wing populist parties experienced by several countries during the last two decades.
1 Related Literature

The theoretical literature on the political economy of immigration policies is vast. While some papers focus on immigration policies related to standards, such as skill requirements (Benhabib, 1996; Ortega, 2005), the most common approach, which this paper takes, involves analysing policies that restrict the number of immigrants, such as immigration quotas (see Preston, 2014 for a survey). These studies emphasise the importance of intergenerational aspects related to the pension system (e.g., Razin & Sadka, 1999; Kemnitz, 2003; Leers et al. 2003; Krieger, 2003; Ben-Gad, 2018), and immigrant fertility (Bohn & Lopez-Velasco, 2019) to explain the determinants of political views towards immigration policies. Most of these papers assume a unidimensional policy space. That is, voters choose the immigration quota but not the fiscal policy.

A key finding in the literature is that the assumption of a unidimensional policy space generates inconsistent predictions. This issue is described in Haut and Peters (1998) and Facchini and Mayda (2009). These papers study a simple economy characterised by a linear income tax and assume that revenues are provided to all citizens as lump-sum rebates. In this setting, the requirement of unidimensionality can be satisfied in two ways. Either (1) the level of public spending or (2) the income tax rate must be exogenously determined. According to Facchini and Mayda (2009), these two alternative assumptions correspond, respectively, to the classes of:

1. Tax adjustment models ($TAM$s; e.g., Scholten & Thum, 1996)

2. Benefit adjustment models ($BAM$s; e.g., Razin & Sadka, 1999, 2000),

These two model types deliver opposite predictions regarding the relationship between age, pre-tax
income, and attitude towards immigration. If immigrants are net fiscal contributors, TAMs show that elderly and low-income citizens are more hostile to immigration than the young and rich citizens; the opposite is true for BAMs.

The intuition that underpins these seemingly contradictory results is as follows. If public spending is exogenously determined, the effect of a rise in the tax base is a fall in the tax rate. Conversely, if the tax rate is unaffected by voter choice, the effect is a rise in public spending per capita. In the former case, immigration mainly benefits young and high-income citizens; in the latter case, the elderly and low-income citizens enjoy the largest share of the gains.

In both models, the endogenous effects of immigration are weakly negative on taxes and weakly positive on public spending when immigrants are net fiscal contributors. Thus, neither of these approaches provides a rationale for the well-documented aversion towards immigration based on its perceived negative fiscal effects.

Preston (2014) argues that the source of this apparent inconsistency lies in how society distributes the gains from immigration and suggests that this puzzle can be addressed by a model that allows for immigration, public spending, and tax policy to be endogenous. Despite this, most studies are based on unidimensional models on account of technical reasons: a Condorcet Winner—a platform that is preferred to any alternative by a majority of voters—does not typically exist if the policy space is multidimensional (Plott, 1967; Grandmont, 1978). This implies that Black’s median voter theorem (1948) does not hold. Thus, voting models that allow for multiple endogenous policy dimensions require the use of a different solution concept.

Several alternative approaches that tackle the multidimensionality issue exist and are widely available in the literature on voting (see Dhillon (2005) and Dotti, (2021) for reviews); e.g., citizen-candidate models (Osborne & Sliwinski, 1996; Besley & Coate, 1997), probabilistic voting (Lindbeck & Weibull, 1987; Banks & Duggan, 2005), and models of endogenous political parties (Roemer, 1999; Levy, 2004, 2005). However, the adoption of such approaches in the literature on the political economy of immigration is extremely limited. To our knowledge, the only attempt to depart from unidimensionality in voting models on immigration policies is Razin et al. (2016). They propose an OLG model similar

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5 Some models in the literature show a negative relationship between immigration and public spending. In Haupt and Peters (1998), for instance, state pensions are decreasing in immigration. This negative relationship is the direct result of an assumption; in our analysis, they are going to be endogenous outcomes of voter choice. Thus, while these models provide interesting predictions, they are unsuitable for the specific questions of this paper.

6 In particular, the existence of a Condorcet winner requires very strong restrictions on voter preferences, implying that the use of the traditional Downesian model is typically restricted to relatively simple problems of redistribution (e.g., Borges and Ratto 2004; Calabrese, 2007). These requirements are generally too restrictive to be satisfied in a dynamic model like the one proposed in this paper. The technical reason for this is illustrated in an online appendix.
to the one used in this paper, in which the native population consists of skilled workers, unskilled workers, and the elderly. They characterise the political coalitions that can prevail among these three types of voters and derive various interesting predictions. Nevertheless, their approach is unsuitable to answer the questions in this paper, as they assume exogenous tax rates. Thus, the implications of their model, in terms of immigration preferences, are the same as those of a standard BAM.

We tackle the multidimensionality issue by using a citizen-candidate model of representative democracy akin to those in the literature (Osborne and Slivinski, 1996; Besley and Coate, 1997). Specifically, we propose a model of electoral competition that extends the static framework from Dotti (2020) to a dynamic setting. The specific choice is a matter of convenience: it ensures tractability, transparency of the mechanisms, and ease of interpretation of the results. It does not, however, shape the main tradeoffs underpinning the prediction of the paper, which would survive alternative assumptions on the nature of the political process.

2 The Model

This section consists of two parts: (1) the economic model of immigration and public spending and (2) a description of the political process.

2.1 The Economic Model

We propose a model of immigration and public spending akin to those in the literature, particularly the model in Razin and Sadka (1999). Unlike their model, however, both public spending and immigration are endogenous in our model.

2.1.1 Demographic Structure

We study an economy lasting $T$ periods. Each period $t = 1, 2, ..., T$ has length normalised to 1 and features a continuum of individual\footnote{In particular, a standard probabilistic voting (Banks & Duggan, 2005) would deliver qualitatively similar predictions.} divided into two generational groups: the working-age population $(Y)$ of size $y_t$ and the elderly $(O)$ of size $o_t$. Within the working-age population are $n_t$ natives and $m_t$ immigrants. All newly arrived immigrants are in the working-age group and there is no return\footnote{The numerical exercise proposed in section 4 makes use of the assumption $T \to +\infty$, delivering an infinite-horizon overlapping-generation (OLG) model, which is further discussed in section 6.2.}
migration: the elderly population includes those individuals who were immigrants in period \( t-1 \). The size of each group is summarised in Fig. 2.

Working-age individuals have exogenous fertility rates: \( \sigma \) for natives and \( \sigma^m = \sigma + \Delta \) for immigrants, with \( \Delta \geq 0 \). The supply of potential immigrants is large.\(^{10}\) At the end of each period, the immigrants and their children are fully assimilated to the native population (i.e. they become identical to natives of the same age group). Under these assumptions, the size of the working-age native population in period \( t \) is given by the formula

\[
n_t = \sigma n_{t-1} + \sigma^m m_{t-1}.
\]

A young individual at time \( t < T \) survives to period \( t+1 \) with probability \( \lambda \in [0,1] \). Thus, life expectancy at birth is \( 1+\lambda \leq 2 \) and the size of the elderly population is

\[
o_t = \lambda (n_{t-1} + m_{t-1}).
\]

Note that \( o_t \) is an increasing function of life expectancy. At the end of period \( T \), all agents die with probability 1 and the economy ends. Lastly, the initial condition of this economy is a working-age population at \( t = 0 \) of size \( y_{p0} \).

### 2.1.2 Citizenship and Voting Rights

We assume that only the \( n_t + o_t \) citizens (i.e., the young natives plus all the elderly) vote - recent immigrants do not. The immigrants acquire citizenship after being resident in the country for one period and such privilege extends to their children; i.e. an “immigrant” is defined solely by their legal status. All the results qualitatively hold under alternative assumptions on voting rights acquisition.\(^{12}\)

### 2.1.3 Individual Preferences

A citizen \( i \) of group \( (Y) \) in period \( t \) has preferences over consumption of private goods \( C^i_t \), the extent of government services \( G_t \), and the share of immigrants in society \( M_t = m_t/(m_t + n_t) \), represented by

\[^{10}\text{This assumption ensures that any immigration quota adopted by the government within the range of available policies is binding, meaning that the number of immigrants is always exactly equal to the quota in each } t.\]

\[^{11}\text{Note that because we assume a continuum of young individuals, the formula for the size of the elderly population cannot be directly derived using a law of large numbers. However, under some non-trivial technical restrictions, one can obtain this formula for } o_t \text{ by assuming a young population which is the limit of a large but discrete number of individuals. Details in Judd (1985).}\]

\[^{12}\text{In particular, one can assume that elderly immigrants and their children do acquire voting rights with no qualitative effect on the results. Details in section 6.2.}\]
the following utility function:

$$U^Y_t \left( \{C^i_{t+r}, M^i_{t+r}, G^i_{t+r}\}_{r=0}^1 \right) = C^i_t + b(G_t) + c(M_t) + 1 \left[ t \neq T \right] \beta \lambda \left[ C^i_{t+1} + d(G_{t+1}) + c(M_{t+1}) \right]$$ (1)

where $\beta$ captures the rate at which an individual discounts future utility and the functions $b$, $d$, and $c$ are strictly concave $C^\infty$ functions. The indicator function simply captures the fact that there is no continuation value after the final period $t = T$.

The function $c$ represents an exogenous taste for immigration and is the same for all citizens, albeit this assumption can be easily relaxed. It captures all non-economic factors affecting voter preferences regarding immigration. Its domain is $[0, \bar{M}]$, where $\bar{M} < 1$ is the level corresponding to fully unregulated immigration. We do not restrict the sign of $c$ and $c'$ for interior values of $M_t$ but we assume $c'(0) \geq 0$ and $c'(\bar{M}) = -\infty$; that is, the citizens are strongly averse to fully unregulated immigration, but don’t mind or may even have a positive taste for a small number of immigrants. The presence of $c$ in the utility function does not shape the mechanisms underpinning the predictions of the paper. It serves the purpose of avoiding corner solutions in which all citizens want completely unregulated immigration or no immigration at all. The framework can be easily extended to allow citizens to have a taste for the size of both recent and past immigration. In section 6.3 we further discuss these aspects and the role played by the function $c$. Additionally, we assume that $G_t \in [0, \bar{G}]$, $b$ and $d$ are increasing, and $b$ satisfies $b'(0) = +\infty$ and $b'(\bar{G}) = 0$.

For retired individuals in period $t$ the direct utility $U^O_t$ is similarly constructed, except it is solely a function of consumption, government services, and immigration in the current period:

$$U^O_t \left( C^i_t, M_t, G_t \right) = C^i_t + d(G_t) + c(M_t)$$ (2)

where the features of $c$ and $d$ are illustrated in the previous paragraphs.

Young individuals who immigrate in period $t$ consume both private goods and government services in the same way natives do; however, their preference specification is irrelevant for electoral outcomes.

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13 All the results hold true as long as the marginal effect of a rise in $M_t$ on the taste component is (1) nondecreasing in income and (2) the value $\bar{M}$ that solves $c'(\bar{M}) = 0$ is the same for all citizens. For instance, we can allow for young natives, old natives, and naturalised immigrants to differ in their exogenous tastes for immigration. Further details in section 6.3.
as they do not vote in that period. Conversely, naturalised immigrants in their old age do vote and have the same preferences as the old natives. This assumption is strong but can be easily relaxed.

2.1.4 Production

Each working-age individual $i$ can be employed either in the private or public sector at a wage rate equal to their productivity. They are endowed with 1 unit of time and their labour supply is perfectly inelastic. This assumption simplifies the analysis, does not drive the tradeoffs that underpin this paper’s predictions, and can be relaxed. Individual $i$’s gross income in period $t$ has formula $y^i_t = \xi \omega^i_t$ with time-invariant average $\bar{y}$, where $\xi$ is an aggregate productivity component and $\omega^i_t$ is $i$’s productivity type.

For simplicity, we assume three productivity types: $\Omega = \{\omega^{\text{Low}}, \omega^{\text{Mid}}, \omega^{\text{High}}\}$, but all the results hold true for a higher number – or even a continuum – of types. The distribution of $\omega^i_t$ is time-invariant with CDF $Q_\rho(\omega_t)$ and mean equal to 1.

The index $\rho$ captures the degree of inequality of the distribution. Specifically, we define a partial order over productivity distributions as follows: $\rho' \geq \rho''$ if and only if the CDFs $Q_{\rho'}$ and $Q_{\rho''}$ satisfy the following conditions:

1. Single-crossing below the median: $Q_{\rho'}(\omega^i_t) \geq Q_{\rho''}(\omega^i_t)$ for all $\omega^i_t \in \Omega$ such that $0 < \omega^i_t \leq \hat{\omega}$, for some threshold $\hat{\omega}$ which satisfies $Q_{\rho'}(\hat{\omega}) \geq \frac{1}{2}$.

2. Mean-preserving: $\int \omega_t dQ_{\rho'}(\omega_t) = \int \omega_t dQ_{\rho''}(\omega_t) = 1$.

In other words, the two distributions have the same mean, but the former exhibits a larger share of relatively low-productivity individuals. As $i$’s income has formula $y^i_t = \xi \omega^i_t$, a higher value of $\rho$ corresponds to higher pre-tax income inequality of non-retired individuals.

Immigrants possess the same average productivity as the natives, which is assumed to be independent of policy choices. This assumption is admittedly restrictive, intended to describe an economy facing a large supply of rather productive potential immigrants that cannot effectively select immigrants based on observable characteristics. The consequences of relaxing this assumption are discussed in footnote 15.

Footnotes:

14 Immigrant preferences do play a role in welfare analysis; this aspect is illustrated in section 3.5.

15 See footnote 13.

16 In particular, all the results hold if the wage elasticity of labour supply is positive for all productivity levels (details in section 6.2 and in the online appendix).

17 The numerical exercise in section 4 makes use of the latter assumption. Further details are provided in section 6.2. Full proofs for this version of the model are provided in the online appendix.

18 If single-crossing holds true on the entire domain of $\omega^i_t$ this implies that the latter distribution is a mean-preserving spread of the former. However, the conditions for $\rho' > \rho''$ is more restrictive than those required for a mean-preserving spread, because condition (1) is more restrictive than second-order stochastic dominance. A similar comparative statics exercise is performed in Dotti (2020) for a model of redistribution.
in section 6.3.

The private sector produces the consumption good using a linear technology and labour as input\(^{19}\) This assumption is imposed for a matter of convenience and can be relaxed (details in section 6.2). It implies that immigration has no effect on the native’s wages and is very restrictive. However, the empirical literature suggests that the size of this effect is generally fairly small (Preston, 2014), meaning that our assumption represents a reasonable approximation. Given this setup, the total production of consumption goods equals the total gross income of private-sector workers.

The public sector produces government services. The uniform quality level of government services \(G_t \in [0, \bar{G}]\) is assumed to be equal to the share of effective labour hired by the public sector or, equivalently, to the ratio of non-pension public spending to output. This means that government services are a partially congested public good\(^{20}\) This assumption is imposed for technical reasons and is admittedly restrictive, but it is not crucial to generate the tradeoff underpinning the predictions of the paper\(^{21}\).

### 2.1.5 Social Security

We assume the existence of a public pension system, which represents a stylised version of the basic old-age state pension scheme adopted by several European countries, including the UK. In each period \(t\) all the elderly, including those who were immigrants in period \(t-1\), are entitled to a net pension (denoted by \(p^*_t\)) provided by the government. The expected size of the pension system is exogenous to electoral choices, but this assumption can be easily relaxed (see below). The pension system possesses three key features.

1. **Pay-As-You-Go (PAYG).** In each period \(t\) the social security expenditures are financed through the fiscal contributions of all the working-age individuals – both natives and immigrants – in the same period. That is, the pension system is not funded. For simplicity, we assume that pension expenditures are financed through general taxation, but our results are robust to alternative

\(^{19}\)This assumption is common in related literature (e.g., Razin & Sadka, 2000). It is justified if one considers that, in a more complex economy, these effects tend to be offset by adjustments in the stock of capital (not explicitly assumed in this analysis) that occurs over the relatively long framework of a generation. This mechanism is considered to be particularly effective for offsetting long-run effects of immigration on wages if firms have access to international capital markets (see: Ben-Gad, 2018).

\(^{20}\)In particular, this assumption implies that the elderly do not cause crowding-out of this imperfect public good. Examples of public services that display these features include public transportation, public offices, and the police.

\(^{21}\)Specifically, it ensures that the marginal cost per taxpayer of government services does not mechanically fall with the size of immigration – an issue that would arise if one assumes instead that \(G_t\) is either a pure private good or a pure public good. This assumption ensures that citizens’ preferences satisfy quasisupermodularity over policy choices, as required in section 3.2 to ensure the existence of an equilibrium.
assumptions, as illustrated in section 6.1. The PAYG assumption is crucial for our analysis because it generates a fiscal surplus from immigration. The mechanism is simple: more immigration translates into a larger working-age population (net contributors on average), but it does not affect the size of the retired population (net receivers). Thus, an increase in immigration allows for the costs of the pension system to be shared among a larger tax base and results, ceteris paribus, in a lower cost per taxpayer.

2. Defined Benefits (DB). The net pension amount \( p_i \) is determined at the end of the working age and is not affected by policy choices that occur after retirement. In particular, it is constant in the size of immigration in the current period. This is consistent with the features of most public pension system in Europe, in which an increase in immigration does not typically translate into higher pension benefits in the short run. This assumption is crucial for the results because it ensures that the elderly do not mechanically benefit from an open immigration policy by receiving more generous pensions (at least in the current period). Without this assumption, the elderly would enjoy large direct fiscal gains from immigration and behave as in a benefit adjustment model: they will be relatively supportive of open immigration.

3. Automatic Balance Mechanism (ABM). The pension amount \( p_i \) received by an old individual mechanically adjusts to anticipated demographic and economic shocks, such that the expected size of the pension system stays constant as long as the immigration policy does not change; i.e.,

\[
E_t \left[ \frac{\text{Total Pension Spending}_{t+1}}{\text{Total Output}_{t+1}} \right] = M_{t+1} = M_t = \gamma \quad \text{for some scalar } \gamma > 0.
\]

This assumption is consistent with the mechanisms adopted by several European countries to preserve the sustainability of their social security systems in the long run.\(^{23}\) It is imposed for a matter of convenience and can be easily relaxed: in section 6.1 we show that under mild additional restrictions all the key results of the paper hold true (and they are even strengthened) if \( \gamma \) is made endogenous to voters’ choices. Thus, (ABM) is not crucial for the existence of the main tradeoff that underpins the results of this paper, which is a direct consequence of the DB and PAYG assumptions.

\(^{22}\)In particular, we study a pension system featuring a self-sufficient pension fund financed through social security contributions and a partially funded one.

\(^{23}\)Automatic balance mechanisms are becoming increasingly common. These mechanisms consist of a formula that translates a change in average life expectancy into a change in monthly pension payments (e.g., Finland after the 2005 reform), or into a change in the retirement age (e.g., Italy after reforms were introduced in 2010 and extended in 2011). Automatic balance mechanisms are also embedded in the pension systems of Canada, Germany, Japan and Sweden. In the UK, the government has made a commitment to review the State Pension age every five years to ensure sustainability, adjusting it to variations in life expectancy and fertility.

\(^{24}\)Specifically, it ensures that the marginal fiscal gains from immigration do not vary with demographics and aggregate productivity. This feature simplifies the derivation of the comparative statics results in section 3.2 by ensuring that citizen’s ideal policies do not vary with \( \lambda, \sigma, \xi \).
Under the assumptions \((PAYG), (DB), \text{ and } (ABM)\) we can show that the pension amount can be written as a function \(p^t_i = p_t(\omega^t_{i-1}, \bar{y}, z_t)\) and the total cost of pensions has formula \(\gamma n_t \bar{y}\), where \(z_t\) is citizens’ old-age dependency ratio \(z_t \equiv \frac{m_t}{m_{t-1}}\) and \(\gamma\) is a constant defined in the previous paragraph (details in appendix B.1). These results ease the derivation of the main findings of the paper presented in section 3.

Lastly, the reader may wonder why young citizens, who represents a majority of the voting population, are not permitted to appropriate the pension benefits allocated to the elderly through taxation. This restriction is justified by the findings in the literature on intergenerational transfers. Specifically, Rangel and Zeckhauser (2001) and Boldrin and Montes (2005) show that in an infinite-horizon OLG model the existence of a public pension system can be the outcome of a self-enforcing intergenerational agreement. These studies indicate that the extent to which working-age people can reduce their net transfers to the elderly through taxation is limited, as the long-run sustainability of the intergenerational agreement depends upon the net benefits young expect to receive in old age. In line with this interpretation, \(p^t_i\) denotes the promised pension net of taxes on social security benefits.  

\[2.1.6 \text{ Public Finance}\]

The public sector raises revenue through a linear tax \(\tau_t\) on labour income and spend it on the provision of government services \(G_t\) and pensions for the elderly. We call the vector \((G_t, \tau_t)\) the fiscal policy in period \(t\).

We assume that the government budget is balanced in every period and we do not allow for public debt.

This assumption is needed to ensure tractability and is common in similar models. The role played by this restriction and the consequences of relaxing it are discussed in section 6.3. Using the aforementioned formulas for the cost of government services to output \(G_t\) and total pension expenditures \(\gamma n_t \bar{y}\), the government budget constraint is constructed as follows:

\[
\tau_t \geq \frac{G_t (n_t + m_t) \bar{y} + \gamma \bar{y} n_t}{(n_t + m_t) \bar{y}} = \frac{\text{Total Spending}_t}{\text{Total Income}_t}
\]

\[25\] Note that \(z_t = \frac{m_t}{m_{t-1}} = 2E_{t-1} \left[ \frac{m_t}{m_{t-1}} \right] M_t = M_{t-1}\), implying that \(p^t_i\) could also be expressed as a function of the expected old-age dependency ratio of the resident population at constant immigration policy \(M_t = M_{t-1}\).

\[26\] Of course, in reality net pension benefits may be affected by changes in the tax policy. Nevertheless, for the reasons summarized in this section, the tax rate on social security benefits is unlikely to be very responsive to endogenous policy changes. For instance, the median retired individual in the UK pays less than 3.5% of their total income in income tax (ONS, 2019). Moreover, social security for the elderly often includes benefits that are exempt from taxes, such as public health insurance (e.g., Medicare in the US) and subsidized home services.

where the right-hand side of (3) is the size of the government.

We perform a simple change of variable by defining the variable *laissez-faire*, denoted by \( L_t \), as the difference between the maximum level of non-pension public spending to output \( G \) and the actual level of such variable in period \( t \); i.e., \( L_t \equiv G - G_t \). This variable change is just a matter of convenience – the reasons behind it is made clear in section 3.

Assuming that the government budget constraint is always satisfied with equality\(^{28}\), we can solve (3) for \( \tau_t \) and define the tax rate function \( \tau(M_t, L_t) \) as follows:

\[
\tau(M_t, L_t) = G - L_t + \gamma (1 - M_t),
\]

(4)

where we assume \( 0 \leq G < 1 - \gamma \) to ensure that \( 0 < \tau(M_t, G_t) < 1 \) for all \( (M_t, L_t) \in [0, M] \times [0, G] \).

The consequences of relaxing this assumption are illustrated in the online appendix.

The formula (4) illustrates a key channel that shapes the results of this analysis. That is, the working-age citizens can ease their tax burden by voting for a less restrictive immigration policy. The mechanism is simple: more immigration translates into a larger working-age population and, in turn, higher aggregate income and tax revenues. However, it does not affect the number of elderly and – thanks to the the DB assumption – the total cost of pensions. Thus, an increase in immigration allows for the costs of the pension system to be shared among a larger tax base, resulting in lower taxes rates.

Under these assumptions, a working-age individual’s private goods consumption is given by her post-tax income, such that \( C_i = [1 - \tau(M_t, L_t)] y_i \).

### 2.1.7 Policy Space

 Voters face a two-dimensional policy space in each period \( t \). Policy platforms consist of an immigration quota \( M_t \) and a degree of laissez-faire \( L_t \). For technical reasons, we restrict our attention to policies \((M_t, L_t)\) with \( M_t \geq M \), where \( M \in [0, M] \) satisfies \( c'(M) = 0 \). This assumption is mostly innocuous.\(^ {30} \)

Thus, the policy space is the set \( X \equiv [M, M] \times [0, G] \) with typical element \((M_t, L_t)\).

\(^{28}\)This must be true at any equilibrium of the voting game. This becomes clear after the equilibrium concept is described in section 3.2.

\(^{29}\)This restriction is crucial for the results in the next section to hold. If the tax rate hits the upper bound, the predictions of the model become those of a standard benefit adjustment model, as illustrated in the online appendix.

\(^{30}\)Such value \( M \) always exists and is unique in \([0, M]\) given the assumptions established in section 2.1.3. Note that this definition excludes from \( X \) all the policies \((M_t, L_t)\) with \( M_t < M \). This assumption is innocuous as long as \( \Delta \) is small (as assumed in section 3), because for \( \Delta \to 0 \) none of these policies are Pareto efficient, meaning they cannot be credibly proposed by any candidate.
2.1.8 Citizens' Objective Function

Let \( \varphi = (\beta, \gamma, \lambda, \sigma, ...) \) be a vector of common parameters. We derive the citizen’s objective functions under the assumption that agents possess perfect foresight regarding future equilibrium outcomes.

Old citizens. Using the formula for \( p_t^i \) into the utility function of an elderly citizen in \( (2) \) we obtain their indirect utility function, which writes \( U_t^{i,O} \left( p_t \left( \omega_{i-1}, y_t, z_t \right), M_t, \bar{G} - L_t \right) = \right) \). Because \( p_t \left( \omega_{i-1}, y_t, z_t \right) \) is constant in \( (M_t, L_t) \), this formula shows that an old citizens’ preferences over \( (M_t, L_t) \) in period \( t \) are independent of their pension levels, income when young, expectations of future policies \( (M_{t+1}, L_{t+1}) \), and history up to period \( t \). Thus, all elderly – regardless of their productivity when they were young – have the same policy preferences; because of this feature we assign the same preference type \( \theta_t^i = -1 \) to all elderly citizens. Their preferences in period \( t \) can be represented by a function \( u_t^{i,O} = u_t \left( \left( (M_{t+r}, L_{t+r}) \right)_{r=0}^1; -1, \varphi, z_t \right) \), which has the formula:

\[
 u_t^{i,O} = d(\bar{G} - L_t) + c(M_t), \tag{5}
\]

This representation dramatically simplifies the analysis: because the productivity type of each elderly citizen is irrelevant for their choices, the state of the economy at the beginning of each period \( t \) is fully summarized by a single aggregate variable: \( z_t \).\(^{31}\)

Note that if the immigrants have the same birth rate as the natives (\( \Delta = 0 \)), then \( z_t \) is constant in \( (M_{t-1}, L_{t-1}) \), implying that voters’ tradeoffs in period \( t \) are unaffected by period\(-1 \) choices. As a result, if \( \Delta = 0 \) the analysis becomes identical to that of a static model.

Young citizens. We set a young citizen preference type \( \theta_t^i \) equal to their productivity parameter \( \omega_t^i \in \Omega \). Then we derive a young citizen’s preferences over policies at time \( t \) as their expected indirect utility

\[
 u_t^{i,Y} = u_t \left( \left( (M_{t+r}, L_{t+r}) \right)_{r=0}^1; \theta_t^i, \varphi, z_t \right). \]

Using formula \( (1) \), this becomes:

\[
 u_t^{i,Y} = [1 - \tau(M_t, L_t)] \xi \theta_t^i + b(\bar{G} - L_t) + c(M_t) + \right)
\]

\[ + 1 \left[ t \neq T \right] \beta \lambda \varepsilon_t \left[ u_{t+1} \left( \left( (M_{t+r}, L_{t+r}) \right)_{r=1}^2; -1, \varphi, z_t \right) \mid (M_t, L_t), h_t \right] \tag{6}
\]

where the indicator function ensures that there is no continuation value in period \( T \), because the

\(^{31}\)Since \( u_t^{i,O} \) is independent of \( \omega_{i-1} \) and \( Q_t^i(\omega_t) \) is time-invariant, the dynamic framework essentially becomes equivalent to one in which an entirely new population of citizens replaces the previous one at the end of each period, such that the age distribution of the “new” fictitious population is determined solely by the citizens’ old-age dependency ratio \( z_t \). Thus, the economy features a unique aggregate endogenous state: \( z_t \). Thus, we do not need to include each citizen’s productivity type in the state space. Note that for finite-horizon versions of the model, one also needs to include the period \( t \) in the state-space as the voter’s dynamic problem is non-stationary.
Formulas (5) and (6) illustrate that elderly \( \theta_i = -1 \) and low-income citizens \( \theta_i = \omega^{Low} \) are less affected by changes in the tax rate \( \tau(M_t, L_t) \) than the young and rich \( \theta_i = \omega^{High} \). As a consequence, the former types of agent prefer a policy that finances public spending through higher income tax rates rather than through a larger number of immigrants. This tradeoff holds despite the net positive fiscal contribution of immigrants, of which the elderly and the poor are net beneficiaries.

We construct the distribution of citizen types \( \theta_t \) in period \( t \) (conditional on history \( h_t \)), which possesses the following CDF:

\[
F_{\rho,t}(\theta_t | h_t) = \begin{cases} 
0 & \text{if } \theta_t < -1 \\
\frac{z_t(h_t)}{1 + z_t(h_t)} & \text{if } -1 \leq \theta_t < 0 \\
\frac{z_t(h_t) + Q_{\rho}(\theta_t)}{1 + z_t(h_t)} & \text{if } \theta_t \geq 0
\end{cases}
\]

(7)

where \( z_t \) is the previously defined citizen’s old-age dependency ratio. Lastly, we use (7) to define the totally ordered set of citizens’ types at time \( t \) as the support of \( \theta_t \), which writes: \( \Theta_t := \{-1\} \cup \Omega \); i.e., \( \Theta_t \) is the set of types that possess non-zero probability mass.

\[\text{32}\]

2.2 Political Process

We adopt a citizen-candidate model of elections with endogenous candidates akin to those in Osborne and Slivinski (1996) and Besley and Coate (1997). Specifically, we propose a version of the citizen-candidate model that adapts the framework in Dotti (2020) to a dynamic economic environment. The specific choice is a matter of convenience and is mostly innocuous for the main results of this paper.\[\text{33}\]

The choice of a citizen-candidate model is not solely motivated by tractability. It also ensures transparency and ease of interpretation of the results. These advantages will become clear in section 3.2, where we illustrate the results for a simple parametric example.

In this section we provide an informal description of the political process. A formal definition of the equilibrium concept is provided in appendix A.1.

Let \( N_t \) denote the set of citizens at time \( t \). In each period \( t = 1, 2, ..., T \) the political equilibrium, named Electoral Equilibrium (EE), is the outcome of a two-stage game.

\[\text{34}\]

\[\text{35}\]

\[\text{36}\]

\[\text{37}\]

\[\text{38}\]

\[\text{39}\]
In the first stage, each citizen $i \in N_t$ simultaneously chooses an action, denoted by $a^i_t$; namely, $i$ decides whether she runs for election as a candidate by proposing a platform $x^i_t = (M^i_t, L^i_t)$ in $X$ or remains inactive ($a^i_t = \emptyset$). Each citizen-candidate $i$ can credibly commit to a platform $x^i_t$ only if it is one of her ideal policies.

In the second stage, voters observe the set of available candidates and elect one member of this set using the method of majority rule. That is, they select a Condorcet winner over the set of available candidates whenever it exists. After the election, the winning candidate implements her policy platform. If no winning candidate exists in period $t$, then a default policy $x^0_t$ — which all citizens strongly dislike — is implemented.\(^{35}\)

Lastly, we restrict our attention to equilibria that satisfy two fairly standard properties.

(i) **Subgame perfection**: equilibrium strategies must be supported by credible beliefs regarding future agents’ behaviour, both on and off the equilibrium path.

(ii) **Markovian strategies**: equilibrium strategies in period $t$ are allowed to be conditional on the state of the economy $z_t$, but not the entire history of the game $h_t$.

Under these assumptions we can show that, even if the EE is typically not unique, the equilibrium policy outcome, denoted by $x^*_t$, is the same in all the equilibria.

The way we model the political process closely resembles that in Besley and Coate’s (1997) citizen-candidate model, specifically the case where the cost of running for elections is set equal to zero. However, our approach differs from theirs because it reduces the set of equilibria in two ways:

(1) It rules out equilibria in which — even if a Condorcet winner exists among the set of alternatives — it is not selected through the electoral process.\(^ {36}\)

(2) It rules out equilibria supported by non-credible beliefs regarding future policy choices off the equilibrium path (non-credible threats).

### 3 Results

In the next section, we provide the reader with an example that illustrates the key mechanism underpinning our main results, which we formally state in section 3.2.

\(^{35}\)Specifically, the policy outcome $x^0_t$ delivers a payoff $-\infty$ to all citizens. If there exist multiple Condorcet winners, a selection rule selects a unique elected candidate. In particular, the selection rule ensures that if the median citizen is not unique, then the rule selects winner a pivotal citizen featuring the lowest type.

\(^{36}\)In particular, our equilibrium concept rules out equilibria of the standard citizen-candidate model (Besley and Coate 1997) that result from voters playing weakly dominated strategies. A similar refinement is imposed in Epple and Romano (2014) and Dotti (2020).
3.1 Illustrative Example

We start from a highly simplified version of the model. The purpose of this exercise is to illustrate how the two-dimensionality of the policy space (2DM) generates the key tradeoff that shapes the predictions of the model, and why such tradeoff does not exist if we constrain the analysis to a unidimensional policy space.

In this example, we set \( T = 2, \Delta = 0, \xi = 1, \) and the parameter capturing the size of the pension system at \( \gamma = 0.2. \) Moreover, we focus on equilibria featuring two candidates only: a young low-income (\( r \)) and a young middle-income citizen (\( l \)). We assume the following set of citizen’s types in each period \( t = 1, 2: \Theta^i_t = \{-1, 0.5, 1, 2\}, \) and we label each element of \( \Theta^i_t \) with superscripts \( Old, Low, Mid, \) and \( High, \) respectively. We choose a parametric utility function featuring utility from public goods in the form \( b(\cdot) = d(\cdot) = \frac{1}{5} \ln(\cdot) \) and a quadratic utility cost \( c(\cdot) = -\frac{1}{2} (\cdot)^2 \) capturing the citizen’s taste for immigration.

We know from section 2 that given appropriate restrictions on the citizen’s beliefs if \( \Delta = 0 \) the expected utility of a young citizen in period 2 is independent of period-1 policy choices; that is, the continuation value \( EV^i_2 \equiv E_1[u((M_2, L_2); -1, \varphi, z_2) | (M_1, L_1), h_1] \) is constant in \( (M_1, L_1). \) Thus, using the definition of \( L_t = G - G_t \) and the proposed functional forms in formulas \( (1) \) and \( (2), \) we obtain the following objective functions in period 1:

\[
\begin{align*}
u^i_1Y &= [1 - \tau(M_1, L_1)] g_t^i + \frac{1}{5} \ln (G - L_1) - \frac{1}{2} M_t^2 + \beta \lambda EV^i_2 \\
u^i_1O &= \frac{1}{5} \ln (G - L_1) - \frac{1}{2} M_t^2,
\end{align*}
\]

for young and old citizens, respectively, where \( \tau(M_1, L_1) \) is as in formula \( (4). \)

**Citizen’s ideal policies and the median voter theorem.** Given the objective functions in \( (8), \) we compute the citizen’s ideal policies.

First, note that the citizen’s objective function in period 1 is strictly concave\(^{37}\) and \( X \) is a compact set. Thus, each citizen \( i \) possesses a unique ideal policy \( (M^i_1, L^i_1). \) The ideal policies in period 1 of each citizen’s type and the corresponding fiscal policies \( (G^i_1, \tau^i_1) \equiv (G - L_1, \tau_1(M^i_1, L^i_1)) \) are summarised in Table 1. **Fig. 3** (left) plots such ideal policies and illustrates the first key feature of the model. That is, the set of citizen’s ideal points \( I_1 \equiv \{(0, 0.2); (0.1, 0.6); (0.2, 0.8); (0.3, 0.87)\} \) is totally ordered in the \( (M_1, L_1) \)-space under the product order \( \leq, \) and such order corresponds to that of the citizen’s

\(^{37}\)See Lemma 2 in Appendix B.1.
Figure 4: Ideal policies of the four types of citizens and of the two candidates $r, l$ (left) and effect of population ageing/rising inequality on the equilibrium policy (right) in the illustrative example.

The intuitive consequence of this ordering property is that even if the citizen’s preferences over the multidimensional choice domain $X$ do not generally satisfy single-peakedness\textsuperscript{39} (Plott, 1967; Grandmont, 1978), those over ideal policies in the set $I_1$ do satisfy such property. In turn, this implies that a (multidimensional) median voter theorem holds true; that is, a Condorcet winner exists over $I_1$ and is the ideal policy of a citizen possessing the median type given the aforementioned ordering over $\Theta_1$. This pivotal voter result – similar to those in Dotti (2020, 2021) – proves particularly useful to derive and interpret three key model implications, which are summarised below.

1. Fiscal Effects. First, we calculate the short-term fiscal effects of implementing the policy platform of the pro-immigration candidate $l$ relative to that of the anti-immigration candidate $r$.

We define the short-term fiscal effect for citizen $i$ as the compensating variation (with sign changed) corresponding to a change in the fiscal policy from $(G^r_1, \tau^r_1)$ to $(G^l_1, \tau^l_1)$; that is, the adjustment in net income that returns a citizen $i$ to the original utility level after the fiscal policy has changed from $(G^r_1, \tau^r_1)$ to $(G^l_1, \tau^l_1)$, everything else – including the size of immigration $M_1 = M^r_1$ – being unchanged.\textsuperscript{40} In this example the formula for the fiscal effects on individual $i$ in period 1 writes:

\textsuperscript{38}This is a consequence of the fact that citizen’s preferences over $(M_1, L_1)$ satisfy quasiusupermodularity in $(M_1, L_1)$ and the (strict) single crossing property in $(M_1, L_1, \theta_1)$ (Milgrom and Shannon 1994). Formal definitions and details are provided in section 3.2 and appendix A.2.

\textsuperscript{39}They also do not typically satisfy other conditions that ensures the existence of a Condorcet winner, such as the unidimensional single-crossing property (Gans & Smart, 1996).

\textsuperscript{40}Note that the fiscal policy $(G^l_1, \tau^l_1)$ may not be (and need not be) feasible given a level of immigration $M^r_1$.
Table 1: Ideal policies of different types of citizens and corresponding fiscal policies \((G_1^i, \tau_1^i)\) in the \(2DM, BAM, \text{and TAM}\).

<table>
<thead>
<tr>
<th></th>
<th>(2DM)</th>
<th>(BAM)</th>
<th>(TAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((M_1^i, L_1^i))</td>
<td>((G_1^i, \tau_1^i))</td>
<td>(M_1^i)</td>
</tr>
<tr>
<td>Old</td>
<td>(0.1, 0.2)</td>
<td>(0.8, 1)</td>
<td>0.2</td>
</tr>
<tr>
<td>Low</td>
<td>(0.1, 0.6)</td>
<td>(0.4, 0.58)</td>
<td>0.2</td>
</tr>
<tr>
<td>Mid</td>
<td>(0.2, 0.8)</td>
<td>(0.2, 0.36)</td>
<td>0.2</td>
</tr>
<tr>
<td>High</td>
<td>(0.3, 0.87)</td>
<td>(0.13, 0.27)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 2: Short-term fiscal effects (in consumption units) of the policy platform of candidate \(l\) relative to that of candidate \(r\) on citizens of different types in the \(2DM, BAM, \text{and TAM}\).

\[
FE_{1,2DM}^i (1, 0.5; \theta_1^i) = \frac{1}{2} - 1 \times (\tau_1^i - \tau_1^r) y_1^i + \frac{1}{2} \ln \left( \frac{G_1^i}{G_1^r} \right).
\]

The general formula is provided in section 3.4.

The short-term fiscal effects for each type of citizen are summarized in the first column of Table 2. The results show that – even if the immigrants are net fiscal contributors – the electoral success of the pro-immigration candidate \(l\) produces negative fiscal effects on the elderly and low-income citizens. As a consequence, the members of those two socioeconomic groups are:

(a) averse to the immigration-friendly policy platform \((M_1^l, L_1^l)\) and supportive of the anti-immigration candidate \(r\) in the elections;

(b) motivated by the negative fiscal effects of the immigration-friendly policy platform.

That is, the model is consistent with both key findings in the empirical literature that motivate this analysis (see section 1).

2. Comparative Statics Second, we compare three scenarios, summarised in Table 3.

(a) Baseline scenario. Society \(S'\) features the distribution of citizen’s types with a 30% share of
elderly citizens in the population. The median citizen is a young and middle-income individual (Mid type). Thus, the median voter theorem implies that the candidate of Mid type (l) is elected and implements her ideal policy \((M^l_1, L^l_1) = (0.2, 0.8)\), resulting in the equilibrium fiscal policy \((G^l_1, \tau^l_1) = (0.2, 0.36)\).

(b) Population ageing scenario. Society \(S''\) features a larger share of elderly citizens relative to society \(S'\): 40% vs. 30%. As a result, the median citizen in society \(S''\) is a young and low-income individual (Low type). Thus, the Low type candidate \(r\) is elected and implements her ideal policy \((M^r_1, L^r_1) = (0.1, 0.6)\), resulting in the equilibrium fiscal policy \((G^r_1, \tau^r_1) = (0.4, 0.58)\). In sum, population ageing translates into a more restrictive immigration policy \(M^r_1\), higher public spending to output \(G^r_1\), and higher tax rates \(\tau^r_1\). This equilibrium policy change is illustrated in Fig. 3 (right).

(c) High income inequality scenario. Society \(S'''\) exhibits higher income inequality than \(S'\). Specifically, the income distribution of working-age citizens in \(S'''\) is a mean-preserving spread of that in \(S'\). This translates into a larger share of low-income citizens in the voting population. As a result, the median citizen in scenario \(S'''\) is a young and low-income individual (Low type). Thus, the Low type candidate \(r\) is elected and implements her ideal policy \((M^r_1, L^r_1) = (0.1, 0.6)\), resulting in the equilibrium fiscal policy \((G^r_1, \tau^r_1) = (0.4, 0.58)\). Thus, increasing inequality translates into a more restrictive immigration policy \(M^r_1\), higher public spending to output \(G^r_1\), and higher tax rates \(\tau^r_1\). This equilibrium policy change is illustrated in Fig. 3 (right).

This simple exercise illustrates the second key mechanism underpinning the results of this paper, which is the following. The elderly and low-income citizens suffer a negative fiscal effect from the implementation of a pro-immigration policy platform. Thus, they support the anti-immigration candidate \(r\), who proposes a more restrictive immigration policy and larger public spending than candidate \(l\). Population ageing and rising inequality increase the share of elderly and relatively low-income citizens in the voting population, causing the median of the distribution of citizen’s types to move towards a weakly lower-income citizen. This mechanism fuels the electoral success of the anti-immigration politician \(r\), resulting in equilibrium in a more restrictive immigration policy, higher public spending to output, and higher tax rates.

3. Comparison with unidimensional models. Lastly, we show that our findings are in sharp contrast

\footnote{This may be a consequence of either higher life expectancy \(\lambda\), or a lower birth rate \(\sigma\), or both.}

\footnote{As the income distribution is (weakly) positive-skewed, a mean-preserving spread typically translates into a larger share of relatively low types in the population. In particular, this is always true under the definition of inequality stated in section 2.1.4. This effect is similar to that of an increase in income inequality in Meltzer and Richard (1981) and several other papers in the literature.}
with the predictions of the two possible unidimensional versions of our model, which correspond to a benefit adjustment model (BAM) and a tax adjustment model (TAM), similar to those in the literature (see section 2). The results for the BAM and TAM are summarised in Tables 1 and 2.

Recall that the immigrants are net fiscal contributors to the receiving country in our model. There are two possible ways to make the policy space unidimensional:

(a) BAM. The tax rate is exogenously fixed at \( \tau_1 = \bar{\tau}_1 \). Thus, an increase in the number of immigrants mechanically translates into larger non-pension public spending \( G_1 \). As a result, the elderly and the low-income citizens – who largely benefit from an increase in \( G_1 \) – neither advocate a more restrictive immigration policy relative to the young and high-income citizens (as shown in Table 1), nor support a relatively anti-immigration candidate in the elections. That is, the BAM is inconsistent with the first key finding in the empirical literature that motivates this analysis.

(b) TAM. The non-pension public spending to output ratio is exogenously fixed at \( G_1 = \bar{G}_1 \). Thus, an increase in the number of immigrants mechanically translates into a lower tax rate \( \tau_1 \). As a consequence, a less restrictive immigration policy has a weakly positive fiscal effect for all types of citizens – including the elderly and the poor – as illustrated in Table 2. This finding implies that in the TAM the elderly and low-income citizens’ aversion towards immigration is motivated by factors other than its (perceived) negative fiscal effects. Such effects are positive, but they are just not large enough to offset other anti-immigration motives that shape the electoral choices of those types of citizens. That is, the TAM is not consistent with the second key finding in the empirical literature that motivates this analysis.

(c) 2DM. As illustrated in paragraphs 1 and 2 of this section, the two-dimensional model can generate tradeoffs that are consistent with both key findings in the empirical literature and deliver, in turn, credible comparative statics results. The next section extends and generalises these results.

### 3.2 Equilibrium Existence and Characterisation

The model presented in section 2 exhibits the following properties:

1. The policy space \( X \) is a compact set and the partially ordered set \((X, \leq)\) is a complete sublattice of \((\mathbb{R}^2, \leq)\).

---

\footnote{For ease of comparison, we set the value of the exogenous variables of BAM and TAM at their equilibrium levels of the 2DM: \( \bar{\tau}_1 = 0.36 \) and \( \bar{G}_1 = 0.2 \). The tradeoffs are unchanged if different values are chosen.}

\footnote{In our example, the aversion towards immigration in the TAM is driven by an exogenous taste for low immigration. In other versions of the TAM proposed in the literature the aversion is driven by other channels, such as the effects of immigration on the wages of low-income natives [Haupt & Peters, 1998].}

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(2) The set of citizen types $\Theta$ is a totally ordered set.

(3) Citizens’ preferences given history $h_t$ satisfy quasisupermodularity (QSM) in $(M_t, L_t)$ and the strict single crossing property (SSC) in $(M_t, L_t; \theta_t)$ (proof in Appendix B.1).

The definitions of QSM and SSC are borrowed from Milgrom and Shannon (1994) and are provided in Appendix A.2. QSM and SSC are widely used in many subfields of Economic Theory and much less restrictive than the conditions that ensure the existence of a Condorcet winner in a multidimensional policy space.\(^{45}\)

Moreover, in the remainder of this paper we maintain the assumption that the difference in fertility rates between immigrants and native is not too large: $\Delta \in [0, \bar{\Delta}]$ for some threshold $\bar{\Delta} > 0$. The formula for the maximum value of $\bar{\Delta}$ is provided in appendix B.1. This assumption eases the derivation of the results by ensuring that the effect of current policy choices on future equilibrium outcomes is small.\(^{46}\) Let $\theta^p_t$ denote the median type over $\Theta^p_t$ and $x^*_t = (M^*_t, L^*_t)$ be the equilibrium policy outcome of the political process. Recall that $z_t$ is the citizen’s old-age dependency ratio. Given the three properties (1), (2), and (3) we can state the following result.

**Proposition 1.** In each period $t = 1, 2, ..., T$ (i) A EE always exists. In any EE (ii) the policy outcome $x^*_t$ is the ideal policy of the pivotal citizen $\theta^p_t$ and (iii) is unique given history $h_t$. (iv) The pivotal citizen’s type $\theta^p_t$ is weakly decreasing in $z_t$.

**Proof.** See Appendix B.1.

In (i) above Proposition 1 establishes the existence of an EE and in (iii), the uniqueness of the equilibrium policy outcome $x^*_t$ (note that the EE is typically not unique). A multidimensional median voter theorem is stated in (ii): in all EE’s the policy outcome is the unique ideal policy of the pivotal citizen; that is, in all equilibria $x^*_t = x^p_t = (M^*_t, L^*_t)$, where the superscript $p$ denotes the pivotal

\(^{45}\)In particular, SSC is a much less restrictive assumption than both the unidimensional single crossing condition in Gans and Smart (1996) and single-peakedness in Black (1948). As a result, this condition alone is insufficient to ensure the existence of a Condorcet winner. In fact, in our economic model, voter preferences satisfy QSM and SSC but, typically, neither single-peakedness nor unidimensional single crossing over $X$. Thus, a Condorcet winner over $X$ generally does not exist. A detailed description of these conditions and proof of non-existence of a Condorcet winner in our setup are provided in the online appendix.

\(^{46}\)We relax this assumption in the computational model presented in section 4 by allowing the immigrants to possess a birth rate larger than $\sigma + \bar{\Delta}$.

\(^{47}\)Formally, $\theta^p_t$ satisfies $\int_{\theta_t}^{\theta^p_t} dF_{\rho,t}(\theta_t | h_t) \geq 0.5$ and $\int_{\theta^p_t}^{\infty} dF_{\rho,t}(\theta_t | h_t) \geq 0.5$. 

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citizen. This result is crucial to derive the main results of the paper stated in section 3.3.

Lastly, in (iv), Proposition 1 captures a key mechanism underpinning the comparative statics results presented in the next section. Namely, a worsening in the citizen’s old-age dependency ratio $z_t$ (due to either a rise in longevity $\lambda$ or a fall in fertility $\sigma$) causes an increase in the share of elderly voters and, in turn, a decrease in the type of the pivotal citizen.

3.3 Effect of Population Ageing, Inequality, and Productivity Shocks

A shock is defined as an unanticipated and permanent change in one (or more) model parameters which occurs in period $t$. We study the effects on the equilibrium policy outcomes of four types of demographic and/or economic shocks, which are defined below.

**Definition 1.**

(a) An increase in longevity is a rise in the life expectancy parameter $\lambda$.

(b) A decrease in fertility is a fall in the birth rate parameter $\sigma$.

(c) An increase in income inequality is a rise in the inequality parameter $\rho$.

(d) An economic depression is a fall in aggregate productivity parameter $\xi$.

The main result of this paper stems from studying the effects of a parameter change of type (a), (b), (c), and/or (d) on the key equilibrium outcomes of this economy. A change in $\lambda$, $\sigma$, $\rho$, or $\xi$ affects the equilibrium outcome in three possible ways:

(i) it changes the demographic composition of the voting population and, in turn, the identity of the pivotal citizen (*political effect*),

(ii) it directly affects the government budget constraint (a smaller tax base, lower taxable income, etc.) (*budget effect*), and

(iii) it affects voter expectations regarding future equilibrium policies, both directly and through the effect of changes in current policy choices (*sophisticated farsightedness*).

The assumption $\Delta \in [0, \Delta]$ on the fertility behaviour of the immigrants implies that the effect (iii) is relatively small. Thus, the results are driven by either the political effect (i) or the budget effect

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48Shocks are permanent in the sense that the value of a given parameter is assumed to change in all future periods; for instance, an increase in the longevity parameter $\lambda$ in period $t$ implies a higher survival probability in all periods $t + r$ with $r = 1, 2, ..., T - t$. 

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Proposition 2. (Effect of population ageing, increasing inequality, and economic depression). (i) An increase in longevity and/or (ii) an increase in income inequality and/or (iii) a decrease in fertility, and/or (iv) an economic depression translate to (1) a less open immigration policy $M_t$, higher non-pension public spending $G_t$, and (3) a larger size of government $\tau_t$ in all periods $t$.

Proof. See Appendix B.3.

The intuition underpinning the results (i), (ii), and (iii) is simple and identical to that of the illustrative example presented in section 3.1. That is, population ageing and rising income inequality result in a decrease in the income of the median type $\theta^p_t$ (political effect). As the pivotal citizen becomes a less productive individual, the equilibrium policy shifts in favour of elderly and welfare-dependent citizens, penalising, in turn, the younger and more productive part of society.

Regarding the effect of a decrease in $\xi$, the intuition underpinning (iv) above in Proposition 2 is that a fall in aggregate productivity reduces the total fiscal gains from immigration, leading to a lower preferred immigration level for all citizens (budget effect).

3.4 Short-term Fiscal Effects

The second key result of the paper concerns the fiscal effects of a policy platform featuring a less restrictive immigration policy on the elderly and the relatively low-income citizens.

The short-term fiscal effect $FE^i_t$ on citizen $i$ of the platform of a candidate $l$ with type $\theta^l_t$ relative to that of a candidate $r$ with type $\theta^r_t$ in period $t$ is defined as the compensating variation - expressed in unit of consumption and changed in sign - of a change in the fiscal policy from $(G^*_t, \tau^*_t) = (G^*_t - L^*_t, \tau^*_t (M^*_t, L^*_t))$ to $(G^l_t, \tau^l_t) = (G^l_t - L^l_t, \tau^l_t (M^l_t, L^l_t))$ at constant immigration equal to $M^r_t$. In words, it is the net transfer (changed in sign) that returns citizen $i$ to their initial utility.
level after a change in the fiscal policy from \((G_i^r, \tau_i^r)\) to \((G_i^l, \tau_i^l)\)\textsuperscript{49}. It has formula:

\[
FE_i \left( \theta_i^l, \theta_i^r; \theta_i^t \right) = \begin{cases} 
\left[ \tau_i^l - \tau_i^r \right] \xi \theta_i^t + b(G_i^l) - b(G_i^r) & \text{for } \theta_i^t \neq -1 \\
\left( G_i^l - d(G_i^r) \right) & \text{for } \theta_i^t = -1 
\end{cases}
\]

for a citizen of type \(\theta_i^t\). Using this formula, we state the following result.

**Proposition 3.** *(Short-term fiscal effects).* In any EE, if there exist two candidates \(r, l\) in period \(t\) such that \(M_i^r < M_i^l\), then there exists a threshold \(\hat{\theta}_i \geq 0\) such that the policy platform of the relatively pro-immigration candidate \(l\) has weakly negative short-term fiscal effect on all individuals featuring type \(\theta_i^t \leq \hat{\theta}_i\) — that is, the old and the relatively poor citizens — with respect to the platform of the relatively anti-immigration candidate \(r\).

**Proof.** See Appendix B.3.

Proposition 3 provides the second key result of the paper. That is, the elderly and the low-income citizens suffer a negative fiscal effect whenever a relatively immigration-friendly policy platform is implemented, even if the immigrants are net fiscal contributors. This result follows the fact that a less restrictive immigration policy is endogenously bundled with a less generous spending policy in the platform of a candidate who represents the interests of the young and high-income part of the native population. As a consequence, the elderly and the low-income citizens oppose open immigration on the grounds of its fiscal effects and support anti-immigration candidates in the elections.

### 3.5 Welfare Analysis

The findings in Proposition 2 do not necessarily indicate that the predicted policy changes are desirable among society as a whole.

In this section, we present a welfare analysis demonstrating that in ageing societies, a marginal tightening in immigration policy from its equilibrium level is typically unambiguously harmful. I use a

\textsuperscript{49}Formally, \(FE_i \left( \theta_i^l, \theta_i^r; \theta_i^t \right)\) is the difference between individual \(i\)'s expenditure function evaluated at fiscal policy \((G_i^l, \tau_i^l)\) and that evaluated at fiscal policy \((G_i^r, \tau_i^r)\) at constant utility level \(\bar{u}^{Y}_t = (1 - \tau_i^r) y_i^r + b(G_i^r) + c(M_i^r) + \beta \Lambda_E \left[ U^{O,1}_{t+1} \left( C^1_{t+1}, h_{t+1}, M_{t+1}^t \right) \right] \) (or \(\bar{u}^{O}_t = d(G_i^r) + c(M_i^r)\) for an old individual) and immigration level \(M_i^r\). Note that the fiscal policy \((G_i^r, \tau_i^r)\) may not be (and need not be) feasible given a level of immigration \(M_i^r\).
social welfare function ($SWF$) as a measure of the societal well-being. The $SWF$ is a weighted average of the utility of citizens at time $t$ and the expected utility of future generations. Let $\Psi_{t+r}(\theta_{t+r})$ denote a function that assigns a weight to individuals of type $\theta_{t+r}$ in period $t+r$.

The $SWF$ in period $t$ is constructed as follows:

$$SWF((M_t,L_t);\varphi \mid h_t,s_t) = E_t \left[ \int_{-\infty}^{+\infty} u_t((M_t,L_t),x_{t+1};\theta_t,\varphi,z_t(h_t)) \, d\Psi_t(\theta_t) + \sum_{r=1}^{T-t} \int_{-\infty}^{+\infty} u_{t+r}((M_{t+r},L_{t+r}),x_{t+r+1};\theta_{t+r},\varphi,z_t(h_{t+r})) \, d\Psi_{t+r}(\theta_{t+r}) \mid h_t,s_t \right]$$

We study the effect of a marginal change in $M_t$ evaluated at $M_t = M_t^*$ on the above measure of aggregate well-being. The idea underpinning this exercise is simple: if at the equilibrium policy $(M_t^*,L_t^*)$ the marginal effect of an increase in $M_t$ on the $SWF$ is greater than zero and $M_t^* < \bar{M}$, there exists a policy $(M'_t,L'_t)$ with $M'_t > M_t^*$ which is welfare-improving.

This means that, in turn, if the immigration policy in equilibrium changes from $M_t^{**}$ to $M_t^*$ with $M_t^* < M_t^{**}$ as a consequence of a marginal change in demographics, the society benefits, $ceteris paribus$, from moving back towards the level $M_t^{**}$; that is, the immigration policy that would have been implemented in the absence of demographic changes. In other words, the society is harmed by the change in the immigration policy at the margin. From this, we can state the following result.

**Proposition 4.** For any Social Welfare Function $SWF((M_t,L_t);\varphi \mid h_t,s_t)$ that assigns a strictly positive weight to each native individual with $\theta^*_i > 0$, there exist thresholds $\hat{\omega}_t > 0$ and $\check{z}_t \in [0,1]$ such that if $\omega^{Low} \leq \hat{\omega}_t$ and $z_t \in [\check{z}_t,1)$, then a marginal loosening in the immigration policy is welfare-enhancing.

**Proof.** See Appendix B.5.

The intuition underpinning this result is as follows. On one hand, the marginal fiscal benefit from immigration for a working-age individual is constant in $M_t$. On the other hand, because the definition of $\bar{M}$ implies $\theta^0(\bar{M}) = 0$, the marginal taste cost of immigration tends to zero as $M_t$ approaches $\bar{M}$.

The value of $\theta^0_t$ is equal to $\omega^{Low}$ if $z_t$ is close enough to 1, meaning that the pivotal citizen possesses

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50The weight to type $\theta_{t+r} > 0$ is strictly positive if $\Psi_{t+r}(\theta_{t+r}) > \max\{\Psi_{t+r}(\theta_{t+r}'),0\}$ for all $\theta_{t+r}' \in \Theta_t$ such that $\theta_{t+r}' < \theta_{t+r}$. Note that we do not account for the welfare of current potential immigrants. This allows us to abstract from a full description of their utility function. Nevertheless, if immigration choices are endogenous, any potential immigrant should be weakly better off if able to immigrate, because they still have the choice between remaining in their country of origin or to emigrating to a different country. Thus, whenever a tightening in the immigration policy is harmful to citizens, this result should hold true if we account for the welfare of potential immigrants.
the lowest income type. If \( \omega^{Low} \) is sufficiently close to zero, then the pivotal voter features near-zero taxable income and is, in turn, almost unaffected by a decrease in the income tax rate caused by any increase in immigration.\(^{51}\)

As a result, if \( \Delta \) is small in magnitude the equilibrium quota \( M^*_t \) approaches \( M \); i.e. the share of immigrants in the working-age population that would be preferred by all citizens on the grounds of the mere taste for immigration embedded in the function \( c \). This implies that at the equilibrium, the marginal aggregate fiscal gains from immigration for the average working-age citizen are substantial relative to its marginal social costs due to taste. As a result, provided that the social welfare function assigns a positive – even if small – weight to young productive citizens, if the citizens’ old-age dependency ratio is sufficiently close to 1, a marginal increase in immigration from its equilibrium level always results in higher social welfare.\(^{52}\)

Proposition 4 suggests that societies characterized by a high old-age dependency ratio are likely to implement excessively restrictive immigration policies. Moreover, it implies that a marginal tightening in the immigration policy caused, for instance, by population ageing may be harmful to society. This result is suggestive in the light of the increasingly controversial restrictions to immigration that have been progressively implemented in countries characterized by rapidly ageing populations, such as the UK and Italy.

### 4 Calibration and Simulated Counterfactuals

The analytical predictions in section 3 are purely qualitative. As such, they do not provide any insight into the magnitude of the effects. Thus, in this section, we parametrise an infinite-horizon version of the model \( (T \to +\infty) \). Over an infinite horizon, the dynamic process becomes stationary and possesses a unique steady-state (provided that \( \Delta \) is not too large in magnitude).\(^{53}\) Moreover, we allow for a continuum of productivity types; i.e., \( \omega_i \in [0, +\infty) \). All the analytical results presented in section 3

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\(^{51}\)This is a sensible scenario if one considers a more realistic tax system in contrast to the simple tax schedule described in section 3. For instance, if the tax system features a personal allowance, as in the UK, the zero taxable income threshold must be adjusted accordingly. The results hold true under the alternative assumption that \( \Omega \) is a continuum featuring a zero lower bound. In such case, \( M^*_t \) tends to \( M \) as \( z_t \to 1 \), but the welfare implications are identical.

\(^{52}\)Conversely, even if \( z_t \) is close to zero, a marginal increase in \( M \) at the equilibrium does not necessarily harm social welfare. Specifically, a threshold \( \underline{z}_t \in [0, 1] \) such that if \( z_t \leq \underline{z}_t \) the society would benefit from a marginally more restrictive immigration policy may not exist for all the possible \( SWFs \) that satisfy the conditions stated above and that assign a strictly positive weight to the elderly. Nevertheless, such threshold \( \underline{z}_t \) exists for some specific functional forms, such as the utilitarian \( SWF \).

\(^{53}\)The proof of existence and uniqueness of the steady-state and its characterisation, as well as those to Proposition 1-2-3-4 for this augmented model are provided in the online appendix. Note that if \( \Delta \) grows large, then the dynamic system may feature multiple steady-states.
hold true in this slightly richer model, whose analytical properties are illustrated in section 6.2.

We calibrate the model to UK data, and then use the calibrated model to simulate key counterfactuals. While the exact quantitative predictions of this numerical exercise should be viewed as purely illustrative, they suggest that the effect of population ageing and rising inequality on immigration policies may be rather large in magnitude. The results are summarised in this section and extensively presented in the online appendix to this paper.

The following utility functions are employed:

\[
U_{i,Y}^t \left( \{C_{i+r}, M_{i+r}, G_{i+r}\}_{r=0}^1 \right) = C_i^t + \delta_1 \ln (G_i) - \delta_2 M_i^2 + \beta \lambda \left( C_{i+1}^t + \delta_1 \ln (G_{i+1}) - \delta_2 M_{i+1}^2 \right) \\
U_{i,O}^t (C_i^t, M_t, G_t) = C_i^t + \delta_1 \ln (G_t) - \delta_2 M_t^2 
\]

(11)

for young and old citizens, respectively. We assume that the pre-tax equivalised income of UK households (among non-retired individuals) possesses a Dagum distribution (generalized log-logistic) and we calibrate the parameters to fit the mean, median, and Gini coefficient in the 2017-2018 UK population (Office of National Statistics, 2019).

The choice of the Dagum distribution is motivated not only by its superior performance in fitting income distributions relative to other commonly used alternatives (lognormal, gamma, etc.) documented in the literature (Kotz and Johnson, 1982), but also by a desirable property that such distribution possesses with respect to inequality. That is, the relationship between the three parameters of the distribution and the implied Gini coefficient of inequality is given by a function whose functional form is known. Thus, this distribution is deemed to be particularly suitable for Political Economy models in which income inequality plays a key role (Glomm and Ravikumar, 1998).

The parameters capturing demographics such as life expectancy at 65 and the fertility rates of natives and immigrants are all consistent with the corresponding values of 2017-2018 (Office for National Statistics, 2019). Lastly, the parameters of the utility function and the pension rate $\gamma$ are calibrated using data on public spending in the UK from the 2018 HM Treasury’s Public Expenditure Statistical Analysis (PESA) report.

I use the calibrated model to simulate the effects of:

1. a permanent increase in life expectancy at 65 (+5 years), and
2. a decrease in the Gini coefficient of equivalised pre-tax income of non-retired households (-10%)

The simulated counterfactuals imply that, in the UK, an increase of 5 years in life expectancy at 65 years old translates to a new steady-state policy featuring 866,768 fewer resident immigrants of working age—equal to 11.27% of the foreign-born working-age population in the UK in 2017–2018 (Fig. 4.A), and a 8.6% increase in (non-pension) public spending per working-age individual (Fig. 4.B). Similarly, a decrease of 10% in income inequality – measured as the Gini coefficient of equivalised pre-tax income of non-retired households – translates to a new policy allowing for 913,800 (+11.88%) additional working-age resident immigrants (Fig. 5.A) and a 9.26% reduction in (non-pension) public spending per individual of working age (Fig. 5.B).

It is important to contextualise these results. In the UK, life expectancy at 65 years old has increased by approximately 6.8 years between 1980 and 2018, and the pre-tax equivalised Gini coefficient for non-retired households has risen by 33.2% over the same period (Office for National Statistics, 2019). Our results suggest that population ageing and rising inequality in the UK over the last few decades may have played a substantial role in shaping the rising levels of aversion towards immigration and the increasingly restrictive immigration policy (see Fig. 3 and DEMIG, 2015). Nevertheless, the reader should be wary about the use of our counterfactual results as means to predict the actual number of immigrants that a given country is expected to receive during a given time frame. This caution is offered because our analysis focuses solely on the demand side, abstracting from the possibility of changes in the supply of potential immigrants, both in quantitative and qualitative terms. For instance, in spite of an increasingly restrictive immigration policy, the UK experienced an unprecedented rise in immigration during the last four decades. This fact should not be interpreted as inconsistent
with our model, because the empirical literature suggests that such rise has been mostly driven by socioeconomic factors affecting the supply of immigrants, such as domestic and foreign GDP growth, unemployment rates, and increasing income inequality (Hatton & Williamson, 2005).

Lastly, we use the calibrated model to perform several additional numerical counterfactual analyses, such as studying the steady-state effect of a shock on the fertility rate, and several robustness checks on our main results. These numerical exercises confirm the robustness of our predictions even when some key assumptions are relaxed; for instance, if the difference in the fertility rates of immigrants and natives $\Delta$ grows large.\footnote{In such a case, the conditions in Proposition 1 may be no longer satisfied, but the main predictions in Proposition 2 and 3 qualitatively hold true for several parametrisations. The only difference is that effect of current policy choices on expected future equilibrium outcomes \textit{(sophisticated farsightedness)} may be non-negligible. However, for significantly large values of $\Delta$ multiplicity of the equilibrium policy outcome may arise, the steady-state may not be unique and a shock may cause a transition to a different equilibrium path. The speed of convergence to the steady-state after a shock decreases in $\Delta$ for all of the parametrisations that generate a unique equilibrium path.}

These exercises are described in detail in the online appendix.

## 5 Empirical Evidence

In this section, we investigate the determinants of British adult residents’ attitudes towards immigration and public spending using data from the British Social Attitude Survey (BSA).\footnote{The BSA does not cover the entirety of the United Kingdom because it does not include respondents from Northern Ireland.} Specifically, we use its 1995, 2003, 2008, 2011, 2013 and 2017 rounds, which include a specific question about attitudes towards immigration.

This type of empirical analysis is not novel. Dustmann and Preston (2007), for instance, used earlier rounds of this survey (1983–1990) to quantify how racial and economic factors shape British
attitudes towards immigration. Facchini and Mayda (2007) and Card et al. (2011) perform similar analyses using different datasets. Their findings are consistent with the predictions of our model. In particular, they find that the preferred number of immigrants is (1) negatively correlated with age, and (2) positively correlated with income, as expected.

Our empirical exercise is similar in nature to those in the existing literature. The key difference between past research and this study – which is more limited in scope – lies in the goal of the analysis. While we do not claim to prove the existence of a causal relationship, we aim to provide suggestive evidence for a key implication of the theoretical model, which is the following.

Our model proposes a channel to explain the strong aversion to immigration exhibited by elderly citizens in survey data. That is, a negative perceived fiscal effect of immigration that occurs after retirement. If such a channel is indeed a key determinant of such attitudes and is substantial in magnitude, then the respondents should tend to become more averse to immigration as they grow old. That is, a citizen’s preferences on the size of immigration should worsen over their life cycle. As a consequence, the positive correlation between age and aversion towards immigration should survive after controlling for cohort effects and the year of the survey. Conversely, if such a correlation is mostly driven by factors that are less likely to vary along the life cycle (e.g., cultural and ideological motives) we should expect it to decrease in magnitude – and possibly vanish – after adding such controls. Our goal is to test this implication of the model.

In the online appendix we provide the results of several empirical analyses performed using this dataset. In particular, we analyse the determinants of attitudes towards public spending financed through taxation. We omit the description of these results in this paper because they are fully consistent with the findings in the empirical literature – as well as with the predictions of our model. That is, the preferred level of taxation to finance public spending is positively correlated with age and negatively correlated with income.

The next section details the data, methodology, and results of this analysis.

5.1 Data and Methods

The dataset accounts for a total of 20,460 observations. The explanatory variables are respondent age (RAge) and household income decile (HHIncD).

56 Only 13,398 observations include information on attitudes towards immigration and only 17,895 observations include information about attitudes towards public spending financed through taxes.

57 The use of household income instead of individual income is justified because the effect of taxes on individual consumption levels typically depends on household income. For instance, for a household in which only one member has
<table>
<thead>
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<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
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<td>5.422016</td>
<td>3.352886</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4: Summary Statistics.

We control for the highest educational qualification attained by the respondent (HEdQual), on a scale from 1 (graduate degree) to 7 (no qualification). Dummy variables capture whether the household includes children (ChildHh), the sex of the respondent (RSex), if they live in rural areas (ResPres), if they are born abroad (BornAbr), if they are religious (Religion), and if they are unemployed (Unempl). The dummy variable Brexit corresponds to the year 2017 (i.e., the only included survey round that was conducted after the referendum on EU membership).

The outcome variable LessImmigr captures the respondent’s attitude towards further immigration. The question is “Do you think the number of immigrants to Britain nowadays should be increased a lot, increased a little, remain the same as it is, reduced a little or reduced a lot?” The respondent selects a value on a discrete scale from 1 (“increased a lot”) to 5 (“reduced a lot”). Thus, the variable LessImmigr measures the degree of aversion towards open immigration policies. The majority of respondents in all periods exhibit a strong aversion to further immigration.

Summary statistics are shown in Table 4.

It is well known that it is not generally possible to separately identify age, cohort and period effects in linear models (Heckman & Robb, 1985). I address this problem by imposing various restrictions on the nature of the cohort and/or period effects, each corresponding to an empirical specification, all of which are detailed in the next section. All results are robust across various specifications.

We use a standard ordered logit model because of the discrete and ordered nature of each outcome variable. The outcome variable LessImmigr can take values \( j \in \{1, 2, 3, 4, 5\} \). A latent variable

positive income, the consumption levels of other family members depends on the income tax rate, even if they do not directly pay an income tax.

The 1995 round of the survey does not include information regarding the respondents’ country of birth or the presence of children in the household. Thus, data from that round are only used in specification [2] in Table 2.

For the 2017 round of BSA, the question changed to “Once Britain has left the EU, do you think immigration into Britain should be increased, reduced, or stay at more or less the same level as now?” Due to this change, we control for the dummy Brexit in specifications [1], [2] and [3] and exclude the most recent data round (2017) in specification [4].

The variable IncreaseTax measures the respondent’s attitude towards public spending financed through taxation. It is the outcome variable the second part of this empirical analysis, whose results are presented in the online appendix. The question is “Suppose the government had to choose between the three options on this card: reduce taxes and spend less on health, education and social benefits, keep taxes and spending on these services at the same level as now, increase taxes and spend more on health, education and social benefits. Which do you think it should choose?” The respondent selects a value on a discrete scale from 1 (“spend less”) to 3 (“spend more”).
LessImmigr is assumed through:

\[ \text{LessImmigr}^*_i = \beta_1 \text{RAge}_i + \beta_2 \text{HHIncD}_i + \beta_3 \text{HEdQual} + \ldots + \epsilon_i \]

The probability of observing the outcome \( \text{LessImmigr}_{it} = j \) conditional on covariates is:

\[ \text{Prob}(\text{LessImmigr}_{it} = j \mid X_{it}) = F(\alpha_j - \text{LessImmigr}^*_i) - F(\alpha_{j-1} - \text{LessImmigr}^*_i) \]

where \( X_{it} \) is the vector of explanatory variables and \( \alpha_{j-1}, \alpha_j \) are the endogenous thresholds on the value of the latent variable that correspond to a switch from choice \( j - 1 \) to \( j \) and from choice \( j \) and \( j + 1 \), respectively. The robust standard errors are clustered at the regional level.\(^\text{61}\)

### 5.2 Determinants of the Preferred Number of Immigrants

Table 5 presents the results of the ordered logit regression with standard errors in parentheses. Table 6 shows the average marginal effects of the regressors of interest with respect to the outcome \( \text{LessImmigr} = 5 \) (i.e., that which corresponds to the strongest hostility towards immigration).

In line with the prediction of the theoretical model, respondent age exhibits a significant positive relationship with the aversion towards immigration. Specifically, an additional year of age results in an approximate average increase of 1 percentage point in the probability of outcome \( \text{LessImmigr} = 5 \). Moreover, the parameter on household income decile and the corresponding marginal effect are negative in all specifications and statistically significant in most, meaning that high-income respondents tend to be less averse to immigration relative to low-income respondents. This is also consistent with the predictions of the model.

Specifications (1) and (2) include time trends and dummies for the respondent’s cohort.\(^\text{62}\) Specifications (3) and (4) includes cohort trends and dummies for the survey year. The coefficient on the dummy Brexit is negative and statistically significant in all the specifications that include this variable.

For illustrative purposes, we simulate the probability of response \( \text{LessImmigr} = 5 \) by an employed, male, UK-born individual in 2017 evaluated at different ages. Fig. 6 plots the effect of age on the

---

\(^{61}\)Clustering for specifications (1) - (3) - (4) is based on a twelve-region partition. For specification (2), which includes data from the 1995 survey round, clustering is based on a six-region partition due to a different classification used prior to 2003.

\(^{62}\)We group the cohorts using intervals of 10 years (1906–1915, 1916–1925, etc.).
### Table 5: Preferred number of immigrants (BSA 1995-2017)

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) LessImmigr</th>
<th>(2) LessImmigr</th>
<th>(3) LessImmigr</th>
<th>(4) LessImmigr</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAge</td>
<td>0.0459***</td>
<td>0.0452***</td>
<td>0.0449***</td>
<td>0.0415***</td>
</tr>
<tr>
<td></td>
<td>(0.0108)</td>
<td>(0.00898)</td>
<td>(0.00606)</td>
<td>(0.00624)</td>
</tr>
<tr>
<td>HHIncD</td>
<td>-0.0233***</td>
<td>-0.0156*</td>
<td>-0.0232***</td>
<td>-0.0127</td>
</tr>
<tr>
<td></td>
<td>(0.00943)</td>
<td>(0.00903)</td>
<td>(0.00918)</td>
<td>(0.00913)</td>
</tr>
<tr>
<td>HEdQual</td>
<td>0.221***</td>
<td>0.217***</td>
<td>0.221***</td>
<td>0.243***</td>
</tr>
<tr>
<td></td>
<td>(0.0201)</td>
<td>(0.0137)</td>
<td>(0.0204)</td>
<td>(0.0249)</td>
</tr>
<tr>
<td>RAge_2</td>
<td>-0.000157</td>
<td>-9.83e-05</td>
<td>-0.000153**</td>
<td>-9.21e-05</td>
</tr>
<tr>
<td></td>
<td>(0.000117)</td>
<td>(9.93e-05)</td>
<td>(7.67e-05)</td>
<td>(7.12e-05)</td>
</tr>
<tr>
<td>RSex</td>
<td>0.0264</td>
<td>0.0595</td>
<td>0.0275</td>
<td>0.0405</td>
</tr>
<tr>
<td></td>
<td>(0.0549)</td>
<td>(0.0375)</td>
<td>(0.0548)</td>
<td>(0.0668)</td>
</tr>
<tr>
<td>Unempl</td>
<td>-0.148</td>
<td>-0.164**</td>
<td>-0.137</td>
<td>-0.140</td>
</tr>
<tr>
<td></td>
<td>(0.0931)</td>
<td>(0.0698)</td>
<td>(0.0904)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>Religion</td>
<td>-0.000976</td>
<td>-0.0936***</td>
<td>-0.00295</td>
<td>-0.0437</td>
</tr>
<tr>
<td></td>
<td>(0.0356)</td>
<td>(0.0265)</td>
<td>(0.0352)</td>
<td>(0.0434)</td>
</tr>
<tr>
<td>ResPres</td>
<td>0.178***</td>
<td>0.178***</td>
<td>0.175***</td>
<td>0.175***</td>
</tr>
<tr>
<td></td>
<td>(0.0543)</td>
<td>(0.0540)</td>
<td>(0.0540)</td>
<td>(0.0563)</td>
</tr>
<tr>
<td>BornAbr</td>
<td>-0.813***</td>
<td>-0.817***</td>
<td>-0.927***</td>
<td>-0.927***</td>
</tr>
<tr>
<td></td>
<td>(0.0937)</td>
<td>(0.0935)</td>
<td>(0.119)</td>
<td></td>
</tr>
<tr>
<td>ChildHh</td>
<td>-0.179***</td>
<td>-0.189***</td>
<td>-0.182***</td>
<td>-0.182***</td>
</tr>
<tr>
<td></td>
<td>(0.0430)</td>
<td>(0.0435)</td>
<td>(0.0537)</td>
<td></td>
</tr>
<tr>
<td>North_En</td>
<td>0.285</td>
<td>0.218</td>
<td>0.285</td>
<td>0.312*</td>
</tr>
<tr>
<td></td>
<td>(0.190)</td>
<td>(0.205)</td>
<td>(0.188)</td>
<td>(0.177)</td>
</tr>
<tr>
<td>South_En</td>
<td>0.0939</td>
<td>-0.00859</td>
<td>0.0947</td>
<td>0.161</td>
</tr>
<tr>
<td></td>
<td>(0.194)</td>
<td>(0.262)</td>
<td>(0.193)</td>
<td>(0.185)</td>
</tr>
<tr>
<td>year</td>
<td>-0.000373</td>
<td>-0.0436***</td>
<td>-1.290***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00916)</td>
<td>(0.00456)</td>
<td>(0.108)</td>
<td></td>
</tr>
<tr>
<td>Brexit</td>
<td>-1.254***</td>
<td>-1.290***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0794)</td>
<td></td>
<td>(0.108)</td>
<td></td>
</tr>
<tr>
<td>Cohort10</td>
<td></td>
<td>0.235***</td>
<td>0.245***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0531)</td>
<td>(0.0684)</td>
<td></td>
</tr>
</tbody>
</table>

Observations: 9,407, 10,303, 9,407, 7,136

Pseudo R²: 0.0774, 0.0368, 0.0772, 0.0556

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1
Table 6: Preferred number of immigrants: marginal effects.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) marginal effect</th>
<th>(2) marginal effect</th>
<th>(3) marginal effect</th>
<th>(4) marginal effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAge</td>
<td>0.00991***</td>
<td>0.0104***</td>
<td>0.00972***</td>
<td>0.00922***</td>
</tr>
<tr>
<td></td>
<td>(0.00234)</td>
<td>(0.00214)</td>
<td>(0.00132)</td>
<td>(0.00136)</td>
</tr>
<tr>
<td>HHIncD</td>
<td>-0.00502**</td>
<td>-0.00360*</td>
<td>-0.00501***</td>
<td>-0.00282</td>
</tr>
<tr>
<td></td>
<td>(0.00204)</td>
<td>(0.00208)</td>
<td>(0.00199)</td>
<td>(0.00203)</td>
</tr>
<tr>
<td>RAge_2</td>
<td>-3.40e-05</td>
<td>-2.27e-05</td>
<td>-3.31e-05**</td>
<td>-2.05e-05</td>
</tr>
<tr>
<td></td>
<td>(2.52e-05)</td>
<td>(2.30e-05)</td>
<td>(1.65e-05)</td>
<td>(1.58e-05)</td>
</tr>
<tr>
<td>Observations</td>
<td>9,407</td>
<td>10,303</td>
<td>9,407</td>
<td>7,136</td>
</tr>
</tbody>
</table>

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Figure 7: Probability of LessImmigr = 5 vs Age: simulated probabilities. Effect of cohort (left) and Brexit (right).

The key finding of this analysis is that the negative relationship between age and attitude towards immigration suggested by the model is supported by this analysis even after controlling for cohort effects and time. In fact, our estimates suggest that cohort effects alone would generate a negative relationship between age and aversion towards immigration. Moreover, time effects do not appear to play a major role in explaining the relationship of interest, with the exception of the Brexit dummy.
Thus, our empirical analysis provides a strong indirect support for the main fiscal mechanism that shapes voters’ preferences in our theoretical model. A much more demanding empirical question is whether population ageing and/or income inequality have an impact on actual immigration policy and, if so, to what extent this is due to a causal link. An attempt to answer this question has been carried out by Boeri and Brucker (2005) for 15 European countries using a variety of data sources and approaches. Their results are mostly in line with the predictions of our model. However, due to the limitations of the existing literature, this remains an open and challenging question for future research.

6 Discussion, Robustness, and Extensions

In this section, we extend and discuss the findings from section 3 and provide some robustness results.

6.1 Robustness: Pension System

The theoretical model outlined in section 3 features rather strong and specific restrictions on the pension system. However, our results are robust to a number of alternative assumptions, which are summarized below. All the results and proofs derived under such alternative assumptions are provided in the online appendix.

(i) **PAYG vs. partially funded pension system.** All the results hold if we depart from a pure **PAYG** pension system by adding a funded component in the form of compulsory savings. However, an increase in the funded portion of the pension system relative to the public component is not innocuous, resulting in more restrictive immigration policies and higher public spending levels.

(ii) **National pension fund.** We can relax the assumption that pensions are financed through general taxation. In particular, all the results hold true if one assumes the existence of a self-sufficient national pension fund financed through social security contributions, provided that the assumptions **PAYG**, Galasso and Profeta (2004) provide empirical evidence of an increasing size of the funded portion of the pension relative to the state pension in several European countries.

64The intuition underpinning this result is that a transition towards a private pension system leads to a fall in the cost of the social security system per taxpayer and, therefore, a decrease in the marginal fiscal gains of immigration per native worker.
Barbara and ABM are maintained.

(iii) Endogenous pensions. We analyse an augmented model in which the expected size of the pension system (parameter $\gamma$ in the baseline model) is made endogenous to voters’ electoral choices in each period, such that the policy space becomes three-dimensional. Given some mild technical restrictions\(^{65}\), and as long as the income distribution of the young exhibits sufficiently high dispersion - all the results in Proposition 1-2-3-4 hold true. Moreover, we show that in this augmented model the endogenous size of the pension system is weakly increasing in $\lambda, \rho$ and weakly decreasing in $\sigma, \xi$; i.e., population ageing, increasing income inequality, and economic depression translate into higher pension spending to output. In turn, this channel exacerbates the negative effects of these sociodemographic shocks on the equilibrium size of the government illustrated in Proposition 2.

6.2 Other Robustness Results and Extensions

The main analytical results of this paper are robust to several alternative assumptions, some of which are outlined below. A detailed description of these additional results is provided in the online appendix.

(i) Voting rights. All the results hold if we depart from the assumption on the naturalisation of immigrants outlined in section 2 (ius soli) and impose the alternative assumption that immigrants and their children never obtain voting rights (ius sanguinis).\(^{67}\)

(ii) Labour supply. As long as the wage elasticity of the labour supply is positive for all workers, all the results carry over if the assumption of perfectly inelastic labour supply is relaxed. This is true, for instance, in the presence of a quadratic utility cost of labour.

(iii) Production technology and endogenous wages. If the degree of concavity of the production function is sufficiently small, all the findings carry over if the assumption of linear production function is relaxed, so that wages decrease with the number of immigrants. This is true, for instance, in an economy with capital and a Cobb-Douglas production function under relatively mild restrictions.

(iv) Endogenous public investment in education. All the main results hold true in an alternative

\(^{65}\)In the online appendix we show that all the results are qualitatively unchanged under the alternative assumption of the existence of a self-sufficient national insurance fund that is formally separated from general taxation and is financed through social security contributions of all working-age individuals. This alternative assumption represents a more realistic description of the UK state pension system, which features a National Insurance Fund financed through social security contributions and required by law to be self-sufficient in expectation.

\(^{66}\)Specifically, we need to assume a continuum of types $\Theta_t = [\bar{\theta}, \tilde{\theta}]$ with smooth c.d.f. $F_t(\theta_t; \rho)$, a weakly progressive pension system, and restrict the curvature of the function $c$ to ensure that the citizen’s objective function is strongly concave. Then all the results hold true if the p.d.f. of the income distribution is sufficiently “flat”: $q_\rho(\theta_t) = \frac{\partial Q_\rho(\theta_t, x_t)}{\partial \theta_t} \leq \bar{q}$ for all $\theta_t \in \Theta$, and some threshold $\bar{q} > 0$.

\(^{67}\)In such a case, voters do not have to consider the impact of their current immigration policy choices on the age profile of the voting population in the following periods (i.e. there are no sophisticated effects), but the state space expands.
setup in which citizens also vote for the level of uniform public investment in education\footnote{This assumption is common in political economy models of intergenerational investment in education. For a review of this type of model, see Dotti (2019).} which determines the average output level in the following period (e.g., $y_{t+1}^i = \xi(e_t)w_{t+1}^i$, where $e_t$ is the level of per-pupil spending in public education).

(v) Dynamics. In order to study the dynamics of the policy adjustment occurring after a demographic and/or economic shock, in the online appendix we study an infinite horizon version of the model outlined in section 2 of this paper. We prove that for $T \to +\infty$ - if $\Delta$ is not too large in magnitude - the economy becomes stationary and possesses a unique steady-state. Moreover, we show that the key qualitative implications of Proposition 1-2-3-4 hold true not only in a static fashion, but also dynamically. Namely, we prove that for an economy at the steady-state, if a shock of type a, b, c, and/or d occurs, the economy converges to a new steady state featuring a more restrictive immigration policy, higher public spending, and a larger government. A detailed description of these findings and their proofs is provided in the online appendix\footnote{Specifically, we provide the proof of existence and uniqueness of the steady-state and its characterisation, as well as the dynamic counterparts of Proposition 1-2-3-4 for the infinite-horizon model. Note that if $\Delta$ grows large, then the dynamic system may feature multiple steady-states.}

6.3 Discussion

In this paper, we purposely abstract from some factors that are likely to play a role in shaping voters' immigration policy choices. These aspects deserve further analysis and represent topics for future research. We discuss some of the most important factors in this section.

(i) Anti-immigration politics and right-wing populism. This paper focuses on anti-immigration politicians and the determinants of their success in elections. The literature in political science and political economy shows that politicians advocating anti-immigration policies are typically members of so-called right-wing populist parties. Moreover, there is compelling empirical evidence that elderly and relatively poor voters are more likely to support these right-wing populist parties than the young and wealthy (Becker & Fetzer, 2017).

Right-wing populist parties are defined in the literature as those that combine vehement anti-elite rhetoric (Acemoglu et al. 2013) with a conservative agenda (e.g., restrictive immigration policy, nationalism, etc.). However, in contrast to traditional conservative parties, populist parties are generally characterised by a commitment to implement simple policies that please voters in the short run, such as overinflating the size of government (Dornbusch and Edwards, 1991) and curbing immigration,
regardless of the long-term consequences of such policies (Guiso et al., 2019).

As a consequence, right-wing populist parties typically exhibit a conservative stance on immigration and a traditionally left-wing fiscal policy platform. This description is consistent with the political trajectories of several parties typically labelled right-wing populist parties in Europe, such as the National Front in France, the United Kingdom Independence Party and the Northern League in Italy.

It is also consistent with the policy platform offered by anti-immigration politicians in our model.

In the light of these considerations, the analysis of the role played by anti-immigration politics in the formation, proliferation and electoral success of right-wing populist parties in several Western democracies represents a promising field of research and a natural extension of the present paper.

(ii) Endogenous selection of immigrants. In section 2, we assume that the supply of potential immigrants is large and the average productivity of the immigrant population is fixed. This means that (1) the government cannot select immigrants based on their skills and (2) we rule out the possibility of the endogenous self-selection of welfare-dependent immigrants based on the number of public benefits provided by the receiving country.

Restriction (1) is strong but unlikely to severely affect our analysis. Several papers in the literature have examined the political economy of qualitative immigration policy based on skill requirements under the assumption that the government can observe the productivity of potential immigrants (Benhabib, 1996; Ortega, 2005). The typical prediction of such models is that the elected government sets a threshold such that only individuals whose productivity exceed such threshold are allowed to legally immigrate. This feature results in a higher average productivity of the immigrants. Thus, for any given level of such threshold, the tradeoffs illustrated in our model regarding the number of immigrants preferred by different types of voters should not be severely affected.

Restriction (2) represents a theoretically important concern, which has been extensively studied in the theoretical literature (Borjas, 1999). In our model, allowing for endogenous self-selection of immigrants would have important consequences, because the fiscal effects of immigration would be-

\footnote{Over the last two decades, these parties have shifted away from their early libertarian economic positions to strong interventionist views (Minkenberg, 2000; Mudde, 2007), particularly regarding certain provisions, including healthcare, social services, and elderly care. For instance, while initially labelled a libertarian party advocating a smaller state, UKIP has consistently proposed a policy platform characterized by a substantial increase in public spending. For instance, the UKIP manifesto for the 2015 national elections pledged “an extra £3bn a year into the NHS in England” and “a commitment to spend 2% of GDP on defence initially, looking to increase it substantially after that”. These figures far exceeded the pledges of their main rivals, the Conservatives and the Labour Party [see: Curtice, 2012]. Other examples include the Freedom Party in Austria, the Danish People’s Party in Denmark, and Fidesz in Hungary. In some cases, right-wing populist parties stem from the evolution of pre-existing far-right statist groups (e.g., the Finns Party in Finland, the Sweden Democrats in Sweden, and Brothers of Italy). Others are entirely new parties founded with their modern populist platforms sometime in the last 20 years (e.g., Independent Greeks in Greece, the Party for Freedom in the Netherlands, and Law and Justice in Poland).}
come a function of the endogenous fiscal policy, potentially affecting the citizen’s tradeoffs regarding immigration. However, the empirical literature suggests that the size of this effect is generally fairly small (Preston, 2014). Thus, for the purpose of this study, we strongly believe that our assumption is a reasonable approximation.

(iii) Non-economic factors. Non-pecuniary factors such as cultural and psychological motivations (Brettell & Hollifield, 2007) are deemed to play an important role in shaping voter’s attitudes towards immigration. In particular, the empirical literature highlights the effect of immigration on compositional amenities (Card et al., 2011). These are public goods whose quality depends on the sociodemographic and ethnic composition of the society. Compositional amenities are typically related to the specific religion, traditions, and language of the receiving society. Because such effects are likely to differ across socioeconomic groups among the natives, this channel may represent a complementary explanation to the stylised facts described in the introduction section of the present paper.

However, our analysis purposely downplays the role of non-economic factors by assuming a common taste for immigration given by the function $c$ in (1) and (2). This assumption is imposed for transparency and ease of interpretation, clearly illustrating that the key tradeoff shaping our results does not depend upon taste differences across age and income groups. However, it is not crucial for our analysis and can be easily relaxed: all the results hold as long as all the citizens possess the same lower-bound $M$ as defined in section 2.1.3. In particular, we could alternatively assume that naturalised immigrants possess a different taste for immigration than natives and that the function $c$ is age-specific. Lastly, we can also allow for native citizens (i.e. excluding naturalized foreign-born individuals) to have a taste not only for the ratio of recent immigrant to total young ($M_t$), but also for that of old foreign-born individuals to total old individuals ($M_{t-1}$) with no qualitative changes in the model predictions (see online appendix). This should reassure the reader regarding the robustness of our predictions.

(iv) Segmented labour markets. Labour market skills may qualitatively differ across workers and not be perfect substitutes of each other, resulting in segmented labour markets. If the production technology is not linear in labour and wages are skill-specific (e.g., high- vs low-education jobs), then the skill composition of the immigrants does matter in shaping the natives’ wages. However, as long as the immigrants possess relative less paid skills on average, the implications of our model should not qualitatively change. That is, the effect of immigration on wages should be more negative for low-income than high-income natives. Thus, this channel should reinforce our result that the poor are more averse to immigration than the rich.
Moreover, the immigrants often possess specific labour market skills that are not available to natives workers (Peri & Sparber, 2009). In particular, in several Western countries immigrant workers perform tasks that benefit mostly the elderly within the native population, such as nursing and home care. This feature may affect the preferences of elderly citizens towards immigration. While the analysis of this channel is beyond the scope of the present paper, it represents a promising topic for future research.

(v) Public debt. In models of voting over public debt, elderly voters – who typically care less about the future than the young – have an incentive to use such policy instrument to increase current public spending and transfer the burden to future generations (Tabellini 1991). This mechanism implies that “[…] the greater fraction of old impatient households relative to young patient households, the more shortsighted is the government, the larger are government deficits, and the faster is government debt accumulation” (Yared, 2019). This dynamic is consistent with the narrative of this paper and strengthen our predictions regarding the effect of population ageing on public spending and the size of government. However, augmenting our model to allow for endogenous public debt implies some non-trivial technical difficulties71 and is beyond the scope of this paper. Such extension represents an interesting topic for future research.

7 Concluding Remarks

This paper investigates the interactions among three key demographic, economic and social processes: ageing, rising inequality and immigration. The aim is to analyse how these processes shape fiscal and immigration policies in democratic countries using the UK as a case study. We detail the effects of increasing life expectancy, decreasing birth rates and rising income inequality on voter’s preferences, policy choices, the political system, and societal wellbeing.

The key novelty of this analysis is that we allow voters to choose both the immigration and fiscal policy (i.e., not only the number of immigrants but also how society divides costs and benefits of immigration). This choice is shown to generate perceived competition between natives and immigrants over welfare benefits – even if no actual competition occurs – because open immigration is endogenously bundled with low public spending in the platform of pro-immigration politicians. As a consequence, such a

71In detail, in the short run an increase in public debt may favour relatively high-income citizens through reduced tax rates. This mechanism represents a major technical obstacle in incorporating public debt in our analysis, because it would jeopardize the total order on citizen’s preferences and, in turn, the existence of a Condorcet winner over the set of citizen’s ideal policies. Such preference pattern would also be inconsistent with the findings in the empirical literature, which suggest the existence of a negative relation between individual income and attitude towards public debt (Heinemann and Hennighausen, 2012). We conjecture that this apparent inconsistency may vanish in a more realistic model with savings and endogenous default on public debt.
platform produces negative short-term fiscal effects on the most welfare-dependent segments of the voting population: the elderly and the poor. In turn, this mechanism causes those types of voters to be strongly hostile to open immigration policies and supportive of anti-immigration candidates in elections.

The first finding of this paper is that population ageing and rising income inequality increase the political pressure to restrict the inflow of immigrant workers and inflate the size of government. This finding suggests that the negative effects of population ageing on public finances due to the increasing costs of the social security system may be exacerbated by endogenous political effects. Direct and indirect effects of the ageing phenomenon may affect the long-run fiscal soundness of the public sector.

The second finding concerns the political effects of these sociodemographic shocks. We show that ageing and rising inequality can help explain the success of anti-immigration politicians and parties in recent years.

The third finding is that the tightening of immigration policy induced by population ageing and rising inequality is generally harmful, though the harm is most severe for young people and future generations. This analysis delivers a pessimistic prediction regarding the ability of our society to adjust to demographic changes and the consequences of such changes on young generations. Population ageing results in an increase in the power of the elderly to shape public policy according to their needs. As a result, young natives and young potential immigrants pay a price. Young natives must support the financial burden of an increasingly large and long-living elderly population, whereas young potential immigrants are prevented from searching for better employment and life opportunities by excessively restrictive immigration policies.

This worrisome no country for young people scenario warrants further research on this topic and constitute a challenge for policy design. The scenario suggests that a key goal of social security reforms in the immediate future should be the promotion of the internalisation of the positive fiscal effects of immigration among elderly and low-income citizens. This could be achieved, for instance, by linking the generosity of the social security system to the expected future old-age dependency ratio of the native population. Reforms in this vein have been attempted in in several European countries over the last two decades, such as Finland in 2005 and Italy in 2010.
Appendix

Appendix A includes formal descriptions of the equilibrium concept and of the two key properties of citizens’ preferences. We maintain the assumption that the difference in fertility rates between immigrants and natives is sufficiently small; i.e. \( \Delta \in [0, \Xi] \) for some threshold \( \Xi > 0 \) whose characterization is provided at the end of section B of this appendix.

A.1 Equilibrium

Let \( a_i^t \in A_i^t \) denote the action chosen by citizen \( i \in N_t \) in a period \( t \in \{1, 2, ..., T\} \) and \( a_i \in A_i \) be the corresponding action profile of all citizens, where \( A_i \) is the set of all possible action profiles.\(^{72}\)

Following Masking and Tirole (2001) we define a strategy \( s^i \) for player \( i \) as a function that, for all periods \( t \) and each history \( h_t \in H_t \), selects an action \( a_i^t \in A_i^t \). Let \( S \) be the set of all possible strategies and \( S_t(h_t) \) denote the set of continuation strategies with typical element \( s_t \). A continuation strategy \( s_t^i \) for player \( i \) is a function that, for each period \( t + r \) (with \( r = t - T \)) and each history \( h_{t+r} \in H_{t+r} \), selects an action \( a_{i+r}^t \in A_{i+r}^t \). For instance, for \( T = 2 \) a continuation strategy in period \( t = 1 \) can be written in the form \( s_t^i(h_1, h_2) = (s_{i,1}^1(h_1), s_{i,2}^1(h_2)) \). Let \( S \) be the set of all possible continuation strategies for all players and \( S_t(h_t) \) denote the set of continuation strategies with typical element \( s_t \) (i.e., the set of all collective strategies in the subgame starting after \( h_t \)).

In order to define the expected payoffs for each player \( i \) from playing strategy \( s^i_t \), let \( a_i^t(s_t | h_t) \) denotes the action selected by citizen \( i \) in period \( t \) given strategy \( s_t \) and history \( h_t \); i.e., the outcome of function \( s^i_t \) in period \( t \) given \( h_t \), and \( x^*_t(s_t | h_t) \) be the equilibrium (two-dimensional) policy implemented at time \( t \). We define the objective function conditional on history \( h_t \) and strategy \( s_t \) of a citizen of type \( \theta_t \in \Theta \), denoted by \( v_t \), as follows:\(^{73}\) \( v_t(x_t; \theta_t, \varphi | h_t, s_t) \equiv E_t [u_t \left( \left\{ x_t, x_{t+1}^* (s_{t+1} | h_{t+1}) \right\}; \theta_t, \varphi, z_t(h_t) \right) | s_t, h_t, x_t] \) for \( s_t \in S_t(h_t) \) and \( x \in X \). Note that this definition capture the fact that forward-looking agents anticipate the effects of current policy choices \( x_t \) on future equilibrium policy outcomes \( x_{t+1}^* (s_{t+1} | h_{t+1}) \).

Using this definition, we can construct the last three key concepts we need, namely:

1. The set of ideal policies of a citizen of type \( \theta_t^i \) given \( s_t, h_t \), which writes \( I_t (\theta_t^i | s_t, h_t) \equiv \arg \max_{x_t \in X} v_t(x_t; \theta_t^i, \varphi | h_t, s_t) \)

\(^{72}\)Thus, \( A_i \) is the Cartesian product of all sets \( A_i^t \) and each element \( a_i \) possesses the cardinality of the continuum.

\(^{73}\)Note that the notation for the objective function of an individual \( i \) becomes \( u_i^{T, \text{type}} = u_i \left( \left\{ (M_t, L_t), (M_{t+1}, L_{t+1}) \right\}; -1, \varphi, z_t \right) \) for a time horizon \( T \).
2. The set of candidates given \( s_t, h_t \), defined as follows: 
\[
C_t(s_t \mid h_t) \equiv \{ i \in N_t \mid a^i_t(s_t \mid h_t) \neq \emptyset \},
\]
where \( \emptyset \) denotes the choice of being inactive.

3. The set of Condorcet winners given \( s_t, h_t \), defined as 
\[
W_t(s_t \mid h_t) = \left\{ i \in C_t(s_t \mid h_t) \mid \int_{\theta_t \in \Theta} 1 \left\{ v_t(x^i_t; \theta_t, \varphi \mid h_t, s_t) \geq v_t(x_j^i_t; \theta_t, \varphi \mid h_t, s_t) \right\} dF_t(\theta_t \mid h_t) \geq 0.5 \forall j \in C_t(s_t \mid h_t) \right\}
\]
That is, \( W_t(s_t \mid h_t) \) is the set of candidates that are weak Condorcet winners over \( C_t(s_t \mid h_t) \). Using
these three definition, we can now state the last two key concepts.

1. **Citizen-candidates (CC):** each citizen can either propose a platform \( x_t \) in \( I_t(\theta^i_t \mid s_t, h_t) \) or be
inactive, i.e. \( a^i_t \in A^i_t(s_t \mid h_t) \) where \( A^i_t(s_t \mid h_t) = I_t(\theta^i_t \mid s_t, h_t) \cup \{ \emptyset \} \); if a candidate \( j \) is elected,
his/her platform is implemented: \( x^i_t(s_t \mid h_t) = a^i_t \).

2. **Majority Rule (MR):** the winning candidate \( w_{s_t|h_t} \) is chosen using a Condorcet Method, i.e.
\( w_{s_t|h_t} \in W_t(s_t \mid h_t) \), where \( W_t(s_t \mid h_t) \) is the set of candidates that are weak Condorcet winners
over \( C_t(s_t \mid h_t) \).

In words, the citizens select one candidate in \( C_t(s_t \mid h_t) \) who is a weak Condorcet winner over
\( C_t(s_t \mid h_t) \). If no candidate is chosen using such method; i.e., \( W_t(s_t \mid h_t) = \{ \emptyset \} \), then we assume
that a default policy \( x^0_t \) is implemented, where \( x^0_t \) is such that \( v_t(x^0_t; \theta_t, \varphi \mid h_t, s_t) = -\infty \) for all
\( x_t \in X \) and all \( \theta_t \in \Theta \). This means that voters strongly dislike outcomes in which no platform is
proposed by any citizen, or in which no stable choice is achieved through majority voting.\(^\text{74}\)
Lastly, we impose the following tie-break rules: (TB1) if \( W_t(s_t \mid h_t) \subseteq W_t(s'_t \mid h_t) \) then \( w_{s_t|h_t} = w_{s'_t|h_t} \); (TB2)
if \( j, k \in W_t(s_t \mid h_t) \) and \( \theta^j_t < \theta^k_t \), then \( w_{s_t|h_t} = j \). These two rule disciplines the collective choice in
the cases in which the set of Condorcet winners \( W_t(s'_t \mid h_t) \) is not a singleton. In particular, (TB2)
addresses those cases in which the median type over \( \Theta_t \) is not unique.

These assumptions imply that (1) the citizens collectively choose a candidate who is a (weak)
Condorcet winner over \( C_t(s_t \mid h_t) \) whenever such a candidate exists\(^\text{75}\) and (2) for any given set of candidates \( C_t(s_t \mid h_t) \), the winning policy is the platform of the winning candidate whenever such a
---
\(^{74}\)Note that the default policy \( x^0_t \) is not a credible platform for any citizen-candidate, because it is not an ideal policy
of any citizen in \( P_t \). Nevertheless, it is a possible off-equilibrium policy outcome if either the set \( C_t(s_t \mid h_t) \) is empty, or
no platform in \( C_t(s_t \mid h_t) \) is a weak Condorcet winner. This assumption can be easily relaxed whenever voter preferences
satisfy quasimodularity and the strict single crossing property, as in the present paper. In particular, all the results
hold true as long as either \( x^0_t \leq x^0_t \) or \( x^0_t \geq x^0_t \) holds true, where superscript \( m \) denotes an individual possessing the
median type in \( \Theta_t \). See Dotti [2020, 2021] for details.

^{75}The method of majority rule ensures that a Condorcet winner is selected whenever one exists. Alternatively, the same
outcome prevails in an election with simple plurality rule and strategic voting if voters do not play weakly dominated
strategies.
candidate exists; i.e., if \( W_t(s_t \mid h_t) \neq \emptyset \), then \( x_t^s(s_t \mid h_t) \in I_t\left(\theta_t^i \mid s_t, h_t\right) \) for some winning candidate \( w_{si|h_t} = j \).

Using this social choice mechanism, we define the payoff function conditional on history \( h_t \) and continuation strategy \( s_t \) of a citizen of type \( \theta_t \in \Theta \), denoted by \( v_t \), as:

\[
\Pi_t\left(s_t^1, s_t^{-i}; \theta_t, \varphi \mid h_t\right) \equiv v_t\left(x_t^s(s_t \mid h_t); \theta_t, \varphi \mid h_t, s_t\right)
\]

for \( s_t \in S_t(h_t) \) and \( x \in X \).

**Definition A.1. (Electoral Equilibrium).** (1) A citizen-candidate equilibrium (CCE) in period \( t \in \{1, 2, \ldots, T\} \) is a continuation strategy profile \( s_t \in S_t \) such that \( \Pi_t\left(s_t^1, s_t^{-i}; \theta_t, \varphi \mid h_t\right) \geq \Pi_t\left(s_t^{-i}, s_t^1; \theta_t, \varphi \mid h_t\right) \) for all \( s_t^i \in S_t^i \) and all \( i \in N_t \). (2) An electoral equilibrium (EE) in period \( t \) is a continuation strategy profile \( s_t^* \) that (i) is Markovian; i.e., \( s(h_t) = s(h_t') \) for all histories \( h_t, h_t' \) such that \( z_{t-1} = z_{t-1}' \); (ii) forms an electoral equilibrium after any history \( h_{t+r} \) in each period \( t+r \) with \( r = 1, 2, \ldots T-t \).

Definition A.1 states that an electoral equilibrium consists of a strategy profile that selects an action profile \( a_t+r \) each period \( t+r \) given the history up to period \( t+r \) and such that each element \( a_t^i \) is an ideal policy of a citizen who has decided to be active, i.e. such that \( a_t^i \neq \emptyset \). The condition for a strategy profile to be an electoral equilibrium is that there exists no citizen \( i \) that (i) possesses a feasible continuation strategy \( s_t^i \) that, given the strategies of other players, can induce in each period \( t+r \) with \( r = 0, 1, \ldots, T-t \) a (ii) new winner \( w_{t+r}' \) and a (iii) new policy outcome \( \left(M_{t+r}', L_{t+r}'\right) \) which make citizen \( i \) strictly better off.\(^{[70]}\)

The Markovian assumption (i) disciplines the beliefs about future equilibrium outcomes conditional on current choices. It is does not play any role in the analysis of the baseline version of the model described in section 3, but it is necessary if \( \Delta > 0 \). Lastly, condition (ii) corresponds to a standard notion of subgame perfection; i.e., agents believe that in any future period \( t+r \) an equilibrium is played given any possible history up to such a period. This rules out equilibria supported by non-credible threats regarding off-equilibrium behaviour.

\(^{[70]}\)The notion of equilibrium is almost identical to the one in Dotti (2020). The three main differences are that in the present paper (a) the equilibrium is defined in terms of candidates rather than policy platforms, (b) candidates may run for election even and (b) it is adapted to a two-period model with forward-looking agents. It is also very similar to that in Epple and Romano (2014). It differs from the latter in some minor details and in one key aspect. Namely, the way the set \( A \) is constructed and condition (iii) in Definition A.1 together ensure that, in the presence of a Condorcet winner among the set of citizens’ ideal policies, the equilibrium of the game is unique and features a single platform, i.e. \( A = \{x^*, Y^*\} \). Conversely, in Epple and Romano (2014) the equilibrium is typically not unique. In the supplementary material of the latter paper the authors propose an alternative equilibrium concept that delivers a unique policy outcome by introducing two political parties that select candidates. All the results in the present paper hold true if the latter definition of equilibrium is adopted.
A.2 Quasisupermodularity and Strict Single Crossing Property

Following Milgrom and Shannon (1994), we define two desirable properties for the conditional objective function \( v_t \).

**Definition A.3.** The function \( v_t \) in period \( t \) for given history \( h_t \) satisfies:

1. **Quasisupermodularity (QSM)** in \( (M_t, L_t) \) if, for any two \((M'_t, L'_t), (M''_t, L''_t) \) \( \in X \), one gets:

   \[
   v_t ((M'_t, L'_t); \theta_t, \varphi | h_t, s_t) - v_t ((M''_t, L''_t); \theta_t, \varphi | h_t, s_t) \geq 0
   \]

   \[
   \Rightarrow v_t ((M'_t, L'_t); \bar{\theta}_t, \varphi | h_t, s_t) - v_t ((M''_t, L''_t); \bar{\theta}_t, \varphi | h_t, s_t) \geq 0;
   \]  \( (13) \)

2. **Strict single crossing (SSC)** in \( (M_t, L_t; \theta_t) \) if, for any two \((M'_t, L'_t), (M''_t, L''_t) \) \( \in X \) with \((M''_t, L''_t) \geq (M'_t, L'_t) \) and \((M''_t, L''_t) \neq (M'_t, L'_t) \) and any two \( \bar{\theta}_t, \bar{\theta}_t \) \( \in \Theta \) with \( \bar{\theta}_t > \theta_t \), one gets:

   \[
   v_t ((M''_t, L''_t); \bar{\theta}_t, \varphi | h_t, s_t) - v_t ((M'_t, L'_t); \bar{\theta}_t, \varphi | h_t, s_t) \geq 0
   \]

   \[
   \Rightarrow v_t ((M''_t, L''_t); \bar{\theta}_t, \varphi | h_t, s_t) - v_t ((M'_t, L'_t); \bar{\theta}_t, \varphi | h_t, s_t) > 0.
   \]  \( (14) \)

QSM and SSC over the complete sublattice \( (X, \leq) \) are desirable properties because they imply that the set of ideal policies \( I(\theta_t | h_t, s_t) \) is monotonic nondecreasing in \( \theta_t \) over \( X \) by theorem 4 in Milgrom and Shannon (1994).

**B Proofs**

Appendix B includes the proofs to the results of the paper.

**B.1 Equilibrium Existence**

**Preliminaries.** First, note that the assumption on the pension system \( E_t \left[ \frac{\text{Total Pension Spending}_{t+1}}{\text{Total Output}_{t+1}} \bigg| M_{t+1} = M_t \right] = \gamma \forall \varphi \in \Phi \) and all \( z_t \in [0, 1) \) implies that the pension \( p_t^i \) must have form \( p_t (\theta_t, \xi, z_t) = \tilde{p}_t (\theta_t) \xi / z_t \) for some increasing function \( \tilde{p}_t \) that satisfies \( \int \tilde{p}_t (\theta_t) dQ_{\rho} (\theta_t) = \gamma \). Using formula \( (6) \) and \( (3) \), the
objective function $v_t$ of a young citizen (i.e. $\theta_t \geq 0$) writes:

$$
v_t ((M_t, L_t); \theta_t, \varphi | h_t, s_t) = \xi \theta_t + \gamma \xi M_t \theta_t + \xi \theta_t L_t + b(\bar{G} - L_t) + c(M_t) + \beta \tilde{p}_{t+1}(\theta_t) \xi \bar{\sigma}_t + A((M_t, L_t); \theta_t, \varphi, z_t(h_t)) + \beta \lambda E \left[ d(\bar{G} - L_{t+1}) + c(M_{t+1}) \left| (M_t, \bar{G} - L_t), z_t(h_t) \right. \right]_{B_{t+1}(M_t, \varphi, z_t(h_t))} \tag{15}
$$

where $\bar{\sigma}_t$ is the average birth rate in period $t$. Notice that given $M_t$ and $\varphi$ the object $\bar{\sigma}_t$ is known, i.e. $\bar{\sigma}_t = \sigma + \Delta M_t$. Also notice that $B_{t+1}(M_t, \varphi, z_t(h_t))$ is independent of $\theta_t$ at time $t$. Using formula (15), the objective function $v_t$ of an old citizen (i.e. $\theta_t = -1$) writes:

$$v_t ((M_t, L_t); -1, \varphi | h_t, s_t) = d(\bar{G} - L_t) + c(M_t) \tag{16}
$$

Using formulas (15) and (16) I can state the following results.

**Lemma 1.** The function $v_t$ satisfies (i) QSM in $(M_t, L_t)$ and (ii) SSC in $(M_t, L_t; \theta_t)$ for all $\varphi \in \Phi$ and after any history $h_t$.

**Proof.** Part (i). QSM in $(M_t, L_t)$. Consider any two elements $(M''_t, L''_t), (M'_t, L'_t) \in X$. A sufficient condition for QSM is Supermodularity (see Milgrom and Shannon 1994). Thus, for condition (13) to hold true it is sufficient that:

$$v_t ((M''_t, L''_t) \lor (M'_t, L'_t); \theta_t, \varphi | h_t, s_t) - v_t ((M''_t, L''_t); \theta_t, \varphi | h_t, s_t) \geq v_t ((M''_t, L''_t); \theta_t, \varphi | h_t, s_t) - v_t ((M'_t, L'_t) \land (M''_t, L''_t); \theta_t, \varphi | h_t, s_t) \tag{17}
$$

after any history $h_t$. Let $\hat{M}_t = \max\{M''_t, M'_t\}$ and $\hat{M}_t = \min\{M''_t, M'_t\}$, $\hat{L}_t = \max\{L''_t, L'_t\}$ and $\hat{L}_t = \min\{L''_t, L'_t\}$, such that $(\hat{M}_t, \hat{L}_t) = (M''_t, L''_t) \lor (M'_t, L'_t)$ and $(\hat{M}_t, \hat{L}_t) = (M''_t, L''_t) \land (M'_t, L'_t)$. Using formula (15), for young citizens the condition above can be written as:

$$
(\xi \theta_t + \beta \tilde{p}_{t+1}(\theta_t) \xi \Delta) \left( \hat{M}_t - M''_t - M'_t + \hat{M}_t \right) + c(\hat{M}_t) - c(M''_t) - c(M'_t) + c(M_t) + \beta \lambda \left[ B_{t+1}(\hat{M}_t, \varphi, z_t(h_t)) - B_{t+1}(M''_t, \varphi, z_t(h_t)) - B_{t+1}(M'_t, \varphi, z_t(h_t)) + B_{t+1}(\hat{M}_t, \varphi, z_t(h_t)) \right] + b(\bar{G} - \hat{L}_t) - b(\bar{G} - L''_t) - b(\bar{G} - L'_t) + b(\bar{G} - \hat{L}_t) + \xi \theta_t \left( \hat{L}_t - L''_t - L'_t + \hat{L}_t \right) \geq 0 \tag{18}
$$

Firstly, either $\hat{M}_t = M''_t$ and $\hat{M}_t = M'_t$, or $\hat{M}_t = M'_t$ and $\hat{M}_t = M''_t$ (a). Secondly, either $\hat{L}_t = L''_t$ and
\( \hat{L}_t = L'_t, \) or \( L_t = L'_t \) and \( \hat{L}_t = L''_t \). Then using results (a) and (b) into formula (18) we get that the left-hand side of (18) always equals zero, which implies that condition (17) is always satisfied for any \( \theta_t \geq 0 \).

For old citizens, using formula (16) the condition in (17) rewrites:

\[
d(\overline{G} - \hat{L}_t) - d(\overline{G} - L''_t) - d(\overline{G} - L'_t) + d(\overline{G} - L_t) + c(M_t) - c(M''_t) - c(M'_t) + c(M'_t) \geq 0 \tag{19}\]

Again, using the fact that either \( M_t = M''_t \) and \( \hat{M}_t = M'_t \), or \( M_t = M'_t \) and \( \hat{M}_t = M''_t \), and that either \( \hat{L}_t = L''_t \) and \( \hat{L}_t = L'_t \), or \( \hat{L}_t = L'_t \) and \( \hat{L}_t = L''_t \), we get that the left-hand side of (19) equals zero, which implies that condition (17) is also always satisfied for \( \theta_t = -1 \). Thus, condition (17) is satisfied for all possible types \( \theta_t \in \Theta \), which implies that \( v_t \) satisfies QSM in \( (M_t, L_t) \).

Part (ii). SSC in \( (M_t, L_t; \theta_t) \). I need to show that for any \( (M''_t, L''_t) \geq (M'_t, L'_t) \) in \( X \) with \( (M''_t, L''_t) \neq (M'_t, L'_t) \) and any \( \hat{\theta}_i > \underline{\theta}_i \in \Theta \) the condition in (14) holds true.

First I compare any types of two young citizens, i.e. any two \( \hat{\theta}_i > \underline{\theta}_i \geq 0 \). A sufficient conditions for (14) to hold true for any two \( \hat{\theta}_i > \underline{\theta}_i \geq 0 \) is the following.

\[
v_t((M''_t, L''_t); \hat{\theta}_i, \varphi | h_t, s_t) - v_t((M'_t, L'_t); \hat{\theta}_i, \varphi | h_t, s_t) > v_t((M'_t, L'_t); \underline{\theta}_i, \varphi | h_t, s_t) - v_t((M''_t, L''_t); \underline{\theta}_i, \varphi | h_t, s_t), \tag{20}\]

which corresponds to the definition of strictly increasing differences in \( (M_t, L_t; \theta_t) \) over \( \{\hat{\theta}_i, \underline{\theta}_i\} \). Use the formula (15), and notice that \( (M''_{t+1}(z_i+1), L''_{t+1}(z_i+1)) \) is independent of each \( i \)'s choice because each \( i \) possesses zero probability mass. Then, using formula (15) into condition (20), the latter writes:

\[
(\hat{\theta}_i - \underline{\theta}_i) \xi [\gamma (M''_t - M'_t) + (L''_t - L'_t)] + \beta [\hat{p}_{t+1} (\hat{\theta}_i) - \hat{p}_{t+1} (\underline{\theta}_i)] \xi \Delta (M''_t - M'_t) > 0 \tag{21}\]

which is always true under the assumptions \( \gamma > 0 \) and \( \Delta \geq 0 \).

Secondly, I compare each type of young citizen with \( \hat{\theta}_i \geq 0 \) to each old citizen with \( \hat{\theta}_t = -1 \). For any old individual, using formula (16) I get:

\[
v_t((M''_t, L''_t); -1, \varphi | h_t, s_t) - v_t((M'_t, L'_t); -1, \varphi | h_t, s_t) =
\]

\[
d(\overline{G} - L''_t) - d(\overline{G} - L'_t) + c(M''_t) - c(M'_t) < 0 \tag{22}\]

where the value of (22) is strictly negative because by assumption \( d \) is strictly increasing and \( c \) is strictly
decreasing for \( M_i > M_i \). Thus, condition (14) is always trivially satisfied for any \( \bar{\theta}_i, \bar{\theta}_t \) in \( \Theta \) such that \( \bar{\theta}_i \geq 0 \) and \( \bar{\theta}_t = -1 \), because the condition \( v_t ((M_i', L_i'); -1, \varphi | h_t, s_t) - v_t ((M_i', L_i'); -1, \varphi | h_t, s_t) \geq 0 \) is never true. Notice that the fact that (22) is always negative also implies that the corresponding alternative condition for SSC: \( v_t ((M_i', L_i'); \tilde{\theta}_i, \varphi | h_t, s_t) - v_t ((M_i', L_i'); \tilde{\theta}_i, \varphi | h_t, s_t) \leq 0 \rightarrow v_t ((M_i', L_i'); -1, \varphi | h_t, s_t) - v_t ((M_i', L_i'); -1, \varphi | h_t, s_t) < 0 \) is also always trivially satisfied, given that the only if part of such condition is always true. Lastly, because condition (14) is satisfied for all \( \bar{\theta}_i, \bar{\theta}_t \) in \( \Theta \), then \( v_t \) satisfies SSC in \((M_i, L_i'; \theta_t)\). Q.E.D.

**Proposition 1.** In each period \( t = 1, 2, ..., T \) (i) A EE always exists. In any EE (ii) the policy outcome \( x_t^* \) is an ideal policy of the pivotal citizen \( \theta_t^p \) and (iii) is unique given state \( z_t \). (iv) The pivotal citizen’s type \( \theta_t^p \) is weakly decreasing in \( z_t \).

**Proof.** Part (i). Suppose an electoral equilibrium in period \( t \) does not exist. Construct a strategy profile \( s_t \) as follows. In each period \( t + r \) for \( r = 0, 1, ..., T - t \) choose the action profile \( a_{t+r} \in A_{t+r} \) such that \( a_{t+r}^i = 0 \) for all \( i \neq j_{t+r}^p \) (where \( j_{t+r}^p \) is a citizen with type \( \theta_{t+r}^p \)) and \( x_{t+r}^p \in I (\theta_{t+r}^p \mid h_{t+r}, s_{t+r}) \). First, notice that in each period \( t + r \) we have \( W_{t+r} (s_{t+r} \mid h_{t+r}) = \{j_{t+r}^p\} \) because there is a unique candidate in \( C_{t+r} (s_{t+r} \mid h_{t+r}) \). Secondly, suppose there exists \( i \in N_t \) and \( \hat{s}_t \in S_t (h_t) \) with \( \Pi_t (s_t^{-1}; \theta_i, \varphi \mid h_t) < \Pi_t (\hat{s}_t^{-1}; \theta_i, \varphi \mid h_t) \), which is equivalent to \( v_t (x_t^p; \theta_t^p \mid h_t, s_t) < v_t (w_{\hat{s}_t \mid h_t}; \theta_t^p \mid h_t, s_t) \). Firstly, \( \theta_t^i \neq \theta_t^p \) because \( x_t^p \in I (\theta_t^p \mid h_t, s_t) \), thus such type of citizen cannot be made strictly better off. In turn, this implies that in any possible alternative action profile \( \hat{a}_t \in A_t \) there must exist at least one citizen with \( \theta_t^k \neq \theta_t^p \) that possesses in his/her set of ideal policies an element \( x_t^k \neq x_t^p \) such that \( x_t^k \) defeats \( x_t^p \) under the majority rule. (A) Say \( x_t^k \in I (\theta_t^k \mid h_t, s_t) \) strictly defeats \( x_t^p \). Recall that Lemma 1 implies that \( v_t \) satisfies (i) QSM in \((x_t; \theta_t)\) and (ii) SSC in \((x_t; \theta_t)\). There are two possible cases.

**Case 1.** \( x_t^k \geq x_t^p \) \((x_t^k \leq x_t^p)\) and \( x_t^k \neq x_t^p \). Optimality and uniqueness of the ideal policy imply \( v_t (x_t^p; \theta_t^p, \varphi \mid h_t, s_t) > v_t (x_t^k; \theta_t^p, \varphi \mid h_t, s_t) \). SSC implies \( v_t (x_t^p; \theta_t, \varphi \mid h_t, s_t) > v_t (x_t^k; \theta_t, \varphi \mid h_t, s_t) \) for all \( \theta_t \leq \theta_t^p \) \( (\theta_t \geq \theta_t^p) \). Because \( \theta_t^p \) is the median type, the citizens with \( \theta_t \leq \theta_t^p \) \( (\theta_t \geq \theta_t^p) \) represent at least half of the voting population. Thus, the tie-break rule TBI implies \( \Pi_t (w_{\hat{s}_t \mid h_t} = j_{t+r}^p \mid h_t) \) implying \( w_{\hat{s}_t \mid h_t} = x_t^p \) and therefore \( v_t (x_t^p; \theta_t^k, \varphi \mid h_t, s_t) \) in \((x_t^k \mid x_t^p)\), which leads to a contradiction.

**Case 2.** \( x_t^k \neq x_t^p \) and \( x_t^k \neq x_t^p \). In this case \( x_t^k \neq x_t^p \). Case 2a: \( \theta_t^k > \theta_t^p \). Because X is a complete lattice, \((x_t^k \mid x_t^p),(x_t^p \mid x_t^k)\) is X (see Milgrom and Shannon, 1994). Optimality and uniqueness of the ideal policy imply \( v_t (x_t^p; \theta_t^k, \varphi \mid h_t, s_t) > v_t (x_t^p; \theta_t^k, \varphi \mid h_t, s_t) \). QSM implies
\[ v_t \left( x_t^k \land x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right) > v_t \left( x_t^p; \theta_t^p, \varphi \mid h_t, s_t \right). \] SSC implies \[ v_t \left( x_t^k \land x_t^p; \theta_t^p, \varphi \mid h_t, s_t \right) > v_t \left( x_t^p; \theta_t^p, \varphi \mid h_t, s_t \right), \]
which implies in turn \( x_t^p \notin I (\theta_t^p \mid h_t, s_t) \), which leads to a contradiction. Case 2.b: \( \theta_t^k < \theta_t^p \). Similarly to 2.a, optimality and uniqueness of the ideal policy imply
\[ v_t \left( x_t^k; \theta_t^k, \varphi \mid h_t, s_t \right) > v_t \left( x_t^k \land x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right). \] QSM implies \[ v_t \left( x_t^k \lor x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right) > v_t \left( x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right). \] SSC implies \[ v_t \left( x_t^k \lor x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right) > v_t \left( x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right), \]
which implies in turn \( x_t^p \notin I (\theta_t^p \mid h_t, s_t) \), which leads to a contradiction.

Part (ii) Suppose there is an electoral equilibrium in period \( t \) such that \( x_{s_t|h_t}^* \notin I (\theta_t^p \mid h_t, s_t) \). This implies \[ v_t \left( x_t^p; \theta_t^p, \varphi \mid h_t, s_t \right) > v_t \left( x_{s_t|h_t}^*; \theta_t^p, \varphi \mid h_t, s_t \right) \] (considering the restriction \( \theta_t^p \) of type \( \theta_t^p \) must not be strictly profitable. This is true only if \( x_{s_t|h_t}^* = x_t^p \), which given \( \bar{x}_t^p = x_t^p \) implies the existence of \( k \in C_t (\bar{s}_t|h_t) \) such that \( \theta_t^k \neq \theta_t^p \) and \[ v_t \left( x_t^k; \theta_t^k, \varphi \mid h_t, s_t \right) \geq v_t \left( x_t^p; \theta_t^k, \varphi \mid h_t, s_t \right \) \) for a majority of voters. Following the same steps as in the proof to part (i) from (A) onward it is easy to show that such \( k \) does not exist. Thus, a deviation \( \tilde{s}_t^p \) with \( \bar{x}_t^p = x_t^p \) is strictly profitable for citizen \( j_t^p: \Pi_t (s_t^p, s_t^{-p}; \theta_t^p, \varphi \mid h_t) > \Pi_t (\tilde{s}_t^p, s_t^{-p}; \theta_t^p, \varphi \mid h_t) \). In turn, the strategy profile \( s_t \) violates the condition stated in Definition A.1, which implies that it is not an EE, leading to a contradiction.

PART (iv). The definition of \( z_t \) implies \( z_t = \frac{\Delta}{\sigma_{t-1}} \). Suppose \( z_t' \geq z_t'' \) but \( \theta_t^p (z_t') > \theta_t^p (z_t'') \). The pivotal voter \( \theta_t^p (z_t') \) (considering the restriction \( TB2 \)) satisfies \( \theta_t^p (z_t') \in \min \{ \theta_t \in \Theta_t \mid Q_\rho (\theta_t^p) \geq \frac{1}{2} (1 - z_t) \} \).

The inequality \( z_t' \geq z_t'' \) implies \( Q_\rho (\theta_t^p (z_t'')) \geq \frac{1}{2} (1 - z_t') \). Lastly, because \( \theta_t^p (z_t') \in \Theta_t \) this implies \( \exists \theta_t \in \Theta_t \) such that \( Q_\rho (\theta_t) \geq \frac{1}{2} (1 - z_t') \) and \( \theta_t < \theta_t^p (z_t') \). In turn, this implies that \( \theta_t^p (z_t') \notin \min \{ \theta_t \in \Theta_t \mid Q_\rho (\theta_t^p) \geq \frac{1}{2} (1 - z_t) \} \), leading to a contradiction. Q.E.D.

Part (iii). The proof requires the following Lemma.

**Lemma 2.** There exists \( \tilde{\Delta} > 0 \) such that if \( \Delta \in [0, \tilde{\Delta}) \), then (i) the function
\[ v_t+r ((M_{t+r}, L_{t+r}); \theta_{t+r}^i, \varphi \mid h_{t+r}, s_{t+r}) \] is jointly continuous in \( (M_{t+r}, L_{t+r}), \theta_{t+r}^i, \Delta \), and strictly concave in \( (M_{t+r}, L_{t+r}) \) for all \( r = 1, 2, ..., t \), and (ii) the equilibrium policy \( z_{t+r}^* (z_t) \) is a continuous function of \( \Delta \) for all \( r = 1, 2, ..., T-t \).

**Proof.** Part (i). Let \( R = T-t \). Because the pivotal voter is unique in each period \( t+r \) given the state \( z_{t+r} \) and continuation strategy profile \( s_{t+r} \) from Proposition 1 (ii), we can define a function \( \theta_{t+r}^p (z_{t+r}) \) that maps \( z_{t+r} \) to the corresponding pivotal citizen's type. Suppose \( v_{t+r} \) is not jointly continuous in \( (M_{t+r}, L_{t+r}), \theta_{t+r}^i, \Delta \) and/or not strictly concave in \( (M_{t+r}, L_{t+r}) \) for some \( r = 1, 2, ..., R \) for all values of \( \Delta \) such that \( \Delta > 0 \). For old individuals \[ v_{t+r} ((M_{t+r}, L_{t+r}); -1, \varphi \mid h_{t+r}, s_{t+r}) = d(G-L_{t+r}) + c(M_{t+r}) \],
thus all these conditions are trivially satisfied given the assumptions on functions \( d, c \). For a young citizen, start from \( r = R \). In such period \( z_{t+R+1} = \lambda/\sigma = \bar{z} \) which is invariant in \( x_{t+R} \). Thus,

\[
v_{t+R} ((M_{t+R}, L_{t+R}); \theta^t_{t+R}, \varphi \mid h_{t+R}, s_{t+R}) = A ((M_{t+R}, L_{t+R}); \theta^t_{t+R}, \varphi, z_{t+R}(h_{t+R})) + +\beta\lambda B_{t+R+1} (M_{t+R}, \varphi, z_{t+R}(h_{t+R}))
\]

(23)

where \( A \) is a jointly continuous function of \((M_{t+R}, L_{t+R}); \theta^t_{t+R}, \varphi \) and strictly concave in \( x_{t+R} = (M_{t+R}, L_{t+R}) \), and \( B_{t+R+1} \) is constant in \( M_{t+R} \). Thus, \( v_{t+R} \) is a jointly continuous function of \((M_{t+R}, L_{t+R}); \theta^t_{t+R}, \varphi \) and strictly concave in \( x_{t+R} = (M_{t+R}, L_{t+R}) \). Strict concavity over a compact set implies that the pivotal citizen in period \( t + R \) has a unique ideal point, i.e. \( I (\theta^p_{t+R} \mid h_{t+R}, s_{t+R}) = \{x^p_{t+R}\} \), which by Proposition 1 (ii) is also the unique equilibrium policy in all equilibria, i.e. \( x^*_{t+R}(z_{t+R}) = x^p_{t+R} \). Moreover, because \( v_{t+R} ((M_{t+R}, L_{t+R}); \theta^t_{t+R}(z_{t+R}), \varphi \mid h_{t+R}, s_{t+R}) \) is jointly continuous in \((M_{t+R}, L_{t+R}); \theta^t_{t+R}, \varphi \) and strictly concave in \((M_{t+R}, L_{t+R}) \), and \( X \) is a convex set, the maximum theorem implies that \( x^*_{t+R}(z_{t+R}) = x^p_{t+R} \) is a jointly continuous function of \( \theta^t_{t+R}, \varphi \). In turn, this implies that \( B_{t+R} (M_{t+R-1}, \varphi, z_{t+R-1}(h_{t+R-1})) = d (L^*_{t+R}) + c (M^*_{t+R}) \) is jointly continuous in \( \theta^t_{t+R}, \varphi \).

Thus, \( v_{t+R-1} ((M_{t+R-1}, L_{t+R-1}); \theta^t_{t+R-1}, \varphi \mid h_{t+R-1}, s_{t+R-1}) = A((M_{t+R-1}, L_{t+R-1}); \theta^t_{t+R-1}, \varphi, z_{t+R-1}) + +\beta\lambda B_{t+R} (M_{t+R-1}, \varphi, z_{t+R-1}(h_{t+R-1})) \) is jointly continuous in \((M_{t+R-1}, L_{t+R-1}); \theta^t_{t+R-1}, \varphi \), and that \( v_{t+R-1} ((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) \) is jointly continuous in \((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}, \varphi \).

Lastly, notice that \( \lim_{t \to 0} v_{t+R-1} ((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) = A((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}(z_{t+R-1}), \varphi, z_{t+R-1}) + +\beta\lambda B_{t+R} (M_{t+R-1}, \varphi, z_{t+R-1}(h_{t+R-1})) \) is jointly continuous and strictly concave in \((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}(z_{t+R-1}), \varphi, z_{t+R-1} \) is jointly continuous and strictly concave in \((M_{t+R-1}, L_{t+R-1}); \theta^p_{t+R-1}(z_{t+R-1}), \varphi \).

Strict concavity implies \( \alpha v_{t+R-1} (x'; \theta^p_{t+R-1}(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) + (1 - \alpha) v_{t+R-1} (x''; \theta^p_{t+R-1}(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) - v_{t+R-1} (\alpha x' + (1 - \alpha) x''; \theta^p_{t+R-1}(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) > 0 \) for all \( x', x'' \in X \) (condition A).

Because \( v_{t+R-1} \) is jointly continuous in \( x, \Delta \), this implies that either (a.) condition (A) is satisfied for all \( \Delta \geq 0 \) and all \( x', x'' \in X \), or (b.) there exists \( \Delta_{t+R+1} > 0 \) such that if \( \Delta < \Delta_{t+R+1} \) \( (B, t + R - 1) \) then \( v_{t+R-1} \) is strictly concave in \( x \). Set \( \Delta \) such that condition \( (B, t + R - 1) \) is satisfied. Then the pivotal voter in period \( t + R - 1 \) has a unique ideal point, i.e. \( I (\theta^p_{t+R-1} \mid h_{t+R-1}, s_{t+R-1}) = \{x^p_{t+R-1}\} \), which is also the unique equilibrium policy in all equilibria given state \( z_{t+R-1} \), i.e. \( x^*_{t+R-1}(g_{t+R-1}) = \)
Moreover, because \( v_{t+R-1}(x_{t+R-1}; \theta_{t+R-1}^p(z_{t+R-1}), \varphi \mid h_{t+R-1}, s_{t+R-1}) \) is jointly continuous in \((M_{t+R-1}, L_{t+R-1}), \theta_{t+R-1}^i, \Delta\) and strictly concave in \( x_{t+R-1} = (M_{t+R-1}, L_{t+R-1}) \), and \( X \) is a convex set, the maximum theorem implies that \( x_{t+R-1}^*(z_{t+R-1}) = x_{t+R-1}^p \) is jointly continuous in \( \theta_{t+R-1}^i, \Delta \). In turn, this implies that \( B_{t+R-1}((M_{t+R-2}, L_{t+R-2}, \varphi, z_{t+R-2}(h_{t+R-2})) = d(L_{t+R-1}) + c(M_{t+R-1}) \) is jointly continuous in \( \theta_{t+R-1}^i, \Delta \). Thus, \( v_{t+R-2}(x_{t+R-2}; \theta_{t+R-2}^i, \varphi \mid h_{t+R-2}, s_{t+R-2}) = A((M_{t+R-2}, L_{t+R-2}, \theta_{t+R-2}^i, \varphi, z_{t+R-2}) + \beta \lambda B_{t+R-1}((M_{t+R-2}, \varphi, z_{t+R-2}(h_{t+R-2})) \) is jointly continuous in \((M_{t+R-2}, L_{t+R-2}, \theta_{t+R-2}^i, \varphi, z_{t+R-2}) \), and that \( v_{t+R-2}(x_{t+R-2}; \theta_{t+R-2}^i, \varphi \mid h_{t+R-2}, s_{t+R-2}) \) is jointly continuous in \((M_{t+R-2}, L_{t+R-2}, \theta_{t+R-2}^i, \varphi, z_{t+R-2}) \), \( \Delta \) and strictly concave in \((M_{t+R-2}, L_{t+R-2}) \) for each \( r = 1, 2, ..., R \). This leads to a contradiction.

Part (ii). Suppose \( x_{t+r}^*(z_{t+r}) \) is not a continuous function of \( \Delta \) for some \( r = 1, 2, ..., R \). From part (i) we know that for \( \Delta \in [0, \bar{\Delta}] \), \( v_{t+r}(x_{t+r}; \theta_{t+r}^p(z_{t+r}), \varphi \mid h_{t+r}, s_{t+r}) \) is continuous in \( x_{t+r} \), \( \Delta \) and strictly concave in \( x_{t+r} = (M_{t+r}, L_{t+r}) \) for each \( r = 1, 2, ..., R \), and \( X \) is a convex set. Thus, Proposition 1 (ii) implies that \( x_{t+r}^*(z_{t+r}) = x_{t+r}^p \) is the unique policy implemented in any equilibrium in each period \( t + r \). Moreover, the maximum theorem implies that \( x_{t+r}^*(z_{t+r}) = x_{t+r}^p \) is a continuous function of \( \Delta \). This leads to a contradiction. Q.E.D.

**Proposition 1.** Part (iii). In any equilibrium the policy outcome \( x_t^* \) is unique given history \( h_t \).

**Proof**  Set the threshold \( \bar{\Delta} \) such that \( \bar{\Delta} \leq \bar{\Delta} \) and therefore \( \Delta \in [0, \bar{\Delta}] \). Under the Markovian assumption, the state of the economy given \( h_t \) is entirely summarized by the states \( z_t, t \). Then the proof is straightforward from Lemma 2 (i) - (ii). Q.E.D.

**B.3 Comparative Statics**

**Proposition 2.** (Effect of population ageing, increasing inequality, and economic depression). (i) An increase in longevity and/or (ii) an increase in income inequality and/or (iii) a decrease in fertility, and/or (iv) an economic depression translate to (1) a less open immigration policy \( M_t \), higher non-pension public spending \( G_t \), and (3) a larger size of government \( \tau_t \) in all periods \( t \).
Proof. We prove this result for any number of periods. Part (i)-(1), -(2). Suppose (i)-(1) or -(2) does not hold true (or both). Consider any $\lambda', \lambda'' \in [\Delta, 1]$ such that $\lambda' > \lambda''$. I define the set $\Phi_\lambda(\varphi) := \{ \hat{\varphi} \in \Phi \mid \hat{\varphi}_j = \varphi_j \ \forall j \neq 3 \}$ and the ordering $\leq_\lambda$ over $\Phi_\lambda(\varphi)$ such that $\varphi' \leq \varphi''$ if and only if $\lambda' \geq \lambda''$.

Consider any two elements $\varphi' = (\beta, \gamma, \lambda', \Delta, \sigma^m, \xi, l, \rho)$ and $\varphi'' = (\beta, \gamma, \lambda', \Delta, \sigma^m, \xi, l, \rho)$ of $\Phi_\lambda(\varphi)$ such that $\varphi' \leq \varphi''$. Lastly, let $z''(h_t) = \lambda'/[\sigma^m - \Delta(1 - M_{t-1})]$ and $t z''(h_t) = \lambda'/[\sigma^m - \Delta(1 - M_{t-1})]$. Consider any two policies $(M''_t, L''_t), (M'_t, L'_t) \in X$ such that $(M''_t, L''_t) \geq (M'_t, L'_t)$. Then $v''_t$ satisfies the single crossing property (SC) in $(M_t, L_t, \varphi)$ over $\Phi_\lambda(\varphi)$ if:

$$v_t ((M''_t, L''_t); \theta''_t (z''(h_t)), \varphi'' | h_t, s_t)) - v_t ((M'_t, L'_t); \theta''_t (z''(h_t)), \varphi'' | h_t, s_t)) + v_t ((M''_t, L''_t); \theta''_t (z''(h_t)), \varphi' | h_t, s_t)) - v_t ((M'_t, L'_t); \theta''_t (z''(h_t)), \varphi' | h_t, s_t)) \geq 0$$

(24)

Recall $z_t \in [0, 1]$ implies $\theta''_t > 0$. Using (13) condition (24) rewrites:

$$\xi [(M''_t - M'_t) + (L''_t - L'_t)] [\theta''_t (z''(h_t)) - \theta''_t (z''(h_t))] + +\beta [\bar{p}_{t+1} (\theta''_t (z''(h_t))) - \bar{p}_{t+1} (\theta''_t (z''(h_t)))] \xi \Delta (M''_t - M'_t) + +\beta \lambda'' [B_{t+1} (M''_t, \varphi'', z''(h_t)) - B_{t+1} (M' , \varphi'', z''(h_t))] + -\beta \lambda' [B_{t+1} (M''_t, \varphi'', z''(h_t)) - B_{t+1} (M' , \varphi'', z''(h_t))] \geq 0$$

(25)

Recall $z_t \in [0, 1)$. Notice that for $\Delta \in [0, \hat{\Delta})$ the LHS of (25) is continuous in $\Delta$ by Lemma 2 (i) and that $\lim_{\Delta \to 0} [B_{t+1} (M''_t, \varphi, z''(h_t)) - B_{t+1} (M' , \varphi, z''(h_t))] = 0$ for all $z''(h_t)$. Thus, either the inequality above is satisfied for all values of $\Delta \in [0, \hat{\Delta})$ for any two $\varphi', \varphi'' \in \Phi_\lambda(\varphi)$ and for all $(M_t, L_t) \in X$, or the intermediate value theorem implies that there exists $\Delta_1 > 0$ such that if $\Delta \in [0, \Delta_1)$, then the inequality above is satisfied for any two $\varphi', \varphi'' \in \Phi_\lambda(\varphi)$ and for all $(M_t, L_t) \in X$. Thus, there exists threshold $\Delta_1 > 0$ such that for $\Delta \in [0, \Delta_1)$ the equilibrium policy $(M_t, L_t)$ is weakly increasing in $\varphi$ over $\Phi_\lambda(\varphi)$, and therefore weakly decreasing in $\lambda$. This leads to a contradiction. Thus, setting the threshold $\hat{\Delta}$ such that $\hat{\Delta} \leq \Delta_1$ is sufficient for the result to hold true. Part (i)-(3) is straightforward from (i)-(1), -(2) given that $z_t = \tau (M_t, L_t)$, which by formula (9) is decreasing in both $M_t$ and $L_t$ and constant in $\lambda$, and that $z_{t+1}$ is decreasing in $M_t$, constant in $L_t$ and increasing in $\lambda$.

Part (ii)-(1), -(2). Suppose (ii)-(1) or -(2) does not hold true (or both). First, I prove that the type of the pivotal voter is decreasing in $\rho$. Suppose $\rho' \geq \rho''$ but $(\theta''_t)'' > (\theta''_t)''$. Recall that by definition of the median citizen $F_{\rho', t} \left( (\theta''_t)'' | h_t \right) \geq 0.5$ and $F_{\rho, t} \left( (\theta''_t)'' | h_t \right) \geq 0.5$. Secondly, the tie-break rule (TB2) and $(\theta''_t)'' > (\theta''_t)''$ together imply $F_{\rho', t} \left( (\theta''_t)'' | h_t, \rho' \right) < 0.5$ and, in turn, $F_{\rho', t} \left( (\theta''_t)'' | h_t \right) < F_{\rho, t} \left( (\theta''_t)'' | h_t \right)$. Using the definition of $F_{\rho, t}$ in (7) and using $z_t < 1$ (which implies $\theta''_t = \omega''_t$), this result implies $Q_{\rho'} \left( (\theta''_t)'' \right) < Q_{\rho'} \left( (\theta''_t)'' \right)$. But the definition of inequality implies that $\rho' \geq \rho''$ only
if \( Q_{\rho'}(\theta_t^i) \geq Q_{\rho''}(\theta_t^j) \) for all \( \theta_t^i \in \Theta_t \setminus \{1\} \) such that \( \theta_t^i \leq \hat{\omega} \) because \((\theta_t^p)''\) is weakly lower than the median productivity, it satisfies \((\theta_t^p)'' \leq \hat{\omega} \). This implies \( Q_{\rho'}((\theta_t^p)'' \geq Q_{\rho''}((\theta_t^p)''\) leading to a contradiction. Then it must be true that \( (\theta_t^p)'' \leq (\theta_t^p)' \). I define the set \( \Phi_\rho(\varphi) := \{ \varphi \in \Phi \mid \varphi_j = \varphi_\land \forall j \neq \ell \} \) and the ordering \( \leq_\rho \) over \( \Phi_\rho(\varphi) \) such that \( \varphi' \leq \varphi'' \) if and only if \( \rho'' \leq \rho' \). Consider any two elements \( \varphi' = (\beta, \gamma, \lambda, \Delta, \sigma^m, \xi, l, \rho') \) and \( \varphi'' = (\beta, \gamma, \lambda, \Delta, \sigma^m, \xi, l, \rho'') \) of \( \Phi_\rho(\varphi) \) such that \( \varphi' \leq \varphi'' \).

Lastly, let \((\theta_t^p)'\) and \((\theta_t^p)''\) denote the type of the pivotal voter under \( \rho' \) and \( \rho'' \), respectively, and note that \( \rho'' \leq \rho' \). implies \((\theta_t^p)'' \geq (\theta_t^p)'\). Consider any two policies \((M_t', L_t''), (M_t', L_t') \in X \) such that \((M_t'', L_t'') \geq (M_t', L_t') \). Then \( v_t \) satisfies \((SC)\) in \((M_t', L_t', \varphi) \) over \( \Phi_\rho(\varphi) \) if:

\[
v_t((M_t'', L_t''), (\theta_t^p)'', \varphi'' \mid h_t, s_t) = v_t((M_t', L_t'), (\theta_t^p)'', \varphi'' \mid h_t, s_t) \geq v_t((M_t', L_t'), (\theta_t^p)', \varphi' \mid h_t, s_t) \geq v_t((M_t', L_t'), (\theta_t^p)'', \varphi'' \mid h_t, s_t)
\]

Using \( (15) \) condition \( (24) \) rewrites:

\[
\xi \left[ \gamma (M_t'' - M_t') + (L_t'' - L_t') \right] ((\theta_t^p)'' - (\theta_t^p)') + 
\beta \left[ \hat{p}_{t+1} \left( (\theta_t^p)' \right) - \hat{p}_{t+1} \left( (\theta_t^p)' \right) \right] \xi \Delta (M_t'' - M_t') + 
\beta \lambda \left[ B_{t+1} \left( M_t'', \varphi'', z_t(h_t) \right) - B_{t+1} \left( M_t', \varphi'', z_t(h_t) \right) \right] + 
\beta \lambda \left[ B_{t+1} \left( M_t'', \varphi', z_t(h_t) \right) - B_{t+1} \left( M_t', \varphi', z_t(h_t) \right) \right] \geq 0
\]

Recall \( z_t \in [0, 1) \). Notice that for \( \Delta \in [0, \hat{\Delta}) \) the LHS of \((27)\) is continuous in \( \Delta \) by Lemma 2 (i) and that \( \lim_{\Delta \to 0} \left[ B_{t+1} \left( M_t''(h_t), \varphi, z_t(h_t) \right) - B_{t+1} \left( M_t', \varphi, z_t(h_t) \right) \right] = 0 \) for all \( z_t(h_t) \). Thus, either the inequality above is satisfied for all values of \( \Delta \in [0, \hat{\Delta}) \) for any two \( \varphi', \varphi'' \in \Phi_\rho(\varphi) \) and for all \( (M_t, L_t) \in X \), or the intermediate value theorem implies that there exists \( \hat{\Delta}_2 > 0 \) such that if \( \Delta \in [0, \hat{\Delta}_2) \), then the inequality above is satisfied for any two \( \varphi', \varphi'' \in \Phi_\rho(\varphi) \) and for all \( (M_t, L_t) \in X_t \). Thus, there exists threshold \( \hat{\Delta}_2 > 0 \) such that for \( \Delta \in [0, \hat{\Delta}_2) \) the equilibrium policy \( (M_t, L_t) \) is weakly increasing in \( \varphi \) over \( \Phi_\rho(\varphi) \), and therefore weakly decreasing in \( \rho \). Using \( \rho'' = 0 \) and \( \rho' = 1 \) this implies in turn that an increase in income inequality from \( Q_{\rho''} \) to \( Q_{\rho'} \) corresponds to a lower equilibrium policy \( (M_t, L_t) \). This leads to a contradiction. Thus, setting the threshold \( \hat{\Delta} \) such that \( \hat{\Delta} \leq \hat{\Delta}_2 \) is sufficient for the result to hold true. Part (ii)-(3) is straightforward from (ii)-(1), -(-2) given that \( \tau_t = \tau (M_t, L_t) \), which by formula \( (4) \) is decreasing in both \( M_t \) and \( L_t \) and invariant to changes in the productivity distribution at constant mean productivity, and that \( z_t \) is decreasing in \( M_t \), constant in \( L_t \) and and and invariant to changes in the productivity distribution at constant mean productivity.

Part (iii)-(1), -(-2). Suppose (iii)-(1) or -(-2) does not hold true (or both). Consider any \( \sigma', \sigma'' \in [0, \sigma_{\text{max}}] \) such that \( \sigma'' > \sigma' \). I define the set \( \Phi_\sigma(\varphi) := \{ \varphi \in \Phi \mid \varphi_j = \varphi_\land \forall j \neq \ell \} \) and the ordering \( \leq_\sigma \) over
\[ \Phi_\sigma(\varphi) \text{ such that } \varphi' \leq \varphi'' \text{ if and only if } \sigma' \leq \sigma''. \]

Consider any two elements \( \varphi' = (\beta, \gamma, \lambda, \sigma'', \Delta, \xi, l) \) and \( \varphi'' = (\beta, \gamma, \lambda, \sigma', \Delta, \xi, l) \) of \( \Phi_\sigma(\varphi) \) such that \( \varphi' \leq \varphi'' \). Lastly, let \( z'_i(h_t) = \lambda / [\sigma' + \Delta M_{t-1}] \) and \( t \in [0, \Delta M_{t-1}] \). Consider any two policies \( (M''_t, L''_t), (M'_t, L'_t) \in X \) such that \( (M''_t, L''_t) \geq (M'_t, L'_t) \). Then \( v^t_i \) satisfies (SC) in \( (M_t, L_t, \varphi) \) over \( \Phi_\sigma(\varphi) \) if

\[
\begin{align*}
v_t ((M''_t, L''_t); \theta^p_t (z''_t(h_t)), \varphi'' | h_t, s_t) - v_t ((M'_t, L'_t); \theta^p_t (z'_t(h_t)), \varphi' | h_t, s_t) \\
v_t ((M''_t, L''_t); \theta^p_t (z''_t(h_t)), \varphi' | h_t, s_t) - v_t ((M'_t, L'_t); \theta^p_t (z'_t(h_t)), \varphi' | h_t, s_t)
\end{align*}
\]

Using (15) condition (28) rewrites

\[
\begin{align*}
\xi & \left[ t (M''_t - M'_t) + (L''_t - L'_t) \right] \theta^p_t (z''_t(h_t)) \sigma'' - \theta^p_t (z'_t(h_t)) \sigma' \\
& + \beta \{ p_{t+1} \theta^p_t (z''_t(h_t)) - p_{t+1} \theta^p_t (z'_t(h_t)) \} \xi (M''_t - M'_t) + \\
& \beta \lambda [B_{t+1} (M''_t, \varphi, z''_t(h_t)) - B_{t+1} (M'_t, \varphi, z'_t(h_t))] + \\
& - \beta \lambda [B_{t+1} (M''_t, \varphi, z''_t(h_t)) - B_{t+1} (M'_t, \varphi, z'_t(h_t))] \geq 0
\end{align*}
\]

Firstly, the first two lines of (29) are strictly positive given that \( \bar{p}_{t+1} \) is weakly increasing. Secondly, recall \( z_t \in [0, 1) \) and notice that for \( \Delta \in (0, \Delta) \) the LHS of (29) is continuous in \( \Delta \) by Lemma 2 (i) and that \( \lim_{\Delta \to 0} [B_{t+1} (M'_t, \varphi, z_t(h_t)) - B_{t+1} (M''_t, \varphi, z_t(h_t))] = 0 \). Thus, either the inequality above is satisfied for all \( \Delta \in (0, \Delta) \), for any two \( \varphi', \varphi'' \in \Phi_\sigma(\varphi) \) and for all \( (M_t, L_t) \in X \), or the intermediate value theorem implies that there exists \( \Delta_3 > 0 \) such that if \( \Delta \in [0, \Delta_3] \), then the inequality above is satisfied for any two \( \varphi', \varphi'' \in \Phi_\sigma(\varphi) \) and for all \( (M_t, L_t) \in X \). Thus, there exists threshold \( \Delta_3 > 0 \) such that for \( \Delta \in [0, \Delta_3] \) the equilibrium policy \( (M_t, L_t) \) is weakly increasing in \( \varphi \) over \( \Phi_\sigma(\varphi) \), and therefore weakly increasing in \( \sigma \). This leads to a contradiction. Thus, setting the threshold \( \Delta \) such that \( \Delta \leq \Delta_3 \) is sufficient for the result to hold true. Part (iii)-(3) is straightforward from (i)-(1), (2) given that \( \tau_t = \tau (M_t, L_t) \), which by formula (4) is decreasing in both \( M_t \) and \( L_t \) constant in \( l \) and decreasing in \( \sigma \).

Part (iv)-(1), (2). Suppose (iv)-(1) or (2) does not hold (or both). Consider any \( \xi', \xi'' \in (0, +\infty) \) such that \( \xi'' > \xi' \). I define the following notation. \( \Phi_{\xi}(\varphi) := \{ \hat{\varphi} \in \Phi | \hat{\varphi}_j = \varphi_j \ \forall j \neq 6 \} \) and the ordering \( \leq_{\xi} \) over \( \Phi_{\xi}(\varphi) \) such that \( \varphi' \leq \varphi'' \) if and only if \( \xi'' > \xi' \). Consider any two elements \( \varphi' = (\beta, \gamma, \lambda, \Delta, \sigma''', \xi', l) \) and \( \varphi'' = (\beta, \gamma, \lambda, \Delta, \sigma'''', \xi'', l) \) of \( \Phi_{\xi}(\varphi) \) such that \( \varphi' \leq \varphi'' \) and any two policies \( (M''_t', L''_t'), (M'_t', L'_t') \in X \) such that \( (M''_t', L''_t') \geq (M'_t', L'_t') \). Then \( v^t_i \) satisfies (SC) in \( (M_t, L_t, \varphi) \).
Lemma 1 and QSM. A citizen’s objective function in period \( t \) is decreasing in \( \theta - \antiimmigration \) candidate \( \theta \) and increasing in \( \proimmigration \) candidate \( l \) such that

\[
M_t \in \Theta, \quad \text{if} \quad \theta - \antiimmigration \land \proimmigration \text{ platforms}.
\]

Thus, the equilibrium policy \((\tau_t)_{t \in \{0, T+1\}}\) is decreasing in \( \Delta \). (Part (ii)-(3) is straightforward from (i)-(1), -(2) given that \( \gamma_t = \tau(M_t, L_t) \), which by formula (4) is decreasing in both \( M_t \) and \( L_t \) and constant in \( \xi \).)

\[
\Delta = \min \left\{ \Delta, \Delta_2, \Delta_3, \Delta \right\}
\]

and note that \( \hat{\Delta} > 0 \). Then setting the threshold \( \overline{\Delta} \) such that it satisfies \( 0 < \overline{\Delta} \leq \hat{\Delta} \), then for any \( \Delta \in [0, \overline{\Delta}] \) all the statements in parts (i)-(ii)-(iii)-(iv) hold true. Q.E.D.

Proposition 3. (Short-term fiscal effects). In any EE, if there exist two candidates \( r, l \) in period \( t \) such that \( M_t^r < M_t^l \), then there exists a threshold \( \hat{\theta}_t \geq 0 \) such that the policy platform of the relatively pro-immigration candidate \( l \) has weakly negative short-term fiscal effect on all individuals featuring type \( \theta_t^r \leq \hat{\theta}_t - \text{that is, the old and the relatively poor citizens—with respect to the platform of the relatively anti-immigration candidate } r. \)

Proof. A citizen’s objective function in period \( t \) is strictly concave by Lemma 2 and \( X \) is compact. Thus, each citizen has a unique ideal policy. Preferences satisfy QSM in \( x_t \) and SSC \( x_t, \theta_t \) by Lemma 1 and \( (X \leq) \) is a lattice. Thus, the ideal policy \( x_t^\star \) is weakly increasing in the citizen’s type \( \theta_t^r \) by Theorem 4 in Milgrom and Shannon (1994). As a consequence, \( M_t^r < M_t^l \) implies \( \theta_t^r < \theta_t^l \) and, in turn, \( L_t^r \leq L_t^l \). The short-term fiscal effect of the platform of candidate \( l \) relative to candidate \( r \) has formula:

\[
\left[ L_t^l - L_t^r + \gamma (M_t^l - M_t^r) \right] \xi \theta_t^r + b \left( G - L_t^l \right) - b \left( G - L_t^r \right)
\]

for a young citizen of type \( \theta_t^r \) and
First, we set $\Delta$ value because $L_{t+1}^i \leq L_t^i$, so (a) all citizens of type $\theta_i^t = -1$ face a weakly negative short-term fiscal effects. The former has weakly positive value at $\theta_i^t = 0$, and is continuous and weakly increasing in $\theta_i^t$. Thus, either (i) the fiscal effects are weakly negative for all $\theta_i^t \in \Theta$, implying that in such case $\hat{\theta}_t$ is trivially the highest type in $\Theta$; or (ii) by the intermediate value theorem, there exists $\tilde{\theta}_t$ (not necessarily an element of $\Theta$) such that (b) $[L_t^i - L_t^i + \gamma (M_t^i - M_t^r)] \xi \hat{\theta}_t + b (G - L_t^i) - b (G - L_t^i) \leq 0$ for all $\theta_i^t \in \Theta$ such that $0 \leq \theta_i^t \leq \tilde{\theta}_t$. Results (a) and (b) together imply that the short-term fiscal effects are weakly negative for all $\theta_i^t \in \Theta$ such that $\theta_i^t \leq \tilde{\theta}_t$ where $\tilde{\theta}_t \geq 0$. Q.E.D.

**Proposition 4.** For any Social Welfare Function $SWF ((M_t, L_t); \varphi | h_t, s_t)$ that assigns a strictly positive weight to each native individual with $\theta_i^t > 0$, there exist thresholds $\tilde{\omega}_t > 0$ and $\tilde{z}_t \in [0, 1)$ such that if $\omega^{Low} \leq \tilde{\omega}_t$ and $z_t \in [\tilde{z}_t, 1)$, then a marginal loosening in the immigration policy is welfare-enhancing.

**Proof.** We proof this result for any $\Delta \in [0, \Delta]$. The $SWF$ has form as in Equation 10. Suppose a marginal increase in $M_t$ evaluated at $M_t^r$ is not welfare-enhancing for some $SWF$ with $\Psi_{t+r}(\theta_{t+r}) > \max \{\Psi_{t+r}(\theta_{t+r}^r), 0\}$ for all $\theta_{t+r}^r \in \Theta_t$ such that $\theta_{t+r}^r < \theta_{t+r}$. I define the marginal social welfare function as follows:

$$MSW ((M_t^*, L_t^*); \varphi | h_t, s_t) := \lim_{M_t^r \to M_t^*} \frac{SWF ((M_t^*, L_t^*); \varphi | h_t, s_t) - SWF ((M_t^r, L_t^r); \varphi | h_t, s_t)}{M_t^r - M_t^*}$$

(32)

First, we set $\Delta = 0$ and we calculate the effect of an increase in $M_t$ on each individual objective function that enters the formula for $MSW$. In this case $v_t$ is differentiable for all citizen’s types, such that for each individual $i$ we get:

$$\lim_{M_t^r \to M_t^*} \frac{v_t ((M_t^*, L_t^*); \theta_i^t, \varphi | h_t, s_t) - v_t ((M_t^r, L_t^r); \theta_i^t, \varphi | h_t, s_t)}{M_t^r - M_t^*} = \gamma \xi \theta_i^t \times 1 \left[ \theta_i^t \geq 0 \right] + c'(M_t^*)$$

(33)

Lastly, consider an individual born in period $t + r$ for $r > 0$. Given $\Delta = 0$, we know that future equilibrium policies are invariant in current policy choices. Thus, I get:

$$E_t \left[ v_{t+r} ((M_{t+r}, L_{t+r}); \theta_{t+r}, \varphi | h_{t+r}, s_{t+r}) | h_t, s_t, (M_t^*, L_t^*) \right] +$$

$$- E_t \left[ v_{t+r} ((M_{t+r}, L_{t+r}); \theta_{t+r}, \varphi | h_{t+r}, s_{t+r}) | h_t, s_t, (M_t^*, L_t^*) \right] = 0$$

(34)
i.e. if $\Delta = 0$ current policy choices do not affect future outcomes. Thus, for $\Delta = 0$, the limit in (33) exists and it simply a derivative. Using the results in (33) and (34) we get:

$$MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t) = \psi \xi \int_{\Theta_t \setminus \{-1\}} \theta_t d\Phi_t(\theta_t) + c'(M^*_t) \int d\Phi_t(\theta_t)$$

Note that given the definition of $\Theta_t$, $\omega^{\text{Low}} \in \Theta_t$ implies $Q_{\rho} (\omega^{\text{Low}}) > 0$, and in turn there exists $z_t < 1$ such that $F_{\rho,t} (\omega^{\text{Low}} \mid h_t) = 0.5$, which implies $\theta^*_t(z_t) = \omega^{\text{Low}}$ for all $z_t \in [\hat{z}_t, 1)$. Thus, using this result into formula (33) we get: $\gamma \xi \omega^{\text{Low}} (z_t) + c'(M^*_t)$ for all $z_t \in [\hat{z}_t, 1)$. Now consider the extreme case $\omega^{\text{Low}} = 0$. This implies that the ideal policy of the pivotal voter is $\arg \max_{(M_t, L_t) \in X} v_t ((M_t, L_t); 0, \varphi \mid h_t, s_t) = (\hat{M}, \hat{L}_T)$, where $\hat{M}$ solves $c'(\hat{M}) = 0$. Set $M^*_t = \hat{M}$ into (32) and $z_t \in [0, \hat{z}_t]$ to get

$$MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t) = \gamma \xi \int_{\Theta_t \setminus \{-1\}} \theta_t d\Phi_t(\theta_t) > 0$$

which is strictly positive for any weight function that satisfies $\Psi_{t+r}(\theta_{t+r}) > \max \{\Psi_{t+r}(\theta'_{t+r}), 0\}$ for all $\theta'_{t+r} \in \Theta_t$ such that $\theta'_{t+r} < \theta_{t+r}$. Note that $MSW ((M_t, L_t); \varphi \mid h_t, s_t)$ is jointly continuous in $(M_t, L_t), \Delta$ because each function $v_{t+r}$ for $r = 0, 1, ..., T - t$ that enters the formula for $MSW$ is jointly continuous in $(M_t, L_t), \Delta, \theta^*_t$ by Lemma 2 and the sum and integration over such functions preserve continuity. Moreover, because by Lemma 2 $v_t ((M_t, L_t); \theta^*_t(z_t), \varphi \mid h_t, s_t)$ is jointly continuous in $(M_t, L_t), \theta^*_t, \Delta$ and strictly concave in $(M_t, L_t)$, and $X$ is a convex set, by the maximum theorem the optimal policy $(M^*_t, L^*_t)$ is jointly continuous in $\Delta, \theta^*_t$, implying that the function $MSW$ evaluated at the optimal policy; i.e., $MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t)$, is itself jointly continuous in $\Delta, \theta^*_t$. Then either $MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t) > 0$ for all $\Delta \in [0, \hat{\Delta})$ and all possible values of $\omega^{\text{Low}}$ that satisfy $\omega^{\text{Low}} > 0$, or the intermediate value theorem implies that there exists thresholds $\Delta > 0$ and $\omega > 0$ such that if $\Delta \in [0, \Delta)$ and $\omega^{\text{Low}} \in [0, \omega]$, then $MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t) > 0$. In particular, we set $\Delta = \min \{\hat{\Delta}, \hat{\Delta}\}$ (see proof to Proposition 3 for $\hat{\Delta}$) to ensures that $\Delta \in [0, \hat{\Delta})$. In turn, $MSW ((M^*_t, L^*_t); \varphi \mid h_t, s_t) > 0$ implies that if $z_t \in [\hat{z}_t, 1)$ and $\omega^{\text{Low}} \in [0, \omega]$, a marginal increase in $M_t$ evaluated at $M^*_t$ is strictly welfare-enhancing for any $SWF$ that satisfies $\Psi_{t+r}(\theta_{t+r}) > \max \{\Psi_{t+r}(\theta'_{t+r}), 0\}$ for all $\theta'_{t+r} \in \Theta_t$ such that $\theta'_{t+r} < \theta_{t+r}$. This leads to a contradiction. Q.E.D.
References


