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# A data-driven and risk-based prudential approach to validate the DDMRP planning and control system

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**Abstract.**

In this paper, we study the single-item dynamic lot-sizing problem in an environment characterized by stochastic demand and lead times. A recent heuristic called *Demand Driven MRP*, widely implemented using modern ERP systems, proposes an algorithm that is will effectively tackle this problem. Our primary goal is to propose a theoretical foundation for such a heuristic approach. To this aim, we develop an optimization model inspired by the main principles behind the heuristic algorithm. Specifically, controls are of the type $(s(t), S(t))$ with time-varying thresholds that react to short-run real orders; in this respect, control is *risk-based and data-driven*. We also consider service levels derived as tail risk measures to ensure fulfillment of realized demand with a predetermined probability; in this respect, our approach is *prudential*. Finally, we use our model as a benchmark to theoretically validate and contextualize the aforementioned heuristic.

**Keywords:** Inventory management, Manufacturing resource planning, Data-driven demand planning, Tail risk measures, DDMRP.

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# 1 Introduction

Managing information flows other than materials along the supply chain, is one of the most challenging tasks in the daily operations of a firm (Zhao et al. (2002)). In an ideal world, information disclosure would help each player to construct optimal strategies for inventory management (Buehler and Halbheer (2011), Christopher (2000), Raghunathan (1999)). In real life, however, the opposite is often the case: Information is a valuable asset, and partners along the supply chain, whether downstream or upstream, tend to keep private as much of their information as possible. In the absence of collaboration, estimates based on experience and forecasting become crucial in setting long-run (risk-based) policies (Simchi-Levi et al. (2005), Zhao et al. (2002)). On the other hand, such estimates must be continuously integrated with actual (real) data to ensure short-run policies and operations adhere to the true reality of the firm (Ptak and Smith (2016)). Moreover, a lot of care must be taken to avoid unpleasant phenomena, often detected in practice, such as the *bullwhip effect* (Lee et al. (1997), Warburton (2004)).

The main goal of this paper is to propose a *practice-oriented* model capable of combining two crucial perspectives: A risk-based approach accomplished using forecasts of future demand and lead times, and a continuous flow of actual data based on short-term reliable orders. These factors are integrated to create an optimal control system for inventory management, consisting primarily of: (i) an original objective function that considers costs referring to an excess of inventory over a target parameter; (ii) data-driven time-varying double-threshold optimal control accounting for peaks in demand; (iii) constraints related to minimal service levels to guarantee customer satisfaction.

To be precise, relying on a probabilistic approach to model both future demand and stochastic lead times, we construct risk measures that are then used to explicitly derive linear constraints representing the minimal service levels to satisfy fulfillment of incoming orders with a predetermined probability. This approach resembles the traditional *prudential approach* to risk management (Jorion (1996)), where tail-based risk measures are used to deal with "*expected risks*". Such risks are the basis of ordinary operations; they can be measured and controlled and, to some extent, managed by partially automatized controls. In contrast, reordering policies are based on a rolling figure of short-term real demand, referred to as *Average Daily Usage* (ADU). Again, such policies are controlled by automated systems that update state variables and adjust control thresholds by relying on a continuous flow of data. We also account for eventual *peaks* of demand, which can be considered to be "*unexpected risks*"; although they are not considered directly in the *automated setup* of thresholds, they contribute to accelerating the pace of orders. As far as the objective function is concerned, besides traditional costs related to holding, reordering and stock-out, we also consider an extra holding cost that penalizes deviations from a target inventory level. This latter cost component aims at preventing overly prompt reactions to exceptional incoming orders, which break the desired *pace* of material flow, reducing the amplification of a bullwhip effect phenomenon. Again, this goal is achieved by setting semi-automated controls that prevent users from reacting sub-optimally to fluctuating customer orders.

Our research originates from a problem in inventory management suggested by practice. In this context, a recent practical approach called Demand Driven MRP (DDMRP) proposes a heuristic algorithm that is quickly becoming a reference planning and control system (Thürer et al. (2020), Villa Hincapie (2018), Shofa and Widyarto (2017), Miclo et al. (2016), Bahu et al. (2019), Dessevre et al. (2019)). So far, however, little attention has been devoted to it in the academic literature. Our aim is to fill this gap by addressing the following question: *What are the objectives and the constraints that mathematically represent at best the "philosophy"behind the DDMRP heuristic?* To this aim, we build a model based on the ingredients described above, which, in turn, are the main qualitative features supposedly focused on by the heuristic. Second, we derive a mixed linear integer version to numerically identify the optimal solution of our theoretical model. Finally, we validate *ex-post* the DDMRP heuristic by means of numerous numerical experiments, representing different instances of the problem for various values of the model's key parameters. Our simulation exercise enables us to show that the algorithm related to the DDMRP heuristic can be related to a well-posed theoretical model. By contrast, we also show that our theoretical model has a practical counterpart, in that its main features are evidently related to the principles guiding the DDMRP approach, widely implemented in recent ERP systems (Miclo et al. (2016)).

Finally, there is a serious concern in management science that a long-lasting divide exists between the academic literature and practice (Bennett (2016), Aguinis et al. (2010)). The former has proposed a number of highly technical and valuable mathematical models to address inventory management and all related issues. The latter is more concerned with providing managers on the ground with reliable heuristics to deal with real-life operations. As already stressed, we believe that our theoretical model is inherently *practice-oriented*, in that it forms a bridge to span this academic/practice divide. On the one hand, our model is theoretically grounded, and complex enough to account for long-run unpredictability and short-run real orders and peaks. On the other hand, it provides a mathematical foundation for the DDMRP heuristic developed recently by practice.

This article is organized as follows. In the next subsection, we highlight how our paper is related to recent literature in inventory management. In Section 2, we present in detail our model for the single-item dynamic lot-sizing problem; in the process, we focus on the following goals: To maintain a target inventory; use real (short-period) order data; and establish a risk-based approach for inventory management. In Section 3, we present and mathematically describe the DDMRP approach. Section 4 contains all simulations used to validate the DDMRP heuristic; notably, we show that it satisfactorily approximates optimal control derived in our setup under several instances of the model's parameters. Finally, in Section 5 we conclude with some final comments.

## 1.1 Related literature

The roots of inventory policies and lot-sizing rules go back more than a century, when Ford Whitman Harris published the Economic Order Quantity (EOQ) model (Harris (1913)). His contribution has helped many firms to immediately find the most economical quantity to make (or order), balancing setup and carrying costs. More recently, this paradigm gave rise to so-called (R,Q) inventory policies. Despite having emerged a long time ago, the (R,Q) inventory policy's variants and real-world applications continue to be studied (Esmaeili et al. (2018)). A modified version of the previous inventory policy is the (s,S) version, which sets a reorder point at $s$ and a target inventory level $S$ (Sapna Isotupa and Samanta (2013), Noblesse et al. (2014), Esmaeili et al. (2018)).

One of the main drawbacks of many cost-based inventory policies is associated with the generation of important fluctuations of reorder quantities, and consequently propagation of instability of demand/orders along the supply chain. According to some authors, this may cause sunk costs and the generation of phenomena such as the bullwhip effect (Hejazi and Hilmola (2006), Potter and Disney (2006), Hussain and Drake (2011)). To overcome these issues, a new heuristic has been proposed under the name of Demand Driven Material Requirements Planning (DDMRP). This heuristic first appeared as a blueprint in 2011, when Carol Ptak and Chad Smith wrote the third edition of Orlicky's Material Requirements Planning and was introduced officially in 2016, when an entire book was dedicated to it (Ptak and Smith (2016)). A number of pioneering academic studies related to this new methodology seem to confirm the heuristic's goodness (Miclo et al. (2015, 2016), Smith and Smith (2013a)). Miclo (2016), using data taken from the famous *Kanban game*, showed that DDMRP can outperform traditional approaches, exhibiting less working capital and more efficient delivery times. Other scholars analyzed various case studies in different fields. Shofa and Widyarto (2017) compared a traditional MRP setting with DDMRP, using a *Discrete Event Simulation Approach* and considering both long lead times and uncertain demand. They found an 11% reduction in inventory, and less inventory fluctuation. Other case studies have been developed: Bahu et al. (2019) undertook a qualitative study analyzing 30 different real implementations of the DDMRP approach; Ihme and Stratton (2015) developed a qualitative study to outline various technical aspects regarding the practical implementation of the DDMRP system. Other ongoing studies address the standardization of the implementation process (Román Cuadra et al. (2017), Bayard and Grimaud (2018), Villa Hincapie (2018), Dessevre et al. (2019)).

As said, we consider both stochastic demand and lead times. Our approach to deal with randomness in demand is similar in spirit to Sox (1997) and Tempelmeier (2007), where a deterministic model formulation with service level constraints is proposed. Also Sodhi (2005) proposes deterministic replenishment-and-planning strategies computed under demand uncertainty. Whereas the literature dealing with stochastic demand is rather vast (see Tempelmeier (2013), Aloulou et al. (2014), Brahimi et al. (2017) for recent surveys on this topic), very few papers address the problem of stochastic lead times. In Alp et al. (2003), a dynamic lot-sizing/vehicle-dispatching problem under deterministic demand and stochastic lead times is analysed. Srivastav and Agrawal (2020) proposes a Gaussian approximation method to deal with the so-called stochastic lead time demand. Only recently, some first attempts have been made to link together stochastic lead times and random demand (see Liu et al. (2021)). We address this issue relying on a prudential approach, where uncertainty enters into a (deterministic) optimization scheme via a tail-risk measure expressed as a service level.

Concluding, as testified by this concise literature review, a number of practical studies have appeared recently, testifying to the good performance of DDMRP. However, a more theoretical investigation capable of identifying the possible objectives and constraints beyond the *philosophy* of DDMRP appears to be lacking.

# 2 The single-item lot-sizing optimization problem

At this stage, our problem could be classified as a *single-item dynamic lot-sizing problem* (SIDLS), specifying when and how much to order throughout a prespecified discrete planning interval with final time $T$. We use the following notations for the model's parameters, variables and controls.

Parameters:

- $\ell$, lead time;
- $h$, holding cost per unit of time;
- $K$, reordering cost;
- $p_{US}$, penalty cost related to *under-stock*;
- $p_{OS}$, penalty cost related to *over-stock*, namely, an excessive quantity of inventory;
- $d = (d_t)_{t\leq T}$, demand.

Variables:

- $x = (x_t)_{t\leq T}$, inventory level (state variable);
- $y = (y_t)_{t\leq T}$, on-order quantity (state variable);
- $u = (u_t)_{t\leq T-\ell+1}$, quantity ordered at time $t$, and made available at time $t+\ell-1$ (control variable).

Generally speaking, given a set of predefined parameters $\theta \in \Theta$, the optimal solution to the general SIDLS problem can be obtained by specifying a suitable parametric target function $f : \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+ \times \Theta \to \mathbb{R}^+$, a class of constraints $\mathcal{C}$, and a class of admissible controls $\mathcal{U}$:

$$\min_{(x,y)\in\mathcal{C},\ u\in\mathcal{U}} f(x, y, u; \theta).$$

As stated in the introduction, our model must be capable of accounting for standard costs (holding, reorder, under-stock penalties), while appropriately penalizing an excess of inventory over a *reference level* chosen by the decision maker. To this aim, we introduce the term "*Average Daily Usage*" (*ADU*):

$$ADU_t = \frac{\sum_{s=t-q_1}^{t-1} \tilde{d}_s + sum_{s=t}^{t-1+q_2} \tilde{d}_s}{q_1 + q_2}. \tag{1}$$

Here, $\tilde{d}_s$ is the *true* demand prevailing at time $s$, possibly corrected to account for exceptional peaks; this is possibly done to avoid abrupt and unmotivated changes in the pace of the ADU itself.[1] Integer quantities $q_1$ (or $q_2$) represent the number of periods considered in the past (future) to compute daily average demand. Specifically, if $q_1 = 0$, we only look at present and future data. In contrast, if $q_2 = 0$, we rely on a rolling average based on recent past data.

[1] An explicit formulation for $\tilde{d}_s$ will be provided when running simulations in Section 4.

The $ADU$ provides a rough measure of daily demand for a resource, computed as an average of past and future demand, and approximates a target inventory level. Our objective function will account for (positive) deviations of the actual stock on-hand from the $ADU$, namely, $(x_t - ADU_t)^+ = \max\{x_t - ADU_t; 0\}$. The optimal policy, as derived under these assumptions, is expected to reduce reorder fluctuations by achieving a stable inventory level (Ptak and Smith (2016), So and Zheng (2003)). There are at least two plausible reasons for explicitly considering this penalty cost when taking decisions. First, it helps to prevent the creation of lumpy demand, which could create problems upstream in the supply chain (Hayya et al. (2006), Gardiner and Blackstone Jr (1995)). Second, we can relate it to financial and accounting issues; it is recognized that inventory management has a positive effect on financial performance and impacts general costs; it is also often related to a loss of opportunities for the shareholders of a company. Put differently, inventory management mediates the relationship between managerial competence and financial performance: Inventory plays a crucial role in a firm's performance and that an out-of-scale inventory results in a net loss of return on equity (Orobia et al. (2020)).

In the remainder of this section, we propose a number of different models characterized by an increasing level of requirements for the class of admissible controls. Our aim is to set a model that at best captures the main features of a prudential and proactive approach. Finally, a linear-integer version of such a model will be presented. This latter linear-integer model will be used as a starting point for the numerical analysis, and to benchmark the DDMRP heuristic against our optimal controls.

## 2.1 Model I - minimal assumptions

Model I presents the target function and the basic minimal requirements for the on-hand quantities. Given the initial condition $x_0 \geq 0$, we have

$$\min \sum_{t=0}^{T} \Big( K\, \delta_{\{u_t > 0\}} + h\, x_t^+ + p_{US}\, x_t^- + p_{OS}\, (x_t - ADU_t)^+ \Big), \tag{2}$$

$$x_{t+1} = x_t + u_{t+1-\ell} - d_t, \quad \forall t,$$

$$u_t \geqslant 0, \quad \forall t.$$

The objective function of Model I considers standard reordering and holding costs, and adds two penalty components for negative inventory and target inventory. The reordering cost $K$ is paid at time $t$ when an order has been placed. The basic holding cost $h$ refers to the need for physical staff, workers, standard equipment and operations such as counting and handling materials. The two penalties $p_{US}$ and $p_{OS}$ add extra holding costs to negative values of $x_t$ and to values exceeding the daily average usage level, respectively penalization for stock-out levels is commonly used in the literature (Jing and Chao (2021)). When it comes to penalization for extra inventory, we indeed consider cost components such as the opportunity cost of capital, costs related to obsolescence, damage and deterioration, the risk of lost or stolen goods, insurance costs related to materials (Azzi et al. (2014), Alfares (2007)).

Equality constraints describe the flow equations of the on-hand quantities. At this level, the lead time is deterministic, and demand $d_t$ is supposed to be known to the decision-maker at any point in time. We will introduce stochastic demand and lead times later in this section. With regard to other constraints, Model I entails the *broader* class of controls: Only non-negativity is imposed. The next models restrict the class of admissible controls with the aim of bringing the optimal solution closer to standard approaches used in practice, such as $(s, S)$ controls and minimal service levels to be satisfied.

## 2.2 Model II - controls of type (s, S)

In Model II, we assume that controls belong to a traditional $(s, S)$ family

$$u_t = \begin{cases} 0 & \text{if } (x_t + y_t - d_t) > s_t \\ S_t - (x_t + y_t - d_t) & \text{if } (x_t + y_t - d_t) \leq s_t \end{cases}, \tag{3}$$

where $y_t$ denotes the on-order quantity at time $t$, and where $s = (s_t)_{t \leq T-l+1}$ and $S = (S_t)_{t \leq T-l+1}$ are time-varying thresholds to be determined optimally. In contrast to Model I, the state variables of the optimization are now $x_t$ and $y_t$. We need to introduce the law of motion for the latter variable. To this aim, we set $y_0 \geq 0$ and, for all $1 \leq t \leq T-1$,

$$y_{t+1} = y_t + u_t - u_{t+\ell-l}. \tag{4}$$

The optimal control problem related to Model II considers the objective function as in (2), and shares all the constraints of Model I; moreover, constraints of type (3), and type (4) and non-negativity constraints for $s$ and $S$ are also added. This then translates the optimal control into an optimal pair of time-varying thresholds $(s, S)$.

## 2.3 Model III - prudential approach

We now introduce a stricter constraint on the values for the lower barrier $s$, which aims at introducing a *prudential approach* to the inventory problem. This figure is related to a risk measure dealing with stochastic future demand and a stochastic lead time $L$. Let us suppose we are at date t; we denote by $D_s$ the future stochastic demand prevailing at time $s = t+1, \ldots, T$. The decision-maker needs to carry out estimations of $D_s$ based on the information he has at time $t$. Specifically, we assume that the decision-maker considers future demand to be log-normally distributed so that $\ln(D_s)$ is Gaussian; moreover, he sets its average at the actual (logarithmic) ADU level as computed in (1), and assumes, for simplicity, a constant standard deviation.[2] Summarizing, $\ln(D_s) \sim \mathcal{N}(\ln(ADU_s), \sigma_D)$ for some constant risk parameter $\sigma_D > 0$. The idea is that the decision-maker bases his decision on a short-term rolling estimate of demand, the ADU, as defined in (1), and on a basic estimation of risks based on a log-normality assumption.The lead time $L$ is also assumed to be log-normal, so that $\ln(L) \sim \mathcal{N}(\ln(\ell), \sigma_L)$. Finally, we assume that demand and the lead time are statistically independent. It is well known that those assumptions make the product $DL$ log-normal. This latter figure, although slightly different *"lead time demand"*, will be crucial for the remainder of the paper. The log-normal nature of lead time demand is acknowledged by different scholars (see, among others, (A), (B), (C), (D)). In addition, log-normality will be useful to set a simple (and deterministic) mixed integer linear

[2] Time-varying standard deviations could also be considered, but this would make notations and derivations more cumbersome.

model for the SIDLS problem, based on few basic statistics of such random variables (Schwartz and Yeh (1982), Das (1983), Tadikamalla (1984), Cobb et al. (2013)).

To account for *rare events*, we also consider a random variable $\Xi$ modeling a *stochastic peak level*, i.e. an event signaling an uneven level of demand in the period $\{t+1, \dots, t+1+q_2\}$, where $q_2$ is short-term planning horizon as defined in (1). We assume that $\Xi$ is a measurable function of (short-period) future demand so that $\Xi = g(D_{t+1}, D_{t+2}, \dots, D_{t+1+q_2})$. We denote by $\xi_t = \mathbb{E}[\,\Xi\,|\mathcal{F}_t]$ the best estimate at time $t$ of such a peak.

Our assumption is that all the information used at time $t$ to make any decision about inventory is based on the aforementioned statistics: First order estimates ($ADU_t$ and $\ell$); second-order estimates of risk ($\sigma_D$ and $\sigma_L$); and a measure of unexpected demand ($\xi_t$). More precisely, first-order and second-order statistics are used to set time-varying semi-automated thresholds $(s, S)$. This reflects a *prudential attitude* to setting safety levels and reorder quantities: The decision-maker sets thresholds in consideration of intrinsic risks related to the stochastic lead time and demand. Moreover, given that those thresholds are time-varying, they suggest that the decision-maker has also a *proactive attitude*,seeking to track the evolution of demand in advance to avoid nasty surprises. On the other hand, $\xi_t$ takes care of the *"unexpected events"* (peaks). Such events, being exceptional in their nature, are not accounted for when setting (semi-automated) thresholds. Rather, they are tackled directly at the control rule level. To introduce a prudential attitude, we consider a *tail measure* of the distribution of the random variables under consideration, namely, the *value at risk* at level $\varepsilon$ (typically, $\varepsilon = 0.05$ or $\varepsilon = 0.01$).[3] Note that this quantity is inherently related to the decision-maker's risk perception. In particular, it may depend on some other parameters of the model, such as the lead time. When the lead time is long, the decision-maker is willing to accept a higher under-stock risk to avoid an inventory out of scale. We will address this issue in further detail when running the simulations. This approach translates into a probabilistic constraint for the threshold $s_t$, which, in turn, determines control $u(t)$. Specifically, for all $t \leq T$, we require

$$\mathbb{P}\left(s_t \geq D_t\, L\right) \geq 1 - \varepsilon. \tag{5}$$

We now show that, thanks to the probabilistic assumption made, we can rephrase this SIDLS problem with stochastic demand and the lead time as a deterministic mixed linear integer program. First of all, we show how to derive a linear constraint on the $s_t$ threshold, by starting from (5).

**Proposition 1** *Consider a stochastic demand such that* $\ln(D_t) \sim \mathcal{N}(\ln(ADU_t), \sigma_D)$ *and a stochastic lead time* $L$ *such that* $\ln(L) \sim \mathcal{N}(\ln(\ell), \sigma_L)$. *Moreover, assume they are independent. Then the constraint expressed by* (5) *translates into the linear constraint*

$$s_t \geq \ell\, ADU_t\,(1 + RF(\varepsilon, \sigma_D, \sigma_L)), \tag{6}$$

*where*

$$RF(\varepsilon, \sigma_D, \sigma_L) = e^{\Phi^{-1}(1-\varepsilon)\cdot\sqrt{\sigma_D^2+\sigma_L^2}} - 1,$$

*is a suitable risk factor.*

[3] $VaR_\varepsilon$ is defined as the (smallest) quantity $x \in \mathbb{R}$ such that $\mathbb{P}(D \geq x) \leq \varepsilon$. The parameter $\varepsilon$ can be fixed considering the decision-maker's risk aversion; for example, if $\varepsilon = 0.05$, then $\Phi^{-1}(1-\varepsilon) \approx 1.64$; if $\varepsilon = 0.01$, $\Phi^{-1}(1-\varepsilon) \approx 2.33$ (Embrechts et al. (2011)). For example, $\varepsilon = 0.01$ means that an out-of-stock event occurs with a probability 0.01%.

*Proof.* Note that

$$\mathbb{P}\left(s_t \geq D_t\, L\right) \geq 1-\varepsilon \iff \mathbb{P}\left(\ln(D_t)+\ln(L) \leq \ln(s_t)\right) \geq 1-\varepsilon.$$

Given that demand and the lead time are independent and log-normal, $Z := \ln(D_t)+\ln(L)$ is Gaussian with distribution $\mathcal{N}(\ln(ADU_t)+\ln(\ell); \sqrt{\sigma_D^2+\sigma_L^2})$. Therefore,

$$\mathbb{P}\left(Z \leq \ln(s_t)\right) \geq 1-\varepsilon$$

is equivalent to

$$P\left(\frac{Z-(\ln(ADU_t)+\ln(\ell))}{\sqrt{\sigma_D^2+\sigma_L^2}} \leq \frac{\ln(s_t)-(\ln(ADU_t)+\ln(\ell))}{\sqrt{\sigma_D^2+\sigma_L^2}}\right) \geq 1-\varepsilon.$$

Thanks to normality, the latter is equivalent to

$$\frac{\ln(s_t)-\ln(ADU_t)-\ln(\ell)}{\sqrt{\sigma_D^2+\sigma_L^2}} \geq \Phi^{-1}(1-\varepsilon) \iff s_t \geq ADU_t\, \ell\, e^{\Phi^{-1}(1-\varepsilon)\cdot\sqrt{\sigma_D^2+\sigma_L^2}}.$$

□

Therefore, it seems convenient to express constraint (6) in the form

$$s_t \geq \ell\, ADU_t\,(1+\,risk\ factor).$$

The first term represents a baseline safety stock, whereas the second term is a *cushion level* to account for risks related to demand and the lead time. In our notations, this risk factor depends on the parameters accounting for risks and is denoted by $RF(\varepsilon, \sigma_D, \sigma_L)$.

Constraints as in (6) can also be considered as *minimum service levels* to be granted to final customers. Notably, $(1-\varepsilon)$ can be considered as the "$\alpha$-service level", namely, the desired probability of not incurring on out-of-stock event (Chen and Krass (2001)). The service level heavily influences the level of in-house material needed to fulfill demand promptly. As suggested by the literature (Radasanu et al. (2016)), our approach is capable of depicting the best trade-off between the stock-out cost (i.e. keeping the customer waiting) and the operational costs of maintaining a high inventory level. Note also that, in setting the service level, we do not involve what we call unexpected peaks. As stated above, we deliberately leave them aside when optimally setting up the semi-automated controls of type $(s, S)$; instead, we consider them directly in the flow equation by slightly correcting the definition of control in Model II. Nevertheless, Equation (6) will also become useful when defining the unexpected peaks of demand.

It is now time to define what we mean by unexpected peaks, which account for exceptional events exceeding a well-defined safety level set at time $t$. Recall that $\Xi = g(D_{t+1}, \ldots, D_{t+1+q_2})$. As a modeling assumption, we assume that $g$ is time-separable; moreover, since $\Xi$ measures peaks of demand, we assume there exists a suitable function $h_t$, observable at $t$, so that

$$\Xi = g(D_{t+1}, \ldots, D_{t+1+q_2}) = \sum_{s=t+1}^{t+1+q_2} \tilde{g}(D_s)\, \mathbb{I}_{\{D_s > h_t\}}.$$

For simplicity, we take $\tilde{g}$ to be the identity function, and assume that

$$h_t = h(ADU_t, \ell, RF(\varepsilon, \sigma_D, \sigma_L)) = \gamma\, ADU_t\, \ell\, RF(\varepsilon, \sigma_D, \sigma_L),$$

where $\gamma$ is a suitable value to be set by the decision-maker. We, therefore, obtain a tractable definition of the peak estimator $\xi_t$, based on Equation (6):

$$\xi_t = \mathbb{E}[\Xi|\mathcal{F}_t] = \sum_{s=t+1}^{t+1+q_2} d_s \, \mathbb{I}_{\{d_s > \gamma \, ADU_t \, \ell \, RF(\varepsilon, \sigma_D, \sigma_L)\}}. \tag{7}$$

Therefore, to account for the new peak estimate, we revise the constraints in (3) as follows

$$u_t = \begin{cases} S_t - (x_t + y_t - (d_t + \xi_t)) & \text{if} \quad (x_t + y_t - (d_t + \xi_t)) < s_t \\ 0 & \text{if} \quad (x_t + y_t - (d_t + \xi_t)) \geq s_t \end{cases} \tag{8}$$

Summarizing, the optimal control problem we seek to solve under Model III shows the objective functional as in Equation (2), with all the constraints of Model I and the generalized $(s, S)$ constraints as in (8) to account for peaks, plus the constraints of type (6) to introduce a minimum service level.

A final remark is due.In some significant situations, we can assume that the risk related to the lead time *dominates* the one related to demand. This can happen, for example, if we have capacity constraints that have a negative effect on supply chain performance (Cannella et al. (2018)). In this case, we can approximate the constraint as in (6) with a new expression that *disentangles* the risks pertaining to the lead time and demand.

**Corollary 1** *If $\sigma_D/\sigma_L$ is small, the constraint expressed in* (6) *can be approximated by*

$$s_t \geq \ell \, ADU_t \, (1 + \alpha \, (1 + \beta)), \tag{9}$$

*where $\alpha = \Phi^{-1}(1-\varepsilon)\,\sigma_L$, and $\beta = 0.5\,\Phi^{-1}(1-\varepsilon)\,\sigma_D^2/\sigma_L^2$.*

*Proof.* We need to show that there exist two positive values $\alpha$ and $\beta$ such that

$$1 + \alpha(1+\beta) \approx e^{\Phi^{-1}(1-\varepsilon)\sqrt{\sigma_D^2 + \sigma_L^2}}.$$

To simplify the notations, let us call $k = \Phi^{-1}(1-\varepsilon)$. If the term $\sigma_D/\sigma_L$ is small (meaning that the risk related to the lead time dominates the one related to demand), the expression $e^{k\sqrt{\sigma_D^2+\sigma_L^2}}$ can be approximated by

$$e^{k\sigma_L\left(1+\frac{1}{2}\frac{\sigma_D^2}{\sigma_L^2}\right)} \approx 1 + k\sigma_L + k\sigma_L \frac{k}{2}\frac{\sigma_D^2}{\sigma_L^2}.$$

By putting $\alpha = k\sigma_L$, and $\beta = 0.5\,k\sigma_D^2/\sigma_L^2$, we obtain (9).

□

Note that $\alpha$ is related only to the risk pertaining to the lead time. When it comes to $\beta$, we can interpret it as the risk pertaining to demand. However, we place it in relative terms to $\alpha$, having assumed that the latter is the predominant one. In the notation of the Proposition 1, we now have

$$RF(\varepsilon, \sigma_D, \sigma_L) \approx \alpha(1+\beta).$$

In the next section, when specifying the final outlook of the model used in simulations, we will rely on this new form of the constraint, where $\alpha$ and $\beta$ take care of the risks for the lead time and demand.

### 2.4 Model IV - mixed linear integer optimization problem

Model III can be transformed into a traditional mixed linear integer programming optimization problem by adding a number of variables and constraints. Specifically, we introduce a constant $M \geq 0$ that is large enough, and the following auxiliary variables

$$w_t = \begin{cases} 1 & \text{if} \quad u_t > 0 \\ 0 & \text{if} \quad u_t = 0 \end{cases} ; \quad z_t = \begin{cases} x_t - ADU_t & \text{if} \quad x_t - ADU_t > 0 \\ 0 & \text{if} \quad x_t - ADU_t \leq 0 \end{cases} ; \quad \delta_t = \begin{cases} 1 & \text{if} \quad z_t > 0 \\ 0 & \text{if} \quad z_t \leq 0. \end{cases}$$

$w_t$ and $\delta_t$ are binary variables accounting, respectively, for the presence at time $t$ of an order $u_t$ and for stock exceeding the ADU, as a first approximation. Recall that controls are expressed in terms of time-varying thresholds $(s, S)$, as explained in Section 2.2, and that $x_0 \geq 0$ and $y_0 \geq 0$ are suitable initial conditions. In this final formulation of the model, we will use the deterministic demand $d_t$, for $t \leq T$ as an input. More precisely, $(d_t)_{t \leq T}$ is a possible realization of the stochastic demand prevailing on the planning horizon. As stated, randomness of demand (and the lead time) is accounted for in this model by means of the risk factor and the service level.

We obtain

$$\min \sum_{t=0}^{T} \Big( K\, w_t + h\, x_t^+ + p_{US}\, x_t^- + p_{OS}\, z_t \Big) \tag{10}$$

$$x_{t+1} = x_t + u_{t+1-l} - d_t \tag{11}$$

$$y_{t+1} = y_t + u_t - u_{t+1-l} \tag{12}$$

$$x_t = x_t^+ - x_t^- \tag{13}$$

$$s_t - (x_t + y_t - (d_t + \xi_t)) \leq M w_t \tag{14}$$

$$s_t - (x_t + y_t - (d_t + \xi_t)) \geq -M(1 - w_t) \tag{15}$$

$$u_t \leq S_t - (x_t + y_t - (d_t + \xi_t)) \tag{16}$$

$$u_t \geq S_t - (x_t + y_t - (d_t + \xi_t)) - M(1 - w_t) \tag{17}$$

$$u_t \leq M w_t \tag{18}$$

$$u_t \geq 0 \tag{19}$$

$$x_t - ADU_t \leq M \delta_t \tag{20}$$

$$x_t - ADU_t \geq -M(1 - \delta_t) \tag{21}$$

$$z_t \leq x_t - ADU_t + M(1 - \delta_t) \tag{22}$$

$$z_t \geq x_t - ADU_t - M(1 - \delta_t) \tag{23}$$

$$z_t \leq M \delta_t \tag{24}$$

$$z_t \geq 0 \tag{25}$$

$$x_t^+ \geq 0, \;\; x_t^- \geq 0 \tag{26}$$

$$s_t \geq \ell \cdot ADU_t \cdot (1 + \alpha\, (1 + \beta)). \tag{27}$$

$$w_t \in \{0; 1\}, \;\; \delta_t \in \{0; 1\} \tag{28}$$

Model IV shares the same objective function as previous models, albeit re written using the binary variable $z_t$. It also shares with previous formulations the equality constraints for the flow equations involving on-hand and on-order quantities. Besides obvious non-negativity constraints, we use constraints (14) and (15) to set $s_t$ if $w_t = 1$ (hence, if at time $t$ an order is switched on). Similarly, constraints (16) and (17) optimally set $S_t$. Constraint (18) is introduced to force $u_t = 0$ when $w_t = 0$. Constraints (20)-(24) are used to set $z_t$ depending on the value of $\delta_t$, in line with the definitions proposed above. Finally, (27) accounts for the service-level constraint in the form exploited in Corollary 1.

Before moving to the numerical findings, in the next section we introduce the main concepts behind the DDMRP heuristic. We will then compare in detail the policy prescribed by such heuristic with the optimal controls suggested by our model.

# 3 DDMRP: A proprietary heuristic and its properties

The Demand Driven Material Requirements Planning (DDMRP) was defined by its creators Ptak and Smith as *"a formal multi-echelon planning and execution method to protect and promote the flow of relevant information and material through the establishment and management of strategically placed decoupling point stock buffers"* (Ptak and Smith (2016)). To promote and protect the flow of materials, the core idea of the DDMRP approach is to strategically place buffers (warehouses) along the bill of material (Smith and Smith (2013b), Ptak and Smith (2016)) and to find their optimal sizing, possibly dynamically to react to change in the information about demand.

DDMRP consists basically of five nested activities: Strategic decoupling; buffer profile and levels; dynamic buffer adjustment; demand-driven planning; and visible and collaborative execution. These activities are all related to the core concept of *buffer*, and address its role in the bill of material, its properties and the company's related reorder policy. In detail, buffers are quantities of inventory or stock that are designed to decouple demand from supply (Ptak and Smith (2016)). Each buffer consists of three zones, each of which takes on a different role and can be identified by a different color. The central *yellow zone* is used for covering the average demand over a supply period; the upper *green one*, is used to dictate the pace of order generation. Finally, the *red zone* represents a security cushion that is integrated into the buffer.

Mathematically, we can define the buffer as a triplet of thresholds $(K_t^G, K_t^Y, K_t^R)$, identifying three different regions: (i) from 0 to $K_t^R$, the red zone; (ii) from $K_t^R$ to $K_t^Y$, the yellow zone; (iii) from $K_t^Y$ to $K_t^G$, the green zone. Therefore, $K_t^G$ represents the upper bound of the entire buffer. The red zone is intended to create the desired inventory cushion level. According to the methodology, it is computed as the sum of two basic sub-zones. The first level is calculated as $ADU_t \ \ell \ \alpha$, where $\alpha$ is a parameter referring to the lead time risk. Note that $\alpha$ plays exactly the role of the parameter we defined in Corollary 1 to account for lead time risks in our model. It should now become clear that the reason for developing this corollary was to enable a comparison of the results of the model and the heuristic. The second sub-zone, called variability factor, adds a security level related to the variability of demand; it is defined as $ADU_t \ \ell \ \alpha \ \beta$, where $\beta \in [0, 1]$ is parameterized on the variability of demand. Again, we recognize the parameters introduced in Corollary 1. Therefore,

$$K_t^R = ADU_t \; \ell \; \alpha \; (1 + \beta).$$

The yellow zone is inherently characterized by the ADU and the *Decoupled Lead Time* (DLT), which is the longest path in the bill of materials between the element under consideration and the first buffer. In our setting, the DLT plays the role of the lead time implemented in the previous sections of this paper. The size of the yellow zone is calculated by

$$K_t^Y - K_t^R = ADU_t \; \ell. \tag{29}$$

Finally, the green zone is related to the reorder quantities, and is calculated by

$$K_t^G - K_t^Y = ADU_t \; \ell \; \alpha_G, \tag{30}$$

where $\alpha_G \in [0, 1]$ is a parameter that the creators suggest should be calibrated in relation to the value of the decoupled lead time.[4] Specifically, they suggest setting a value close to zero (low) for $\alpha_G$ if the lead time is long; in contrast, for short lead times, they propose a value close to one (long). As evident from (30), a high value of such parameter increases the reordered quantity. This could sound counter-intuitive at first glance; however, this policy is based on the following rationale. In practice, when dealing with short lead times, traditional schemes of inventory management (as well as DDMRP) require the frequent reordering of a small amount of material. This has the drawback of breaking the pace of inventory accumulation due to overly frequent orders. To circumvent this issue, the heuristic suggests topping up the quantity reordered, which is expressed exactly by the correction when $\alpha_G$ is long. In contrast, when dealing with long lead times, traditional approaches require that big quantities are ordered to minimize reordering costs. Instead, the DDMRP heuristic has an opposite perspective to the way in which long lead time parts are traditionally handled, seeking to avoid the persistent problems and shortages caused by big and infrequent orders. The heuristic forces as frequent and small orders as possible to reduce the risk of disrupting the entire supply by applying a small $\alpha_G$ as a correction to the green zone. Our simulations we will verify that this requirement helps to reduce costs in case of short lead times.

Summarizing, the three thresholds turn out to be defined as follows:

$$K_t^R = ADU_t \; \ell \; \alpha \; (1 + \beta), \tag{31}$$
$$K_t^Y = ADU_t \; \ell \; (1 + \alpha + \alpha\beta), \tag{32}$$
$$K_t^G = ADU_t \; \ell \; (1 + \alpha + \alpha\beta + \alpha_G). \tag{33}$$

A final remark is due on the parameters $\alpha$ and $\alpha_G$. In the DDMRP methodology, they are imputed as a single parameter $\alpha$ that should be taken high (close to one) if the lead time is short and close to zero if it is long.

We now explain in detail how the reordering control is set. DDMRP is based on the idea of controlling the *Net Flow Position* (NFP) of the material present along the supply chain.

[4]According to the DDMRP methodology, the green zone can also take into account the presence of minimum order quantities or order cycles. For more details on these aspects, see Slack et al. (2010) or Jacobs et al. (2011).

In order to face the problem of supply order generation, a flow-based approach is defined. Specifically,

$$NFP_t = x_t + y_t - (d_t + \xi_t), \tag{34}$$

where $x$, $y$, $d$ are as defined in Section 2, and where $\xi_t$ represents an unexpected peak as defined in Section 3. In the context of DDMRP, the peak can be further specified in relation to the red threshold defined above:

$$\xi_t = \sum_{s=t+1}^{t+1+q_2} d_s \mathbb{1}_{\left\{d_s \geq 0.5 \cdot K_t^R\right\}}.$$

Finally, DDMRP stipulates the following control policy, based on the buffer levels defined above

$$u_t = \begin{cases} K_t^G - NFP_t & \text{if} \quad NFP_t < K_t^Y \\ 0 & \text{if} \quad NFP_t \geq K_t^Y \end{cases} . \tag{35}$$

Note that the control suggested by DDMRP is one particular instance of the admissible controls of Model III defined in Section 2.3, where $(s_t, S_t)$ are expressed in terms of the three thresholds of the buffer and of the NFP. For example, the threshold level to determine peaks, $0.5\, K_t^R$, is obtained in the model by inserting $\gamma = 0.5$ in to Equation (7). However, there is still a huge difference between the optimization model and the heuristic in terms of the information used to compute the reorder policy. The former is set to find an optimal control over the entire planning horizon $T$; in doing so, it uses all the available information, specifically, the entire demand vector $(d_t)_{t \leq T}$. In contrast, the DDMRP heuristic establishes its reorder policy relying on the three quantities defined in Equations (31)-(33). Therefore, demand influences the reorder policy at time $t$ only by means of short-period real orders (either in the past or in the future). Under this perspective, the heuristic is *less powerful* in that its information is limited compared to the theoretical model. In this respect, when comparing the performance of the reorder policy specified by DDMRP with our optimal control, we also test whether the DDMRP approach can be considered as a *good heuristic approach*, based on less information, to the solution of the proposed optimization model.

# 4 Validation of the DDMRP heuristic

In this section, we run a number of simulations to compare the optimal solution of our linear-integer optimization scheme with the heuristic specified by DDMRP. As steted above, the main objective of these simulations is to validate the algorithm proposed by DDMRP by showing that it proposes a reordering policy that is in line with the optimal one derived by the model developed in Section 2.4. Notably, this procedure also emphasises how well our model represents the original SIDLS problem.

First of all, we specify the exact form of the ADU used in the simulations. For simplicity, we choose $q_2 = 0$, and $q_1 = \ell$, where $\ell$ is the average lead time. In this way, we compute the ADU by looking at a short-period rolling average taken over the last $\ell$ periods. Under those assumptions, $ADU_t$ is defined as:

$$ADU_t = \frac{1}{\ell} \sum_{s=t-\ell}^{t-1} \tilde{d}_s.$$

We now specify the exact formulation of $\tilde{d}$ used in our simulations; we set $\tilde{d}_s = \min\{d_s; 0.5K_s^R\}$. Note that the value $0.5K_s^R$ corresponds to the threshold used to define the peak in Formula (7), with $\gamma = 0.5$.[5] By doing this, we exempt the decision-maker from considering exceptional values of demand when setting the reference level for inventory.

Concerning the parameters of the model, we set $T = 52$, mimicking the simulation of one year of inventory based on weekly periods. We take $d_0 = 20$ as the average demand per period. In line with the model, we simulate time series of demand by drawing $d_s : s = 1, \ldots, T$ from a log-normal distribution such that $\log(D) \sim \mathcal{N}(d_0, \sigma_D)$, where $\sigma_D = 0.5$.[6] As for the lead time, given that, this variable is crucial for discussing the impact of the DDMRP approach, we consider three different cases: Short ($\ell = 3$), medium ($\ell = 12$) and long ($\ell = 21$) average lead times.[7] Concerning the lead time risk, $\sigma_L$ is fixed at the level $\sigma_L$=0.8; note that $\sigma_L > \sigma_D$, since we believe that the risk related to the lead time has a greater impact than the risk related to the variability of demand. As for the initialization of the state variables, we set $y_0 = 0$ and $x_0 = d_0 \cdot \ell$.

We now discuss the cost parameters pertaining to the objective function. These have been fixed arguing on a *magnitude scale*. We first devote our attention to *standard costs*; starting from the less impacting to the largest impacting costs, we have: the holding cost ($h = 1$, expressed in dollar terms per unit of material a per unit of time), the penalty for under-stock ($p_{US} = 10$), and the reordering cost ($K = 100$, expressed in dollars per single order). The ratio of 1:100 between holding and reordering costs is in line with the standard literature, especially when referring to productions that require high set-up costs.[8] It remains to discuss the value of the over-stock penalty, which is crucial in shaping the goodness of the DDMRP heuristic since, by construction, its goal is to prevent excessive deviation from a reference level of on-hand materials. We therefore test an instance where we do not consider such a penalty term (i.e., $p_{OS} = 0$), and instead consider two different values for that parameter, one small compared to $p_{US}$ but equal to $h$ (i.e., $p_{OS} = 1$) and the other larger than $h$ but still smaller than $p_{US}$ (i.e., $p_{OS} = 5$).

The service level considered by the decision-maker, as discussed in Section 3, may depend on the lead time: When the lead time is long, the decision-maker may be willing to accept a rather higher under-stock risk to avoid huge orders, which could cause the immobilization of inventory. To account for this issue, we connect the service level to the lead time as follows: If the lead time is short, we take a 90% service level, namely, $\varepsilon = 0.1$; for an intermediate lead time, 80% ($\varepsilon = 0.2$) and for a long lead time 70% ($\varepsilon = 0.3$). As a consequence, relying on Corollary 1, we derive for each scenario the corresponding values for the *risk parameters* $\alpha$ and $\beta$. Specifically, we obtain the following specifications: For $\varepsilon = 0.1$, $\alpha = 1.03$ and $\beta = 0.25$; for $\varepsilon = 0.2$, $\alpha = 0.67$ and $\beta = 0.16$ and for $\varepsilon = 0.3$, $\alpha = 0.42$ and $\beta = 0.10$. We stress that the possibility of creating a formal connection among the service level, the lead time and the related risk parameter is possible thanks to the new framework we have developed in Section

[5]Having set $q_2 = 0$, the computation of the peak involves only one period into the future (see Equation (7)).

[6]The seed in the numerical simulation is fixed to ensure the comparability of all instances of the different experiments.

[7]Depending on the lead time, each simulation is run over a time period defined as $T + \ell$. Ex post, we neglect the first $\ell$ periods to reach a stationary behavior of inventory, ensuring the comparability of the three experiments.

[8]As an example, consider the production of small components of great precision (such as lenses for personal devices), requiring ultra precision and micro machining, or high expedition costs for delive (Jáuregui et al. (2010)).

2.3. On the other hand, DDMRP recommends relating $\alpha_G$ to the lead time, conveying that this is due to a "service level" issue. In this respect, our model allows us to elicit this *common-sense* through a mathematical dependence as emerging by the practice.

Finally, we briefly discuss the parameters that refer to the DDMRP heuristic. Two among the three risk parameters, $\alpha$ and $\beta$, are in line with that discussed above, and correspond to what we set for our theoretical model. Concerning $\alpha_G$, the guideline of the DDMRP manifesto suggests taking values between 0 and 1. In our simulations, we will then consider the two extreme cases to see how the heuristic behaves.

Summarizing, all fixed parameters of the model are reported in Table 1.

| $T$ | $x_0$ | $y_0$ | $d_0$ | $h$ | $K$ | $p_{US}$ |
|---|---|---|---|---|---|---|
| 52 | $d_0 \cdot \ell$ | 0 | 20 | 1 | 100 | 10 |

Table 1: Values of the parameters are kept fixed.

In contrast, Table 2 contains all varying parameters. Overall, by considering all values of the varying parameters, we identified nine different and significant specifications of the optimization problem (three different lead times against three different values for $p_{OS}$). For each such specification, we have two alternative parametrizations of DDMRP, depending on the value of $\alpha_G$.

| $\ell$ | $p_{OS}$ | $\alpha_G$ |
|---|---|---|
| $\{3; 12; 21\}$ | $\{0; 1; 5\}$ | $\{0; 1\}$ |

Table 2: Values of the parameters varying across the different experiments.

Finally, Table 3 shows the different values of the service level and the related values of the risk parameters as a function of the lead time which, as stated, varies across different experiments.

| $\ell$ | $\varepsilon$ | $\alpha$ | $\beta$ |
|---|---|---|---|
| 3 | 0.1 | 1.03 | 0.25 |
| 12 | 0.2 | 0.67 | 0.16 |
| 21 | 0.3 | 0.42 | 0.10 |

Table 3: Values of the service level and the related risk parameters depending on the lead time.

In the remainder of this section, we present a number of remarks and facts related to the strategy proposed by DDMRP and its comparison with the theoretical model described in Section 2.4.

*First: Benchmarking DDMRP with the theoretical model.* To benchmark the DDMRP heuristic with the theoretical model, we first discuss the results in terms of the corresponding objective functions. These results are presented in Table 4 for the three cases where $p_{OS} = 0$, $p_{OS} = 1$ and $p_{OS} = 5$. We denote by $f_{opt}$ the objective function of the linear-integer model evaluated under the optimal strategy and by $f_{DD}\,(\alpha_G = 0)$ and $f_{DD}\,(\alpha_G = 1)$ the total costs obtained by implementing the heuristic suggested by DDMRP under the two scenarios. To ease comprehension, the last two columns of Table 4 show the relative difference between $f_{DD}$ and

$f_{opt}$ in the two cases where $\alpha_G = 0$ and $\alpha_G = 1$, respectively. The smaller the difference, the better the heuristic in approximating optimal control. A quick glance at this table shows that for all the parameters displayed, the heuristic with $\alpha_G = 0$ outperforms the one with $\alpha_G = 1$, and this difference in performance increases with both $p_{OS}$ and $\ell$. Specifically, for a short lead time and penalization cost (first row in the table), we see that the two instances of the heuristic perform almost equivalently. By increasing either $\ell$ or $p_{OS}$, the performance of the heuristic with $\alpha_G = 0$ improves. The opposite happens in the case $\alpha_G = 1$: in the last row of the table, we see that the cost related to such heuristic approximately doubles the optimal cost.

| $p_{OS}$ | $\ell$ | $f_{opt}$ (model) | $(f_{DD} - f_{opt})/f_{opt}$ ($\alpha_G = 0$) | $(f_{DD} - f_{opt})/f_{opt}$ ($\alpha_G = 1$) |
|---|---|---|---|---|
| | 3 | 8004 | 17.64% | 18.49% |
| 0 | 12 | 14756 | 8.78% | 57.74% |
| | 21 | 30998 | 0.00% | 92.39% |
| | 3 | 12888 | 8.17% | 29.39% |
| 1 | 12 | 26809 | 3.59% | 67.92% |
| | 21 | 59652 | 0.00% | 96.18% |
| | 3 | 31401 | 2.03% | 44.73% |
| 5 | 12 | 74293 | 0.48% | 77.66% |
| | 21 | 174269 | 0.00% | 98.89% |

Table 4: The objective function $f$ computed under the optimal control of Model III and the relative performances of the policy suggested by DDMRP for different values of $p_{OS}$ and different lead times.

The same effect can be seen by looking at the corresponding inventory levels. Figure 1 summarise the inventory of optimal control (black line, marked crosses) and the one suggested by DDMRP (red line, marked by empty dots) under different specifications of the model, in case $p_{OS} = 0$. Panels 1a and 1b represent the inventory for a short lead time ($\ell = 3$); Panels 1c and 1d show the case of a medium lead time ($\ell = 12$) and, finally, Panels 1e and 1f represent the case of a long lead time ($\ell = 21$). Moreover, the inventory of DDMRP in case $\alpha_G = 0$ and $\alpha_G = 1$ is shown on the left and right, respectively. The graphs confirm what was previously stated: In the case $\alpha_G = 0$ (left panels), the inventory levels related to optimal control and DDMRP get closer as long as the lead time increases. In contrast, when $\alpha_G = 1$, by increasing the lead time, we clearly see that the inventory proposed by DDMRP becomes disproportionately large.

*Second: A focus on the case with a short lead time and high reordering costs.* We now concentrate on the case of a short lead time ($\ell = 3$) and $p_{OS} = 0$. As noted above, in this case DDMRP with $\alpha_G = 1$ is closer to optimal control than the other situations. In this respect, the DDMRP methodology suggests that the parameter $\alpha_G$ plays a crucial correction role by affecting the frequency of orders, especially when the lead time is short and the reorder cost is high. To substantiate this claim, we now test different values of $K$ in the specific case of $\ell = 3$ and $p_{OS} = 0$; our goal is to identify the value of $K$, which makes DDMRP with $\alpha_G = 1$ outperform the case with $\alpha_G = 0$. In Table 5, we report the values of $f_{opt}$ and the relative differences of the two $f_{DD}$ functions, depending on the value of $K$. In the two cases where $K = 150$ and $K = 200$, the cost associated with the DDMRP policy with $\alpha_G = 1$ is closer to the optimal cost compared to the case $\alpha_G = 0$. Moreover, the cost related to these instances of the model is nearly accurate in approaching the optimal cost. This result confirms the stipulation of the DDMRP methodology: with a short lead time and a high reordering cost, it is preferable to place slightly higher orders to diminish their frequency. To our knowledge, this is the first time that an explicit link among reorder costs, lead times and pace of inventory is made through a theoretical model.

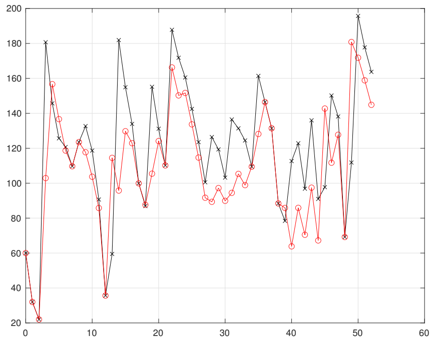
| $K$ | $f_{opt}$ (model) | $(f_{DD} - f_{opt})/f_{opt}$ ($\alpha_G = 0$) | $(f_{DD} - f_{opt})/f_{opt}$ ($\alpha_G = 1$) |
|---|---|---|---|
| 100 | 8004 | 17.64% | 18.49% |
| 150 | 8875 | 27.50% | 13.62% |
| 200 | 9531 | 38.66% | 12.10% |

Table 5: The objective function $f$ computed under the optimal control of Model III and under the policy suggested by the DDMRP for different values of $K$. Here $\ell = 3$ and $p_{OS} = 0$.

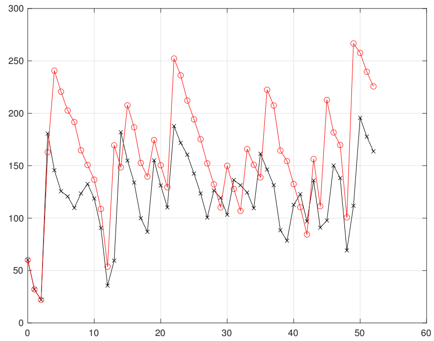
Once more, we can confirm the important role of $\alpha_G$ by looking at Panels 1a and 1b of Figure 1 it is evident that, when the lead time is short, by putting $\alpha_G = 1$ (right top panel), we increase the ordered quantities and the number of orders decreases, thus keeping a better pace of inventory. Again, this is desirable only for short lead times and high reordering costs. Otherwise, for long lead times, a large value of $\alpha_G$ would suggest excessive order quantities, as shown by Panels 2f and 3f, where inventory becomes uselessly huge.

*Third: DDMRP as a "good" heuristic for the formal optimization model.* Recall that the reorder policy stipulated by DDMRP is an admissible control for the optimization problem proposed in Section 2.4. In this respect, our numerical analysis suggests that DDMRP can be considered, under many instances of the problem, as a valuable heuristic approach to the solution of the optimization problem. Put differently, in case the decision-maker has the objectives formalized in (2), he will be satisfied by implementing the DDMRP approach in that it matches his expectations well in terms of inventory. Furthermore, Table 4 shows that the costs of optimal control and the one related to the heuristic are closer (if not exactly equal) for large values of $p_{OS}$ and/or large values of the lead time. Concerning the former, as stated in Section 3, the DDMRP heuristic promises to guarantee a *pace* of inventory, keeping close to the desired reference level and avoiding precipitous orders pushed by prompt reactions to out-of-scale peaks. This assertion is validated by our experiments: When accounting for a penalty $p_{OS}$, the inventory suggested by DDMRP is always very close (if not equal) to optimal control as suggested by the theoretical model. This can be seen also by looking at the left panels of Figures 2 and 3. The reason why a longer lead time makes the DDMRP policy closer to the one suggested by the model is less clear. As stated above, one relevant difference between the two schemes is that our Model III uses all the information about demand when solving for optimal control, whereas DDMRP relies only on short-period real orders. In this respect, when the lead time is long, DDMRP takes advantage of a larger horizon when computing the ADU, in that, by assumption, $q_1 = \ell$ and $q_2 = 0$. Put differently, when $\ell$ is short, the heuristic uses *less information* which cause a *less precise* solution compared to the full-information optimal control provided by Model III.

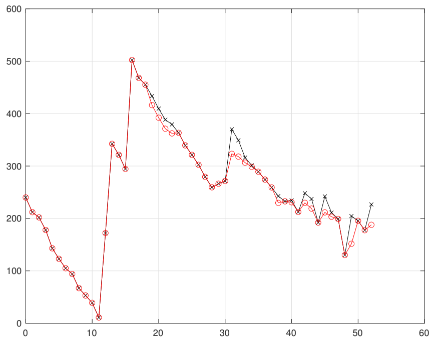
*Fourth: On the role of $p_{OS}$ to keep the inventory more stable.* By increasing the value of $p_{OS}$, the optimal cost increases: the decision-maker adds a penalty due to high values of inventory. In this regard, we can compare Panel 1a (where $p_{OS} = 0$) and Panel 3a (where $p_{OS} = 5$), which both refer to a short lead time. We can see how both the number of peaks and their value, as obtained by our model (black line, marked by crosses), are higher in the first case compared to the latter. We can interpret this fact as a better output in terms of pace of inventory, albeit requiring a higher cost. The difference between the two optimal costs can be interpreted as the *disclosure of a hidden cost.* Since the DDMRP heuristic does not depend on the cost structure,



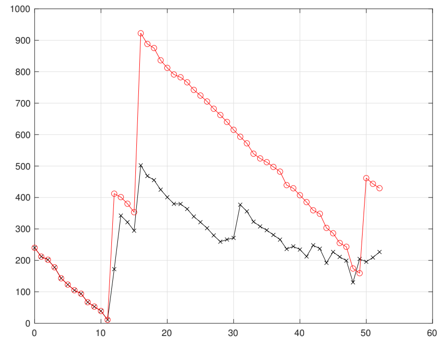
(a) $\ell = 3,\ \alpha_G = 0$



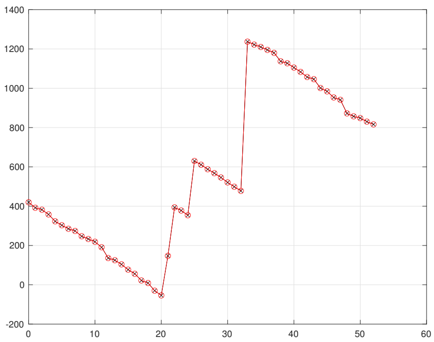
(b) $\ell = 3,\ \alpha_G = 1$



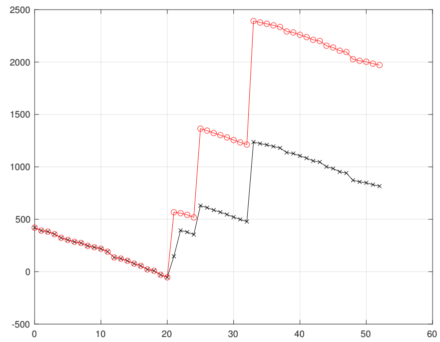
(c) $\ell = 12,\ \alpha_G = 0$



(d) $\ell = 12,\ \alpha_G = 1$

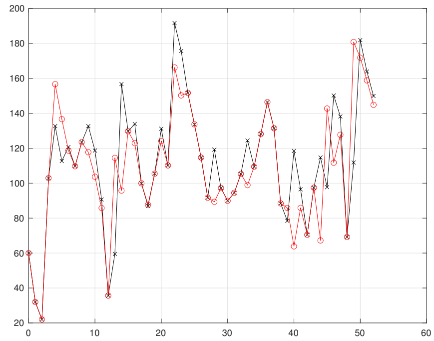


(e) $\ell = 21,\ \alpha_G = 0$

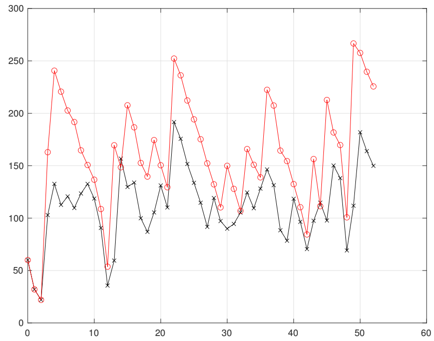


(f) $\ell = 21,\ \alpha_G = 1$

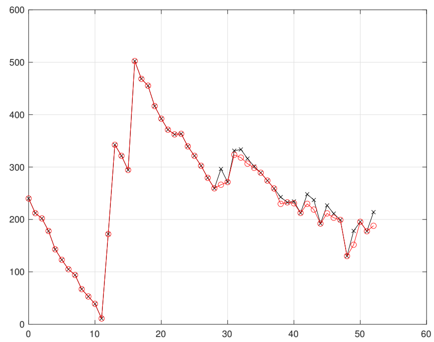
Figure 1: Inventory level for optimal control (black line) and the DDMRP heuristic (red dashed line). The three left panels show the inventory level for DDMRP with $\alpha_G = 0$; the right panels, for $\alpha_G = 1$. Here, $p_{OS} = 0$.



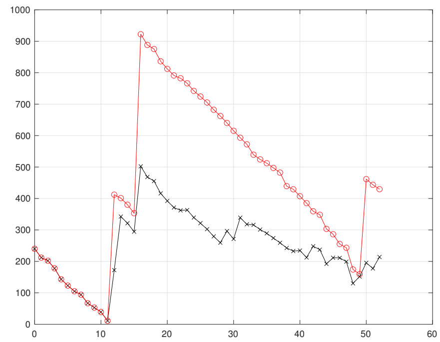

(a) $\ell = 3,\ \alpha_G = 0$



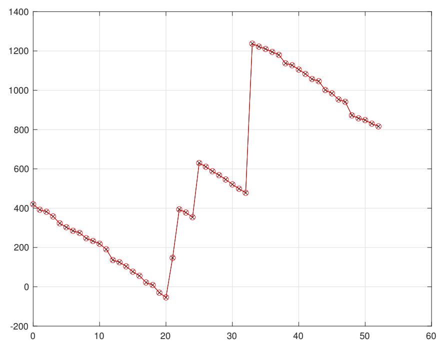

(b) $\ell = 3,\ \alpha_G = 1$



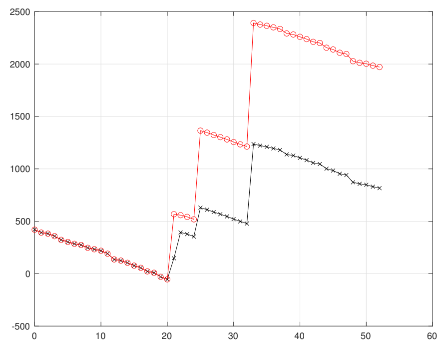

(c) $\ell = 12,\ \alpha_G = 0$




(d) $\ell = 12,\ \alpha_G = 1$

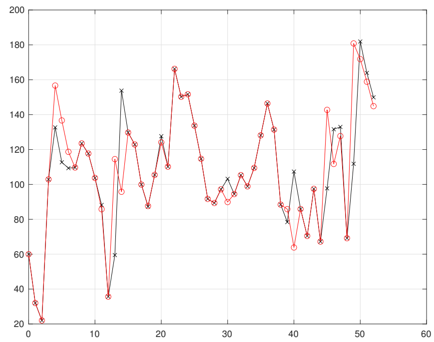



(e) $\ell = 21,\ \alpha_G = 0$

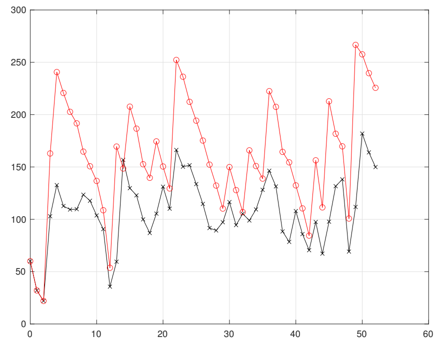



(f) $\ell = 21,\ \alpha_G = 1$

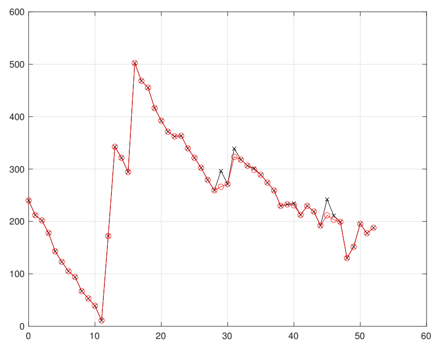
Figure 2: Inventory level for optimal control (black line) and the DDMRP heuristic (red dashed line). The three left panels show the inventory level for DDMRP with $\alpha_G = 0$; the right panels, for $\alpha_G = 1$. Here, $p_{OS} = 1$.



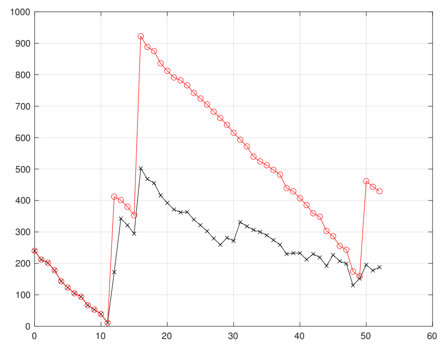
(a) $\ell = 3,\ \alpha_G = 0$



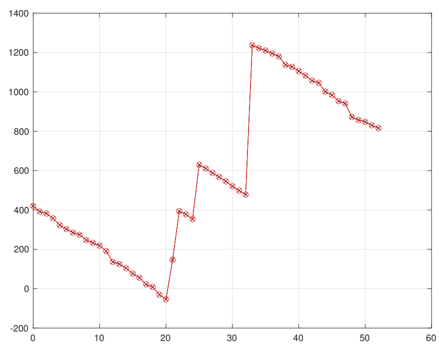
(b) $\ell = 3,\ \alpha_G = 1$



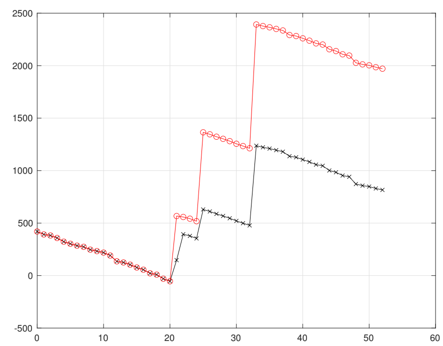
(c) $\ell = 12,\ \alpha_G = 0$



(d) $\ell = 12,\ \alpha_G = 1$



(e) $\ell = 21,\ \alpha_G = 0$



(f) $\ell = 21,\ \alpha_G = 1$

Figure 3: Inventory level for optimal control (black line) and the DDMRP heuristic (red dashed line). The three left panels show the inventory level for DDMRP with $\alpha_G = 0$; the right panels, for $\alpha_G = 1$. Here, $p_{OS} = 5$.

the inventory level is exactly the same in both cases (see red line marked by empty dots). In fact, as stated, DDMRP already includes this dimension in its policy; in some sense, it behaves *as if* the penalty $p_{OS}$ has been considered.

# 5 Conclusions

In this paper, we studied a single-item dynamic lot-sizing problem by considering both stochastic demand and lead times. Our approach was based on a prudential principle aiming at setting an inventory cushion that proactively reacts to short-horizon real orders and unexpected peaks. Specifically, our theoretical probabilistic model comprehends: (i) a service level constraint, expressed as a risk measure and (ii) a penalty cost for large deviations from a reference inventory level. In doing so, we obtained two significant properties of the optimal policy derived by implementing the model: (i) the presence of an optimal buffer level $s(t)$ and (ii) a reordering value $S(t)$, which has been proved to inhibit excessive reorder values, especially when the lead time is short. These two factors ensure an inventory "pace", escaping from overly impulsive reactions to peaks in demand while maintaining a buffer to avoid undesired stock-out. This is a traditional trade-off between two opposite risks: stock-out and excessive immobilization of resources.

In addition, our model was developed by taking into consideration the *philosophy* at the base of a recent heuristic proposed by practice, referred to as *Demand Driven MRP*. Despite its widespread use, this heuristic lacks a theoretical foundation a model capable of providing evidence on the goodness of the policy prescribed by the algorithm. We ran various experiments with the aim of benchmarking the policy suggested by the DDMRP heuristic with our optimal control. By doing so, we *validated* such a heuristic by showing that, under several specifications of the model, it matches the optimal policy. To be precise, we saw that by following the stipulations of the DDMRP approach in terms of the setting of the main parameters, the heuristic is capable of maintaining an inventory pace for both short and long lead times.

To run the validation exercise, we translated the original SIDLS problem with stochastic demand and lead times into a mixed linear integer optimization scheme. Despite being deterministic, the latter formulation reflects the inherent randomness of the problem through a short-period time-varying average demand (the ADU); a risk factor related to the variability of demand and lead times, based on a minimum service level (the Value at Risk of the lead time demand) and a time-varying measure of future peaks. We showed that such statistics is sufficient to determine the optimal control of the model, and, in turn, the policy suggested by the DDMRP heuristic. In this respect, our research seems to suggest that such statistics, based on real and short-term orders, are key to determining a reorder policy that satisfies for a decision-maker following the "philosophy"of DDMRP, i.e. decision-makers who consider not only standard inventory costs, but also sunk-costs related to high inventory fluctuations.

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