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Abstract
Is inequality good or bad for innovation? I study an endogenous growth model with heterogeneous agents; due to credit frictions, inequalities in wealth lead to misallocation of talent. A more unequal reward scheme incentivises innovation in any given period, but it leads to a more unequal distribution of opportunities that may exacerbate the misallocation of talent in the next period. Empirically, I show that the flow of patents in a US state is negatively correlated with inequality of opportunity, but positively with inequality of effort; and that the elimination of state death taxes, as a proxy for an increase in the financial incentives towards risky activities, had a positive short-term but a negative long-term effect on the growth rate of patents.

Keywords
Occupational Choices; Adverse Selection; Bequests; Theil's Index; Death Taxes

JEL Classification: D15, D53, D58, D82, H23, O31

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Alessandro Spiganti
Max Weber Fellow, 2018-2019 and 2019-2020

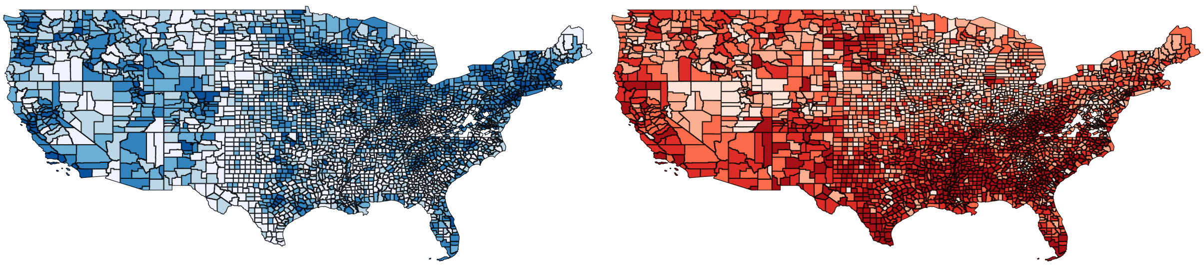
1 Introduction

Inequality has been rising in many wealthy countries in recent decades. For example, the average income of the richest 10% is almost ten times that of the poorest 10% across OECD countries, up from seven times a quarter of a century ago (Keeley, 2015); in the US, the share of all income accruing to the top 1% increased by 11 percentage points from 1979 to 2017 (Saez, 2019). In the years since the financial crisis, concerns about this increase have entered the political and economic mainstream, sparking a new wave of economic literature on the following question: is inequality good or bad for growth? Opinions are still very much divided, and many possible mechanisms have been proposed. For example, it has been argued that inequality may negatively impact growth if, in the presence of credit market imperfections, it causes misallocation of talent (e.g. Galor and Zeira, 1993, Banerjee and Newman, 1993, Krueger, 2012); conversely, it may enhance growth by providing the financial incentives for agents to embark on risky activities, exert unobservable effort, and work hard (e.g. Mirrlees, 1971, Okun, 1975, Mankiw, 2013). In this paper, I contribute to this open debate in three ways. First, by studying the relationship between inequality and innovation, one of the major drivers of economic growth (Barro and Sala-I-Martin, 1995). Second, by dividing total inequality into two components, which may affect innovation in different ways. Following Roemer (1993, 1998), I consider total inequality as a composite measure of *inequality of opportunity*, i.e. inequality stemming from circumstances beyond individual responsibility (like socio-economic background), and *inequality of effort*, i.e. inequality that results from causes that are within their control (like unequal exerted effort). Third, by taking a dynamic perspective: the interplay between both types of inequality and innovation not only contributes to observed differences in outcomes in any given period, but these differences are also likely to be transmitted across generations, thus shaping the future playing field.

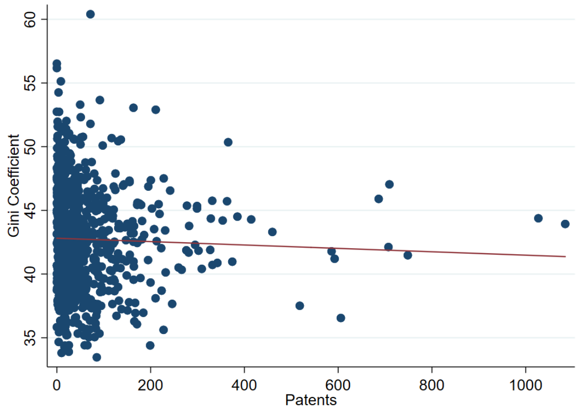
Anecdotally, there seems to be no clear relationship between total inequality and innovation, but a negative correlation between inequality of opportunity and innovation. The first two panels of Figure 1, for example, map the average per capita number of patents filed in a given US county and the corresponding income Gini index, respectively. By comparing the two panels, one can see that there exist groups of counties that rank high on one index and low in the other, and groups that maintain the same relative position.[1] Consistently, Figure 1c shows a near-zero correlation between these two measures for metropolitan counties. Conversely, Figure 2 presents two measures of inequality of opportunity at the US county level: panel 2a maps the Opportunity Index, an annual report developed by Opportunity Nation and Child Trends (2019) that summarises data on

[1] For example, most counties in the Great Lakes regions have a relatively high number of patents and rather low inequality, whereas in the West South Central the opposite happens. Counties in Florida and the San Francisco Bay area tend to score high on both patents and inequality, whereas those in the middle of the West North Central division score low on both statistics.

education, health, community, and economy to show what "opportunity looks like in the United States", whereas panel 2b maps a proxy for intergenerational income mobility, a concept closely related to inequality of opportunity (Brunori *et al.*, 2013). By comparing these with the number of patents in Figure 1a, there seems to be an overlap: relatively innovative counties tend to also score high on opportunities and social mobility, whereas counties with low mobility and opportunities also tend to have a low number of patents per capita.[2] This is confirmed by the last two panels of Figure 2, which depict a clear positive relationship between these two measures and the number of patents per capita for metropolitan counties. Armed with this anecdotal evidence, this paper investigates the following questions: What is the relationship between different types of inequality and innovation? To what extent is there an optimal degree of inequality for innovation? Can countries be both innovative and have equal societies?



(a) Patents (b) Gini Coefficient

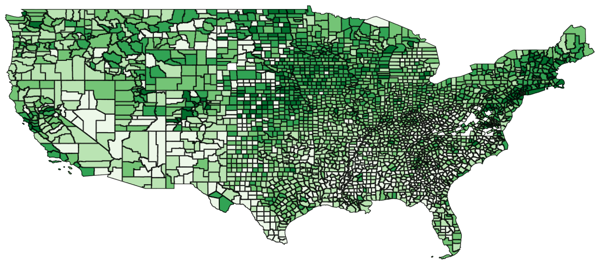


(c) Inequality and Innovation

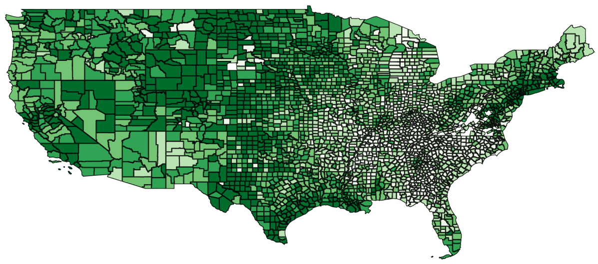
Figure 1: Patents and Total Inequality by US Counties

Notes. The fist panel reports quintiles of the average number of utility patents (per hundred thousand residents) granted between 2000 and 2010 by the United States Patent and Trademark Office to a patent inventor resident in the county (data elaborated from United States Census Bureau, 2016a, United States Patent and Trademark Office, 2019). The second panel reports quintiles of the 2000's income Gini index (United States Census Bureau, 2016b). For both panels, darker colours represent relatively higher indexes. The third panel reports the scatter-plot of these two measures for metropolitan counties only (as defined by Ingram and Franco, 2014).

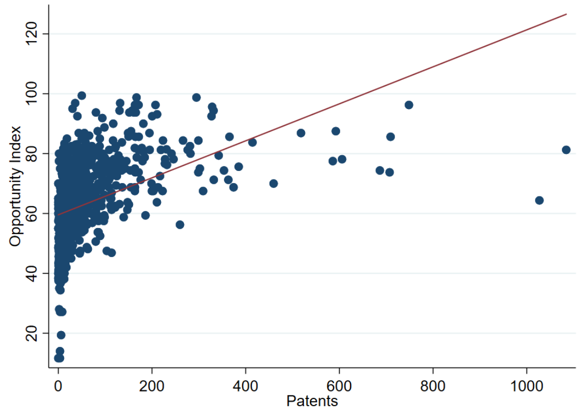
[2]For example, counties in the New England, Middle Atlantic, and Pacific divisions score high on all indicators, whereas counties in the South Central, the South Mountain, and part of the West North Central divisions tend to score relatively low.



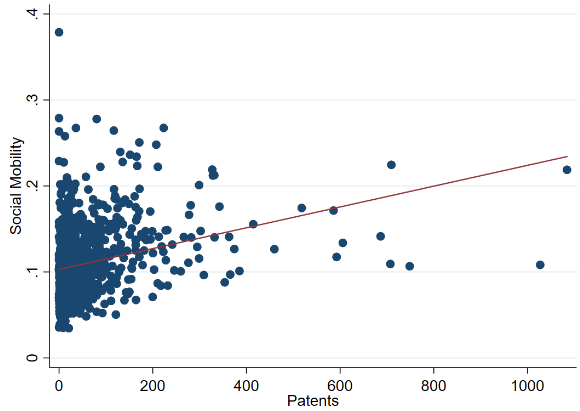
(a) Opportunity Index



(b) Social Mobility



(c) Opportunities and Innovation



(d) Social Mobility and Innovation

Figure 2: Patents and Opportunities by US Counties

Notes. The fist panel reports quintiles of the 2011 - 2018 average Opportunity Index (data elaborated from Opportunity Nation and Child Trends, 2019). The second panel reports quintiles of the fraction of children with parents in the 25^{th} percentile of income who grew up in a given county and then in adulthood had an individual income in the top 20% in adulthood (Chetty *et al.*, 2018). Darker colours represent higher indexes. The last two panels report scatter-plots of the average Opportunity Index and the measure of social mobility, respectively, with the average number of utility patents (per hundred thousand residents) granted between 2000 and 2010 to a patent inventor resident in the county (data elaborated from United States Census Bureau, 2016a, United States Patent and Trademark Office, 2019), considering metropolitan counties only (as defined by Ingram and Franco, 2014).

To investigate these questions, I first construct a theoretical model with three components: (i) Schumpeterian innovations, (ii) heterogeneous agents, and (iii) credit frictions. The foundation of the model is a canonical Schumpeterian framework, where growth is the result of a random sequence of quality-improving innovations. The novelty is that agents are assumed to be heterogeneous in unobservable talent and observable wealth, and must choose between working for a wage or becoming inventors. Incentivised by the prospect of monopoly rents, inventors exert effort to create better machines; workers are instead employed by a representative firm. If capital markets worked perfectly, only talented agents would become inventors. However, becoming an inventor requires an initial investment and, since talent is unobservable by the lenders, collateral is used to screen borrowers. As a consequence, inequalities in wealth translate into unequal opportunities that lead to a misallocation of talent: poor talented agents must become workers, and are thus displaced by relatively wealthier but untalented inventors.

On the one hand, I show that a more equal distribution of observable wealth is always associated with an increase in the number of talented innovators, as long as the economy is relatively wealthy. Indeed, this translates into a more equal distribution of opportunities, a more widespread access to credit, and less misallocation of talent. Conversely, a more unequal wealth distribution, and thus more inequality of opportunity, may be beneficial for innovation when the economy is poor, since it allows at least some agents to overcome setup costs that are large in relation to average wealth. On the other hand, I show that the number of innovations in any given period positively depends on the relative reward to a successful inventor. Indeed, a more unequal reward scheme (more inequality of effort) increases the financial incentives for agents to embark on the risky innovation process and exert effort.

I then extend the model dynamically, by assuming that old agents are periodically replaced by a new generation, to whom they bequeath wealth. As a consequence, the distribution of opportunities, the evolution of the quality of the machines, and the relative rewards to different occupations become endogenous. The relative profits of the innovators, for example, partly depend on the quality of the machines, whose evolution, in turn, depends on the occupational choices of the agents; these are shaped by the inherited wealth distribution, and thus by the rewards to different occupations in the previous generation. In such a setting, initial conditions, like the initial wealth distribution or the initial quality of the machines, have long-run effects on the growth prospect of the economy. For example, I show that poor and/or technologically disadvantaged countries may be stuck in a no innovation trap. Moreover, an intertemporal trade-off emerges: a more unequal reward scheme incentivises innovation in any given period, but in the presence of bequests this translates into a more unequal playing field for the next generation, which may hamper innovation in the future.

In the second part of the paper, I empirically test the main implications of the theoretical model. First, I investigate the relationship between innovation and various measures of inequality at the US state level. I measure innovation using data on patents and citations from the United States Patent and Trademark Office, going back to 1976. I use labour income data since 1968 from the Panel Study of Income Dynamics to calculate the total inequality rate at the state level. I employ a widely-used technique in the inequality of opportunity literature to separate total inequality in each state and year into inequality between socio-economic groups (classified by race and parental education) and inequality within groups. The first component represents a proxy for inequality resulting from circumstances beyond the individual's control, and is thus used in the social sciences as a lower bound for inequality of opportunity; by controlling for these circumstances, the latter becomes a proxy for inequality that arises from an individual's conscious choices, and is thus used in the social sciences as an upper bound for inequality of effort. By regressing innovation on these lagged measures of inequality, I find that, whereas innovation is uncorrelated with total inequality, this is the result of a negative significant correlation with inequality of opportunity and a positive significant correlation with inequality of effort.

Second, I provide some indicative evidence of the dynamic trade-off highlighted above. I exploit the wide variation in the chronology of elimination of US state estate, inheritance, and gift taxes as a proxy for state-level changes in the political attitudes to economic inequality. In particular, and consistently with the arguments often put forward by critics and supporters of state "death" taxes, I consider their elimination as indicative of a short-term increase in the incentives to undertake risky activities (i.e. an increase in inequality of effort) but a long-term decrease in equality of opportunity. I use an event study to compare the growth rates in the flow number of utility patents granted in a given state with the annual growth rate at the US level. I find that most states experienced a positive abnormal growth rate in the five years immediately following the elimination of the taxes, but a negative cumulative one twenty years down the line.

The remainder of this paper is organised as follows. Section 2 reviews previous literature on the relationship between inequality and innovation. Section 3 presents the theoretical model. Section 4 describe its static equilibrium, a series of implications, and the dynamics. Section 5 outlines the empirical analyses. Finally, Section 6 concludes.

2 Previous Literature

This paper bridges the Schumpeterian growth theory literature pioneered by Aghion and Howitt (1992) with the literature on the effect of misallocation on growth (see Murphy *et al.*, 1989, Banerjee and Newman, 1993, Galor and Zeira, 1993, for some seminal contributions). More broadly, this paper is also related to the literature on the consequences of occupational choices for inequality (Kambourov and Manovskii, 2009), on innovation incentives (e.g. Holmström, 1989, Aghion and Tirole, 1994, Manso, 2011, Spiganti, forthcoming), and on occupational persistence across generations (e.g. Caselli and Gennaioli, 2013, Lo Bello and Morchio, 2016).

More specifically, this paper belongs to a growing literature on the relationship between inequality and innovation.[3] Recently, Aghion *et al.* (2019) and Jones and Kim (2018) have built Schumpeterian models that link the dynamics of top income inequality to innovation, and showed that creative destruction makes growth more inclusive. Acemoglu *et al.* (2017) positively link innovative activities of an economy to a more unequal reward structure, whereas Spiganti (2018) finds a non-monotonic relationship between

[3] In this paper, I study neither the effect on income inequality of the introduction of new technologies (see Violante, 2008, for a brief survey on skill-biased technological change), nor the effect of inequality on the incentives to innovate through demand composition (Murphy *et al.*, 1989, Zweimüller, 2000, Foellmi and Zweimüller, 2006, 2017, Hatipoğlu, 2012).

wealth inequality and innovation. Differently from these papers, I focus on both wealth and income inequality, and the feedback effect between them due to the presence of intergenerational linkages.

The long term effect of the misallocation of talent in innovative activities when there are intergenerational linkages is also the focus of Jaimovich (2011) and Celik (2018). However, there are several differences between our papers. First, in Celik's (2018) quantitative model, inventors are skilled workers employed by firms for a fixed wage, whereas in Jaimovich (2011), horizontal innovation occurs when agents open up new sectors matching their intrinsic skills. In this paper, innovation is vertical and Schumpeterian: inventors are entrepreneurs who are willing to face the risk of failure to discover better vintages of existing machines and pursue monopoly rents. This allows me to study how the occupational choice into innovation is shaped by an endogenous reward structure. Second, in Celik (2018) everyone would like an innovative job, as it pays exogenously better than routine jobs, but the number of training opportunities necessary to become skilled is scarce and subject to a tournament mechanism; in Jaimovich (2011), there is no occupational choice as everyone is an entrepreneur, but adverse selection may prevent credit flowing to the most productive sectors (i.e. with a better match between an entrepreneur's skill and a sector's characteristics). Here, observable wealth is used by banks to screen different borrowers: as a consequence, endogenous wealth classes arise in equilibrium, each associated with different occupational choices.[4] Since the wealth distribution affects the composition of the wealth classes and the occupational choices of the agents, it affects the resulting growth rate of the economy. Moreover, this indirectly affects the new wealth distribution. This allows me to study how the number of innovators and their average quality change vis-à-vis the state of the economy.

Empirically, there is a very recent and flowering literature on the relationship between inequality and innovation. For example, Akcigit *et al.* (2017), Aghion *et al.* (2018), Celik (2018), and Bell *et al.* (2019) merge individual income data with individual patenting data and find a positive relationship between parental resources and the probability of becoming an inventor. Conversely, Aghion *et al.* (2019) find a positive effect of patenting on top income inequality, using a US state level panel. In this paper, I follow techniques that are widely used in the inequality of opportunity literature (see Roemer and Trannoy, 2016, for a review) to decompose total inequality at the US state level into inequality of opportunity and inequality of effort. This allows me, for the first time, to investigate the relationship between innovation and both components, whereas the above papers consider only one.

[4] On the modelling side, the interplay between adverse selection, moral hazard, and occupational choices in this paper is reminiscent of e.g. Grüner (2003), Ghatak *et al.* (2007), Inci (2013), and Spiganti (2018). Differently from these papers, I extend this framework to a dynamic setting and embed it into a Schumpeterian model of innovation.

3 The Model

Time is discrete and infinite, $t = 1, 2, \ldots, \infty$. In any given period t, there is a continuum of one-period lived agents of mass one, indexed by h, with the same instantaneous utility function,

$$u(c_{t,h}, b_{t,h}) = c_{t,h}^{1-\delta} b_{t,h}^{\delta}, \tag{1}$$

where $\delta \in (0, 1)$, $c_{t,h}$ is consumption of the final good, and $b_{t,h}$ is bequest in period t.[5]

Agents are heterogeneous in two dimensions. First, they differ in their innovative ability a: in any period t, a proportion $\lambda \in (0, 1)$ of agents is talented, the remaining proportion $1-\lambda$ is untalented. Second, agents differ in their wealth endowment, $A_t \in (0, \infty)$, which is distributed according to the (continuously differentiable) cumulative distribution function $\Phi_t(A)$, whose probability density function is $\phi_t(A)$.[6] Let $\bar{A}_t = \int_0^\infty A_t d\Phi_t(A)$ be average (and total) wealth in t. For simplicity, I assume that ability and wealth are uncorrelated: this implies that abilities are also intergenerationally uncorrelated and that there is an equal proportion of talented and untalented agents for every wealth level. At the beginning of their life, agents receive their wealth in the form of a bequest from their parent.

3.1 Final Good Production

Agents consume an homogeneous final good, y_t. This is produced competitively by a representative firm combining unskilled labour and a continuum of machines indexed on the interval $[0, 1]$ according to

$$y_t = f(l_t, x_{t,m}) = l_t^{1-\alpha} \int_0^1 Q_{t,m}^{1-\alpha} x_{t,m}^{\alpha} dm, \tag{2}$$

where $\alpha \in (0, 1)$, l_t is labour, $Q_{t,m}$ is the quality of machine of type m used, and $x_{t,m}$ is the quantity of this machine.[7] Let $Q_t \equiv \int_0^1 Q_{t,m} dm$ be the average quality of the machines, an aggregate quality index of the economy.

[5] In line with the "warm glow" or "joy of giving" literature that follows from Andreoni (1989, 1990), I assume that bequests, rather than offspring's utility, enter the utility function directly. Under this assumption, utility is linear in end-of-period wealth, and this makes the model more tractable (see e.g. Galor and Zeira, 1993, Banerjee and Newman, 1993, Jaimovich, 2011, for similar assumptions).

[6] In the terminology of Becker (1993), individuals differ with respect to both "opportunities" and "abilities".

[7] Similar formulations of this multisector Schumpeterian model of endogenous growth (i.e. where growth is generated by a random sequence of vertical improvements) appear in Aghion and Howitt (2009, Ch. 4) and Acemoglu *et al.* (2012). Note that there is nothing of importance lost by having l_t and $Q_{t,m}$ raised to the same power (see Aghion and Howitt, 2009, Ch. 4, problem 2), and that I am implicitly assuming that production uses only the highest quality machine for each type.

The profit-maximization problem of the final good producer is

$$\max_{l_t,\{x_{t,m}\}_{m=0}^1\geq 0} p_t f(l_t, x_{t,m}) - w_t l_t - \int_0^1 r_{t,m} x_{t,m} dm, \tag{3}$$

where p_t is the price of the final good, w_t is the wage rate, and $r_{t,m}$ is the price of machine of type m used. The first order conditions are

$$(1-\alpha)p_t l_t^{-\alpha} \int_0^1 Q_{t,m}^{1-\alpha} x_{t,m}^{\alpha} dm = w_t \tag{4a}$$

$$\alpha p_t l_t^{1-\alpha} Q_{t,m}^{1-\alpha} x_{t,m}^{\alpha-1} = r_{t,m} \tag{4b}$$

and thus the following iso-elastic demand curves are obtained (for ease of reading, I am ignoring that some of the right-hand side variables are policy functions):

$$l_t(p_t; w_t, r_{t,m}) = \left(\frac{p_t(1-\alpha)}{w_t} \int_0^1 Q_{t,m}^{1-\alpha} x_{t,m}^{\alpha} dm\right)^{\frac{1}{\alpha}} \tag{5a}$$

$$x_{t,m}(p_t; w_t, r_{t,m}) = \left(\frac{\alpha p_t}{r_{t,m}}\right)^{\frac{1}{1-\alpha}} Q_{t,m} l_t. \tag{5b}$$

3.2 Innovation

Innovation in each machine takes place as follows. Becoming an inventor requires an exogenous sunk cost of I_t. An innovator is then matched randomly with one machine (one to one, no congestion). Producing one unit of any machine costs ψ units of final good.

Innovation is stochastic, with probabilities of success depending on the innovative talent of the agent. Untalented agents always result successful with probability ρ_L. Talented individuals can raise the probability of success to ρ_H by working hard, but this comes at a positive cost e_t, which is measured in monetary units.[8] Hereafter, effort-exerting talented agents are denoted by H (mnemonic for high-ability), whereas untalented and shirking talented agents are denoted by L (for low-ability). The talent of the agents and their effort level are known only by them, but the distribution of talent in every wealth level is public information.

In case of success, an innovator increases the quality of the machine from $Q_{t-1,m}$ to $Q_{t,m} = (1+\gamma)Q_{t-1,m} > Q_{t-1,m}$ and, in line with the endogenous technical change literature, becomes the sole producer of the machine m. The profit-maximization problem

[8] The talent distribution in the population can thus be thought of as a distribution of the cost of effort, which is prohibitively high for untalented individuals. Intuitively, everyone in this economy is born untalented: some individuals have the potential to undertake some costly activity to increase their talent, whereas those that remain lack the natural ability. Similarly to Grüner (2003), Inci (2013), and Spiganti (2018), moral hazard is necessary to have some poor talented workers in equilibrium (since wealth is assumed to be non-negative).

of the inventor of a new machine m is

$$\max_{r_{t,m}^{M}, X_{t,m}^{M} \geq 0} (r_{t,m}^{M} - \psi) X_{t,m}^{M} \qquad \text{s.t. } X_{t,m}^{M} \geq x_{t,m} \text{ in (5b)}, \tag{6}$$

where $r_{t,m}^{M}$ and $X_{t,m}^{M}$ are the price and quantity supplied of the monopolistically-produced machine m in t. Since demand is iso-elastic, the monopoly price is a constant mark-up over marginal cost, $r_{t,m}^{M} = \psi/\alpha$,[9] and thus the equilibrium demand function for monopolistically-produced machines, $x_{t,m}^{M}$, becomes

$$x_{t,m}^{M} = \left(\frac{\alpha^2 p_t}{\psi}\right)^{\frac{1}{1-\alpha}} Q_{t,m} l_t. \tag{7}$$

Here, I make a further simplifying assumption. Similarly to Aghion and Howitt (2009, Ch. 6), I assume that the starting quality for any given machine m at date t has the average quality parameter Q_{t-1} across all machines last period, rather than the quality parameter $Q_{t-1,m}$ of that machine last period.[10] Therefore, an innovator that is successful in inventing a new machine, would profit

$$\pi_t\left(p_t; w_t; Q_{t-1}\right) \equiv \left(\frac{\psi}{\alpha} - \psi\right) \left(\frac{\alpha^2 p_t}{\psi}\right)^{\frac{1}{1-\alpha}} (1+\gamma) Q_{t-1} l_t \tag{8}$$

from selling the machine.

With probability $1-\rho_i$, $\forall i = \{H, L\}$, the innovation does not materialise. In such case, the old machine is produced competitively. Let $X_{t,m}^{C}$ be the quantity of the competitively-produced machine m in t. Since the unsuccessful innovator prices the machine at the marginal cost, $r_{t,m}^{C} = \psi$, the equilibrium demand function for competitively-produced machines is

$$x_{t,m}^{C} = \left(\frac{\alpha p_t}{\psi}\right)^{\frac{1}{1-\alpha}} Q_{t-1} l_t. \tag{9}$$

The unsuccessful innovator breaks even.

3.3 Credit Contracts

In each period, there are several banks competing à la Bertrand, each owned equally by all agents. Workers deposit their wealth in the banks for a risk-free rate of return, R_t: an investment of one unit in t yields a return of R_t units at the end of the period. All agents take this rate of return as given when making their occupational choices. Banks use these deposits to lend money to innovators who ask for it: without loss of generality,

[9] I am implicitly assuming, for simplicity, that innovation is drastic, in the sense of Tirole (1988): the monopolist can charge any price she wants without fearing entry from potential competitors.

[10] I make this assumption to avoid further complications arising from having to include the quality of the machine in the optimal contract derived below.

I assume that an agent with wealth A_t only borrows up to $I_t - A_t > 0$ to finance the set-up cost; conversely, rich innovators deposit $A_t - I_t > 0$.[11] In the next sections, unless otherwise specified, I focus on the credit constrained agents when deriving the optimal contracts, as unconstrained agents are free to take their first-best choice.

Banks observe the wealth of the borrowers, and whether they succeeded or not in discovering a new machine, but ability is unobservable.[12] They take prices, including the riskless rate of return, as given, and can offer a distinct menu of contracts for every wealth level. Banks hold the same beliefs, which they form simultaneously, about how agents decide when offered a given menu of contracts. This menu of contracts consists of a repayment schedule, given the factor prices and qualities of the machines, contingent on the outcome of the innovation process and the announced type. I assume limited liability protects the agents, in the sense that an innovator cannot be left with negative end-of-period payoff. As a consequence, and since in the failure state innovators break even, they will be able to pay back a positive amount only in the case of success.[13] A loan contract offered by a given bank then takes the following form (since it does not generate confusion, I shall drop the subscript indicating a given bank and t):

$$\boldsymbol{\sigma}(A, \Omega) = \begin{bmatrix} \sigma_H(A, \Omega) \\ \sigma_L(A, \Omega) \end{bmatrix} = \begin{bmatrix} D_H^S(A, \Omega) \\ D_L^S(A, \Omega) \end{bmatrix}, \tag{10}$$

where σ_i is the contract designed for the i-type agents with wealth A and Ω is a vector of prices, interest rate, and average quality of the machine at t (i.e. the *state* of the economy). These contracts set the repayments to the bank by the i-type agent in the success state, D_i^S.

Therefore, with probability ρ_i, the innovator is successful in inventing a new machine, and thus profit $\pi(\Omega)$ from selling the machine. She will then pay $D_i^S(A, \Omega)$ to the bank. Let the subsequent realised net payoff of an i-type innovator in the success state be given by V_i^S; limited liability implies $V_i^S \geq 0$. Conversely, an unsuccessful innovator would break even from the production of machines, and the financiers would recover zero income. Thus, the expected payoff of an innovator is $\mathcal{V}_i(A, \Omega) \equiv \rho_i V_i^S - e_i$, $\forall i = \{H, L\}$, where $e_H = e$ and $e_L = 0$. Conversely, the payoff of an agent who becomes a worker is

[11] It is well-known, see e.g. DeMeza and Webb (1987), that there must be maximum self-finance in equilibrium, because this comes with better terms than borrowing for high ability agents (thus, if there are agents who are not using their entire wealth, they must be untalented).

[12] Since abilities are intergenerationally uncorrelated, parents' historical outcomes provide no useful information to the banks. In real life, business plans are also likely to be used as screening devices in bank loan applications: however, adding a three-signal structure, where the probability of getting the better signals increases with ability, to the current model would not qualitatively change the partial static equilibrium.

[13] This means that the repayment in case of failure cannot be positive, but, in principle, it may be the case that banks offer money to unsuccessful innovators. Given risk-neutrality, however, imposing the repayments to be zero in the failure state is without loss of generality. Appendix A.2 presents the proofs without imposing this.

$\mathcal{W}_i(A, \Omega) \equiv w + RA, \forall i = \{H, L\}$: I call this payoff "the outside option" to innovation.

4 Partial Equilibrium Analysis

In this section, I first analyse the static counterpart of the model: I focus on a given period, and thus the ability and wealth distributions, as well as the average quality of the machines, are given. I thus derive the set of credit contracts offered by banks and the optimal occupational choices of the agents. Later, I study the dynamic evolution of the economy. Throughout this section, I abstract from general equilibrium effects in the credit and labour markets, i.e. I take the risk-free interest rate and the wage rate as exogenously given.[14]

I take the standard assumption that talented innovation is efficient, whereas untalented innovation is not. This means that, if agents could self-finance completely, only talented agents would find it profitable to enter the innovation sector. However, untalented agents may still find it profitable to become innovators if cross-subsidised by talented agents. This is formalised as follows,

Assumption 1 (Static Efficiency). $\rho_H \pi(\Omega) - e > w + RI > \rho_L \pi(\Omega) > w + RI(\rho_L/\bar{\rho})$,

where $\bar{\rho} = \lambda\rho_H + (1-\lambda)\rho_L$ is the Bayesian probability of success of a random applicant.

I impose a Bertrand-Nash equilibrium concept in the static framework. As it is well-known from Rothschild and Stiglitz (1976), this may lead to non-existence of a competitive screening equilibrium. I circumvent this by restricting the set of feasible contracts to loan contracts only, i.e. non-negative repayments made by entrepreneurs to the banks.[15]

Definition 1. *Assume banks are Bertrand-Nash players following pure strategies, offering loan contracts, and paying an interest R on deposits, which they take as given. A static partial equilibrium consists of choices for individuals $(c_h^\star, b_h^\star)$, the final good producer $(l^\star, \{x_m^\star\}_{m=0}^1)$, and the innovators $(\{r_m^\star, X_m^\star\}_{m=0}^1)$; prices $(w^\star, p^\star)$; profits for banks, final good producer, and innovators; sector allocations and effort decisions; and an individually rational and incentive compatible menu of contracts for each wealth class, such that: (i)*

[14] For example, this would be the case if the economy is small and with access to perfect international capital and labour markets. Financial intermediaries (depositors) would be able to draw (deposit) liquid funds from (in) an international credit market, with a perfectly elastic supply and demand at the international rate of return R. Firms would be able to hire workers with a perfectly elastic supply.

[15] Practically, this means that banks cannot lure in additional depositors by increasing the interest rates they offer to lenders. As explained below, this restriction means that banks will make positive profits on the contracts offered to particular wealth classes, like in Jaimovich (2011). This is, however, a Bertrand-Nash equilibrium as there are no profitable deviations in the set of feasible contracts. One could, alternatively, enlarge the set of feasible contracts and then either impose a Bertrand-Wilson's (1977) anticipatory equilibrium concept, as in Inci (2013), or allow banks two rounds of play, like in Hellwig (1987). In any case, the qualitative results of the model do not change. See Appendix A.2 for more detail.

banks earn non-negative profits at every wealth level, (ii) the machine and final good producers maximise their profits, (iii) the menu of contracts is a Bertrand-Nash equilibrium, (iv) the machines and final good markets clear, (v) individuals choose the occupation that maximises their expected end-of-period wealth, and (vi) talented individuals choose innovation if indifferent between the two occupations, whereas untalented individuals choose wage-earning when indifferent.

Note that this definition lacks market clearing conditions for the credit and labour markets, since I am analysing the partial equilibrium counterpart.

4.1 The Equilibrium Under Full Information

Suppose information about agents' talent were complete, so that in equilibrium, banks would charge an interest rate that accurately reflects an agent's intrinsic risk of failure. Since there cannot be any cross-subsidisation in an equilibrium without adverse selection, talented individuals become innovators, facing a rate of return equal to R_t/ρ_H, and an expected end-of-period payoff of $\mathcal{V}_t^{FB}(A_t, \Omega_t) = \rho_H \pi(\Omega_t) - R_t(I_t - A_t) - e_t$. Conversely, untalented agents become workers, with an end-of-period payoff of $\mathcal{W}_t^{FB}(A_t, \Omega_t) = w_t + R_t A_t$. Note that, by Assumption 1, the talent premium is positive, $\mathcal{V}_t^{FB}(A_t, \Omega_t) - \mathcal{W}_t^{FB}(A_t, \Omega_t) > 0$. As shown below, it reaches its maximum value when information asymmetries are absent.

Under full information, the average probability of success of the innovators is equal to ρ_H, and there are λ innovators each period. Thus, the expected number of successful innovations in any given period is given by $\rho_H \lambda$. Conversely, the expected number of unsuccessful innovations is $1 - \rho_H \lambda$. Since innovations increase the quality of the machines to $(1+\gamma)Q_{t-1}$, whereas failure leaves the quality equal to Q_{t-1}, the average quality of the machines increases over time with a constant growth rate of $g^{FB} = \gamma \rho_H \lambda$.[16]

4.2 Static Equilibrium

Below, I intuitively derive the static partial equilibrium of the model, i.e. the contracts offered and the subsequent occupational choices of the agents, taking the state of the economy and the wealth distribution as given. The proofs are formally given in Appendix A.2. For readability, I drop the time subscript.

Following the literature on adverse selection, one should expect two types of equilibrium contracts: pooling, in which types remain undistinguishable, or separating, in which types reveal their unobservable ability by selecting different terms. Here, one can

[16]Note that, in first best, all machines would be produced competitively, whereas here there is still an inefficiency due to the presence of monopoly rights granted to the successful innovators. The resulting underutilisation of machines can easily be corrected with a subsidy in the use of (new vintages of) machines, such that their net price is identical to the marginal cost.

easily exclude that there exists a separating loan contract, for a given wealth class, such that both talented and untalented agents become innovators. To see this, consider the following zero-profit conditions from separating contracts,

$$\rho_H \left(\pi\left(\Omega\right) - V_H^S\right) = R(I - A) \tag{11a}$$

$$\rho_L \left(\pi\left(\Omega\right) - V_L^S\right) = R(I - A), \tag{11b}$$

where the first line refers to talented agents, and the second one to untalented agents with wealth A. The implied levels of $\mathcal{V}_H$ and $\mathcal{V}_L$ suggest that this menu of contracts cannot be incentive compatible, as the untalented agents would always prefer the contract designed for the talented innovators.

Hence, an equilibrium contract must be either a pooling contract or a separating contract that only the talented type accepts.[17] In a zero-profit pooling contract, the repayment of a random borrower with wealth A in the success state is given by $D^S = R(I - A)/\bar{\rho}$. For a given state of the world Ω, an i-type agent would accept this contract if her participation constraint is satisfied,

$$\rho_i \left(\pi\left(\Omega\right) - \frac{R\left(I - A\right)}{\bar{\rho}}\right) - e_i \geq w + RA. \tag{12}$$

At the same time, talented agents would be willing to exert effort only if

$$\rho_H \left(\pi\left(\Omega\right) - \frac{R\left(I - A\right)}{\bar{\rho}}\right) - e_H \geq \rho_L \left(\pi\left(\Omega\right) - \frac{R\left(I - A\right)}{\bar{\rho}}\right). \tag{13}$$

Solving these for A reveals that talented agents exert effort with a pooling contract if their wealth is greater than a threshold A_e, and they enter the innovation sector if their wealth is greater than a threshold A_H. Conversely, untalented agents become innovators only if their wealth is lower than a threshold A_L. These thresholds are given by, respectively,

$$A_e \equiv I + \frac{\bar{\rho}\left(e - \left(\rho_H - \rho_L\right)\pi\left(\Omega\right)\right)}{R\left(\rho_H - \rho_L\right)} \tag{14a}$$

$$A_H \equiv \frac{\bar{\rho}(w + e) + \rho_H\left(RI - \bar{\rho}\pi\left(\Omega\right)\right)}{R\left(\rho_H - \bar{\rho}\right)} \tag{14b}$$

$$A_L \equiv \frac{\rho_L\left(\bar{\rho}\pi\left(\Omega\right) - RI\right) - \bar{\rho}w}{R\left(\bar{\rho} - \rho_L\right)}. \tag{14c}$$

Talented agents are only willing to accept a pooling contract if the amount they need to borrow is small: indeed, since banks underestimate their probability of success, they are obliged to subsidise the untalented agents. Conversely, untalented agents only accept

[17] It is easy to prove, given the probabilities of success, that it can never be the case that a loan contract, for a given wealth level, attracts the untalented agent but not the talented agent.

pooling contracts if they can enjoy large cross-subsidies.

Note that by Assumption 1, $A_L > 0$. Throughout this paper, I focus on the most interesting case by further assuming that $A_L > A_e > 0$: this ensures that there is both adverse selection and some poor talented workers in equilibrium. This in turn implies that $A_e > A_H$. Wherever A_H lies, for agents with $A < A_e$, this pooling contract cannot be offered: indeed, the average probability of success of the innovators in this class would be ρ_L, resulting in negative profits for the banks. As a consequence, the only contract that can be offered for these wealth levels is the one on the zero-profit condition from untalented innovation, with $D^S = R(I - A)/\rho_L$. Since everyone is treated as untalented, by Assumption 1 all agents in this wealth bracket prefer to become workers.

The other possibility is that, for a given wealth class, only the effort-exerting talented agents enter the innovation sector. A putative separating contract on the zero-profit condition entails $D^S_H = R(I-A)/\rho_H$: I refer to this contract as "the zero-profit separating contract". Obviously, this contract can be offered only if an untalented agent with the same wealth does not have any incentive to imitate the talented agent, i.e. if

$$w + RA \geq \rho_L \left(\pi(\Omega) - \frac{R(I - A)}{\rho_H} \right). \tag{15}$$

This condition requires that her initial wealth is higher than a threshold A_{HH} given by

$$A_{HH} \equiv \frac{\rho_L (\rho_H \pi(\Omega) - RI) - \rho_H w}{R(\rho_H - \rho_L)}, \tag{16}$$

where $A_{HH} \in (A_L, I)$ by Assumption 1.

Since for $A \in (A_L, A_{HH})$ the zero-profit separating contract above does not satisfy the incentive compatibility constraint of an untalented agent with identical wealth, these talented agents will have to receive a different contract. The solution involves raising the interest rate demanded of the talented agents in such a way that makes the untalented agents indifferent between entering the innovation sector and becoming workers. This is achieved by imposing $\rho_L \left(\pi(\Omega) - D^S_H\right) = w + RA$, or, equivalently, $D^S_H = \pi(\Omega) - (w + RA)/\rho_L$. I refer to this contract as "the profitable separating contract".

Given the set of contracts offered to each wealth class, it is easy to derive the resulting occupational choices of the agents. Proposition 1 outlines the static partial equilibrium that ensues.

Proposition 1 (Static partial equilibrium)**.** *Banks offer contracts on the zero-profit condition from untalented innovation to agents with wealth in $[0, A_e]$, pooling contracts to agents in $[A_e, A_L]$, profitable separating contracts to agents in $[A_L, A_{HH}]$, and zero-profit separating contracts to agents in $[A_{HH}, I]$. This is associated with the following occupational choices: all agents in $[0, A_e]$ become workers, all agents in $[A_e, A_L]$ become innovators, talented agents with $A \geq A_L$ become innovators, whereas their untalented* *counterparts become workers.*

4.2.1 Wealth Classes

Proposition 1 underlines that we can split the population into different pools of borrowers depending on their wealth level. Indeed, the contractual structure of the lending market endogenously introduces four wealth classes, that I label working, lemons, rich, and unconstrained.

The working-class agents have wealth between $[0, A_e]$. Given the size of the loan that they would need, talented agents do not apply for loans, and thus the only offer banks can make is an interest rate of R/ρ_L. As a consequence, every agent in this class becomes a worker, with an end-of-period wealth of $w + RA$. The lemons-class agents have wealth in $[A_e, A_L]$. Banks offer only pooling contracts, with an interest rate of $R/\bar{\rho}$, and both types of agents become innovators. The expected end-of-period wealth of an i-type agent in this class is $\rho_i\left(\pi - R(I - A)/\bar{\rho}\right)$. The rich-class agents have wealth in $[A_L, A_{HH}]$. Banks can offer separating contracts to agents in this class, but with an interest rate that is slightly higher than the one consistent with the risk profile of the talented innovators.[18] Given the terms of the optimal contract, the expected end-of-period wealth of a talented agent in this class is $\rho_H\left(w + RA\right)/\rho_L$. Finally, the unconstrained-class agents have $A \geq A_{HH}$. Only talented agents in this wealth class become innovators, with an expected income of $\rho_H\pi - R(I - A)$, whereas untalented agents become workers.

Denote by $U_i(A, \Omega)$ the expected income level achieved by an i-type with wealth A in an economy with state Ω. From the end-of-period wealth of the agents in the static partial equilibrium, this lemma follows.

Lemma 1. *Let* $\Delta(A, \Omega) \equiv U_H(A, \Omega) - U_L(A, \Omega)$. *Then: (i)* $\Delta(\cdot) \geq 0$, $\forall A$. *Moreover, (ii)* $\Delta'_A(\cdot) = 0$, $\forall A \in (0, A_e)\bigcup(A_{HH}, \infty)$,*; (iii)* $\Delta'_A(\cdot) > 0$, $\forall A \in [A_e, A_{HH}]$,*; (iv)* $\Delta(A, \Omega) \equiv \mathcal{V}^{FB}(A, \Omega) - \mathcal{W}^{FB}(A, \Omega) + e$, $\forall A \geq A_{HH}$. *Furthermore, (v)* $\Delta'_Q(\cdot) = 0$, $\forall A \in (0, A_e)\bigcup(A_L, A_{HH})$*; (vi)* $\Delta'_Q(\cdot) > 0$, $\forall A \in [A_e, A_L]\bigcup(A_{HH}, \infty)$.

The talent premium, $\Delta(A, \Omega)$, is weakly increasing in wealth and machines' quality, and only reaches its full information counterpart in the unconstrained class. This means that talented agents benefit more from an increase in wealth and/or average quality than untalented agents.

Three recent empirical studies have found that an individual's propensity of becoming an innovator increases with parental resources (Akcigit *et al.*, 2017, Aghion *et al.*, 2018, Bell *et al.*, 2019). Here, one could split the population in two groups: one consisting of working- and lemons-class agents, and one with rich- and unconstrained-class agents. The propensity of becoming an innovator of the former group would be

[18] Precisely, banks ask for an interest rate of $\left(\rho_L\pi - w - RA\right)/\left(\rho_L\left(I - A\right)\right)$ on these loans.

$[\Phi(A_L) - \Phi(A_e)]/\Phi(A_L)$, whereas the propensity of the latter group would be equal to $\lambda[1 - \Phi(A_L)]$: therefore, this empirical prediction can be matched for certain parameter values.[19] Moreover, Bell *et al.* (2019) find that inventors from unconstrained groups are, on average, of higher talent than from the discriminated group, which is consistent with this paper.

4.2.2 Number of Innovators and Equilibrium Growth

Proposition 1 implies that the number of talented, n_H, and untalented innovators, n_L, in the static partial equilibrium are given by, respectively,

$$n_H(\Omega) = \lambda[1 - \Phi(A_e(\Omega))] \tag{17a}$$

$$n_L(\Omega) = (1-\lambda)[\Phi(A_L(\Omega)) - \Phi(A_e(\Omega))]. \tag{17b}$$

The total number of innovators and their average quality are, respectively,

$$n(\Omega) = n_H(\Omega) + n_L(\Omega) \tag{18a}$$

$$\rho(\Omega) = \frac{\rho_H n_H(\Omega) + \rho_L n_L(\Omega)}{n(\Omega)}. \tag{18b}$$

The expected number of successful innovations is given by ρn: for each of these machines, initial quality improves by a factor $1+\gamma$. Conversely, the expected number of unsuccessful innovations is $1 - \rho n$: for these machines, quality does not increase. As a consequence, in the static partial equilibrium, the growth rate of average quality is given by $g = \gamma\rho n$. The following lemma outlines some comparative statics of the static partial equilibrium.

Lemma 2. *(i) Consider two identical economies, but for the expected reward for innovation, such that $\pi > \pi'$. Then $n_H \geq n'_H$ and $n_L \geq n'_L$, therefore $g \geq g'$; (ii) Consider two identical economies but for the initial wealth distributions, $\Phi(A)$ and $\Phi'(A)$, such that $\Phi(A)$ first-order stochastically dominates $\Phi'(A)$. Then $n_H \geq n'_H$.*

The intuition for Lemma 2 is straightforward. Part (i) exploits the fact that, whereas the wealth threshold A_e is strictly decreasing in the profit of the successful innovator, A_L (and A_{HH}) is strictly increasing in it: the economy with a higher reward for innovation (both absolutely and relatively to the wage) will have a larger lemon-class and a smaller working-class, and thus more innovators. Since the growth rate is increasing in the number of innovators, this economy grows faster, even if some of the additional innovators are

[19]These studies, however, find that the relationship is highly non-linear and particularly steep at high levels of parental resources: the prediction of the model would be improved by allowing for e.g. more types of agents, risk aversion, variable project size, and correlation between wealth and ability, at the cost of added complexity.

of low ability.[20] Among other things, this implies that more technologically advanced economies grow faster, as the expected reward of the innovator is increasing in the average quality of the machine. Part (ii) says that, other things equal, wealthier economies tend to have more talented innovators. This is because, as the economy becomes wealthier, more agents will find themselves in the upper classes, where the adverse selection problem turns into an efficient redistribution (rich-class) or disappears (unconstrained-class).[21]

4.2.3 Innovation and Inequality of Opportunity

Here, I focus on the effect of the initial level of inequality on the static partial equilibrium. In particular, I consider two identical economies but for the initial wealth distributions, such that one is obtained through a single mean-preserving spread of the other (á la Rothschild and Stiglitz, 1970). Using a mean-preserving spread amounts to ranking distributions with the same average (and, here, total) wealth by second-order stochastic dominance. Since Atkinson (1970), second-order stochastic dominance has become a standard way in which to rank distributions in terms of inequality. Indeed, it is equivalent to (generalized) Lorenz dominance, the most commonly used ordering in the literature on the comparisons of income and wealth distributions.

Lemma 3. *Consider two identical economies but for the initial wealth distributions, $\Phi(A)$ and $\Phi'(A)$, such that $\Phi'(A)$ is obtained by a single mean-preserving spread of $\Phi(A)$: thus, $\Phi(A)$ crosses $\Phi'(A)$ only once, and from below. Denote this crossing as $\tilde{A}$. Then, (i) if $\tilde{A} < A_e$, $n_H < n'_H$; (ii) if $\tilde{A} = A_e$, $n_H = n'_H$ and $n_L > n'_L$; (iii) if $A_e < \tilde{A} < A_L$, $n_H > n'_H$ and $n_L > n'_L$; (iv) if $A_L < \tilde{A}$, $n_H > n'_H$.*

Indirectly, lemma 3 establishes the presence of a threshold for average wealth above which equality-enhancing redistributions are always associated with an increase in the number of talented innovators. Below this threshold, the contrary holds true: more inequality is beneficial when the economy is poor, since it allows at least some (talented) individuals to overcome setup costs that are large in relations to average wealth (an effect already stressed by e.g. Barro, 2000). The exact position of this threshold depends on the

[20]Indeed, it is possible for the growth rate in the constrained equilibrium to be greater than the growth rate under full information. This happens if $\rho_H(\lambda - n_H) - \rho_L n_L$, a measure of the cost of having displaced the poor talented innovators, is negative. That having too many innovators may hurt the economy becomes clearer when we consider total output. Since the number of successful innovations corresponds to the number of monopolistically produced machines with demand given by (7), whereas the remaining machines have demand given by (9), the production of final good in the static partial equilibrium is equal to $y = l\,(\alpha p/\psi)^{\frac{\alpha}{1-\alpha}}\,\hat{Q}$, where $\hat{Q} \equiv \left(\rho n \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) Q$ is the average corrected quality of the machines at the end of the period, which takes into account that certain machines are produced competitively and others monopolistically. Whereas Q is monotonically increasing in ρn, $\hat{Q}$ is hump-shaped.

[21]Whether this is also associated with fewer untalented innovators depend on the particular wealth distribution i.e. on the relative flows of agents in and out of the lemon class. In a simplified version of the model with no effort, and thus no working class, the number of untalented innovators would always decrease as the economy gets richer.

particular wealth distribution. In many commonly used income and wealth distributions subject to a single mean-preserving spread, like shifted Pareto and Lognormal, the single-crossing point is not smaller than the mean, $\tilde{A} \geq \bar{A}$, and the distance $\tilde{A} - \bar{A}$ is weakly increasing in the mean-preserving spread. Thus, as long as the average agent is not among the poorest individuals of the working class, in these distributions, more equal wealth distributions are always associated with more talented innovators.

4.3 Dynamics

The analysis in the previous section has been conducted within a static framework, as the quality of the machines and the wealth distribution at the beginning of the period were taken as given. Since these are actually endogenous, and reciprocally influence each other over time, here I present the dynamics of Q_t and $\Phi_t(A)$.

Given the utility function in (1), individuals will optimally bequeath a fraction δ of their end-of-period income to their offspring. This amount will in turn fully determine the initial wealth of the new individuals. Henceforth, I split the population of agents in lineages indexed by $h \in [0, 1]$. Since types are intergenerationally uncorrelated by assumption, the wealth transition equations for any lineage are given by

$$A_{t+1,h} = \delta\left[w_t + RA_{t,h}\right] \qquad \text{if } A_{t,h} < A_{t,e};$$

$$A_{t+1,h} = \begin{cases} \delta\left[\pi(\Omega_t) - R_t(I_t - A_{t,h})/\bar{\rho}\right], & \bar{\rho} \\ 0, & 1-\bar{\rho} \end{cases} \qquad \text{if } A_{t,h} \in [A_{t,e}, A_{t,L}];$$

$$A_{t+1,h} = \begin{cases} \delta\left[\left(w_t + R_t A_{t,h}\right)/\rho_L\right], & \lambda\rho_H \\ 0, & \lambda(1-\rho_H) \\ \delta\left[w_t + R_t A_{t,h}\right], & (1-\lambda) \end{cases} \qquad \text{if } A_{t,h} \in [A_{t,L}, A_{t,HH}];$$

$$A_{t+1,h} = \begin{cases} \delta\left[\pi(\Omega_t) - R_t(I_t - A_{t,h})/\rho_H\right], & \lambda\rho_H \\ 0, & \lambda(1-\rho_H) \\ \delta\left[w_t + R_t A_{t,h}\right], & (1-\lambda) \end{cases} \qquad \text{if } A_{t,h} \in [A_{t,HH}, I_t];$$

$$A_{t+1,h} = \begin{cases} \delta\left[\pi(\Omega_t) + R_t(A_{t,h} - I_t)\right], & \lambda\rho_H \\ \delta\left[R_t(A_{t,h} - I_t)\right], & \lambda(1-\rho_H) \\ \delta\left[w_t + R_t A_{t,h}\right], & (1-\lambda) \end{cases} \qquad \text{if } A_{t,h} \geq I_t.$$

The dynamic path of the economy is dictated by the following system:

$$Q_t = (1 + \gamma\rho_{t-1}n_{t-1})Q_{t-1} \tag{20a}$$

$$\Phi_t(A) = \Gamma_{t-1}\left[\Phi_{t-1}(A)\right], \tag{20b}$$

where the operator $\Gamma_{t-1}[\cdot]$ maps the wealth distribution in $t-1$ into the initial wealth distribution in t, given the transition equations above. This operator evolves over time, as the transition equations depend on the average quality of the machines. The dynamic evolution of Q_t, in turn, depends on the wealth distribution, through the occupational choices of the agents. As a consequence, the dynamic system in (20) is non-stationary, and thus complicated to study analytically. Nevertheless, under certain conditions, I can show the existence of a balanced growth path equilibrium.

4.3.1 Balanced Growth Path Equilibrium

Growth in this economy is driven by the improvements in the machines done by successful innovators. In this section, I focus on the balanced growth path equilibrium where aggregate variables grow at the constant rate $g = \gamma\rho n$. In order to do so, I make the following simplifying assumptions: the initial investment is a linear function of the average quality of the machines, $I_t = \iota Q_t$; the effort cost is given by $e_t = \epsilon\left(\bar{\rho}\pi_t - RI_t - w_t\right), \forall t$, where ϵ is a function of parameters (this ensures that $A_{t,L} > A_{t,e} > 0, \forall t$, as shown in Appendix B.4); the interest rate is constant, $R_t = R, \forall t$; and the number of workers and the price of the final good are normalised to one, $l_t = 1$, and $p_t = 1, \forall t$. In Section D, I let markets determine R_t and w_t as well, and provide numerical results.

Proposition 2 (Balanced Growth Path)**.** *The balanced growth path equilibrium of the economy has the following form: (i) the number of innovators and their average quality are time-invariant, i.e. $n_t = n$ and $\rho_t = \rho, \forall t$, (ii) aggregate variables $\bar{A}_t, Q_t$, and y_t, and rewards w_t and π_t grow at constant rate $g = \gamma\rho n$.*

There are three generic cases depending on the equilibrium number of innovators and their average talent: (i) a steady-state with no innovators and zero growth rate, (ii) an equilibrium in which agents do not need to borrow, and thus the growth rate is the same as under perfect information, and (iii) an equilibrium with both talented and untalented innovators, and a positive growth rate.

The first steady state represents a *no innovation trap.* For this situation to arise, it must be the case that, in a given period, all agents are in the working class. Since there are no innovators, the average quality of the machines does not increase, and the wealth thresholds remain constant. If $\delta(w+RA_e) < A_e$, even the offspring of the wealthiest agent will find herself in the working class in the next period, and so in all future generations. In the long-run, wealth converges to a degenerate distribution, with $A_{t,h} = \delta w/(1-\delta R), \forall t, h$.

For the second case, consider a situation in which all agents are able to self-finance themselves in a given period. As explained previously, in such an economy agents self-select in the efficient occupations. The offspring of an unsuccessful innovator inherits a positive amount: depending on parameters, she can be wealthier than her parents and able to self-finance, and so are the offspring of workers and successful innovators. In this case, all future generations of agents are also able to self-finance themselves, and $g_t = g^{FB}, \forall t$.

In the last case, the number of innovators is positive, and thus the average quality of the machines grows at a positive rate. However, the flows of agents entering and exiting the lower classes match across generations, and thus misallocation of talent persists over time.

5 The Empirical Analysis

In this section, I run two empirical analyses. First, I show that the relationship between innovation and inequality depends on which type of inequality is considered. Then, I study the dynamic effects of changes in inequality on innovation.[22]

5.1 Innovation and Different Types of Inequality

In this section, I empirically analyse the relationship between inequality and innovation at the US state level. I first calculate various measures of inequality using the Panel Study of Income Dynamics (PSID). I then measure innovation in each state using data from the United States Patent and Trademark Office (USPTO). Finally, I empirically characterize the effect of inequality on innovation.

5.1.1 Inequality Measures

It is well-known that measuring inequality is empirically challenging (e.g. Keeley, 2015). Moreover, inequality measures are seldom comparable across countries. Data on wealth inequality, especially, is hard to come by; when available, it does not go back in time very far. On the contrary, data requirements to study the long-term effects of inequality are very stringent: one not only needs comparable measures of inequality but also information for at least two distant periods in time, generally ten years (e.g. Marrero and Rodríguez, 2013). Given these limitations, I carry the analysis at the US state level, using data from the PSID; moreover, instead of focusing on wealth inequality, I construct a more general measure of inequality of opportunity.

The PSID is the world's longest running household panel survey: it started in the 1968, with over 18,000 individuals living in 5,000 families in the United States, and it is still running. I use the weights supplied by the PSID to make the sample representative at the national level, by compensating for unequal selection probabilities and differential attrition.[23] My analysis, however, is run at the state level. Unfortunately, PSID samples

[22]The analysis is run using Stata 15 by StataCorp (2017) and the user-written program by Liao (2016c).

[23]For longitudinal consistency, I disregard the Latino sample, that was added to the PSID data only between 1990 and 1995.

may not be representable at the state level, and state sample sizes are small. To limit the impact of these problems, I drop states with fewer than 50 observations in a given year. This results in an unbalanced panel of 32 states distributed throughout the whole US territory: West (Arizona, California, Colorado, Oregon, Utah, Washington), South (Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, South Carolina, Tennessee, Texas, Virginia, plus District of Columbia), Midwest (Iowa, Illinois, Indiana, Michigan, Minnesota, Missouri, Ohio, Wisconsin), and Northeast (Connecticut, Massachusetts, New Jersey, New York, Pennsylvania).

Since some information is not available for wives in all waves, I restrict my attention to individuals who are household heads (male in married family unit, but also female otherwise). I only consider individuals aged between 18 and 65 at the time of the interview. I calculate my measures of inequality based on the labour income of the respondents, given that information about other sources of income is not consistently available in the PSID. To account for composition effect, I first regress gross labour income on a second-order polynomial of potential experience. I then collect residuals from these regressions, and since they are centered around zero, I add a constant to match the minimum of the series.

Recycling notation from the theoretical model, let x_i be the so-calculated gross income of individual $i = \{1, \ldots, N\}$ in a given state s and year t (for ease of reading, I drop these subscripts below), $\bar{x}$ the weighted mean income of the state-year sample, and f_i the sampling fraction of i in the state-year sample (i.e. i's sampling weight over the weight's sum for state s in year t). For each state s in year t, I estimate the corresponding Theil's T index. This is defined as

$$T = \sum_{i=1}^{N} f_i \frac{x_i}{\bar{x}} \ln\left(\frac{x_i}{\bar{x}}\right). \tag{21}$$

Theil (1967) argued that this measure of entropy, or degree of disorder, provides a useful device for measuring inequality.[24] Theil's measure has been widely used in social science: one reason for its popularity is that, unlike the Gini coefficient, the total amount of inequality can be additively decomposed into a between-group component and a within-group component (see e.g. Liao, 2016a and 2016b). For this purpose, I partition the individuals in a given state and year into a mutually exclusive and exhaustive set of types, based on their father's education (i.e. no education, primary, secondary, and tertiary education) and race (i.e. white and non-white). I thus obtain (up to) eight types: all individuals in each type m share the same *circumstances*. Race and parental education, as proxies for more general socio-economic background (e.g. wealth, transmission of ability and connections, investments in human capital) are the circumstances most widely used in the empirical literature. The between-group inequality component for a given state in

[24]Hereafter, Theil index refers to Theil's first measure, or Theil's T. I estimate Theil's T and its decomposition non-parametrically, like in Marrero and Rodríguez (2013), rather than parametrically, like in Ferreira and Gignoux (2011), given the structure of the database.

a given year is calculated as

$$T_b = \sum_{m=1}^{M} y_m \ln\left(\frac{\bar{x}_m}{\bar{x}}\right), \tag{22}$$

where y_m is type m's weighted income share expressed as a proportion of the weighted sample total income, and $\bar{x}_m$ is the weighted mean income of group m.

To summarise, T_b measures inequality due to differences between circumstances: since these are beyond the individual's control, T_b is used in the social sciences as a proxy for inequality of opportunity. Conversely, the within-group component, $T_w = T - T_b$, expresses inequality within groups, and is thus seen as a proxy for inequality due to individual's choices or effort (over which the individual has control). Since one can realistically control for only a limited set of circumstances, T_b is actually a lower-bound on the real inequality of opportunity (Ferreira and Gignoux, 2011, Marrero and Rodríguez, 2013).[25]

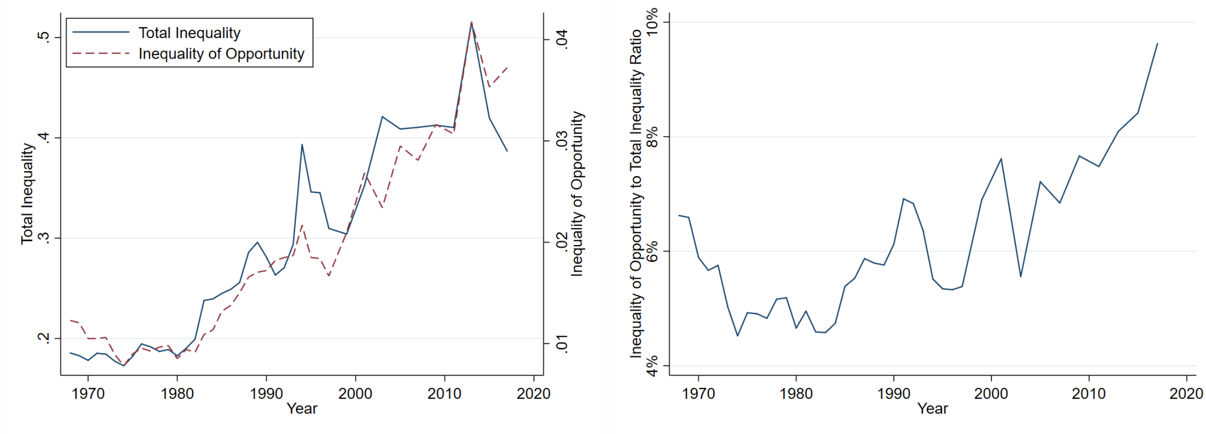
5.1.1.1 Inequality in the US

Figure 3a shows that both total inequality and inequality of opportunity at the US level were relatively stable up until the 1980s, while they have increased since then. The trend for total inequality is consistent with well-known facts, see e.g. US Census Bureau and Solt (2019). Figure 3b shows that the percentage of total inequality due to different opportunities (arising from race and parental education) is modest but not insignificant (similarly to e.g. Ferreira and Gignoux, 2011, for six countries in Latin America, Marrero and Rodríguez, 2012, for 23 European countries, and Marrero and Rodríguez, 2013, for 26 US states), and seems to have been moving upwards in recent decades.

Figure 4 shows 50-year average of the estimated inequality indexes for all available US states, sorted from the most to the least unequal. Figure 5 maps these values. Broadly speaking, these show that there are both groups of states whose positions remain basically unchanged across the different indexes and states that rank high on some index but low on another one. For example, Maryland, Florida, and New Jersey are at the top of all rankings, whereas Connecticut, Iowa, and Oregon are at the bottom. Conversely, Massachusetts, California, and Pennsylvania score relatively high on inequality of effort but low on inequality on opportunity, whereas the contrary holds for Louisiana and Alabama.

Consistently, Figure 6 shows the relationship between total inequality and the estimated inequality of opportunity index, and between inequality of effort and inequality of opportunity indexes, respectively, using decade averages for each state. Their coefficients of determination is 0.46 for the former relationship, positive but far from unity, and it

[25] Though luck plays an important role in the theoretical model, it is not considered in the decomposition of the Theil's index. See Lefranc *et al.* (2009) for a model of equality of opportunity that encompasses circumstances, effort, and luck, and how these can be empirically identified.



(a) Values (b) Ratio

Figure 3: Time Evolution of Inequality, US

Notes. The total inequality index (Theil's T) and its decomposition are calculated by the author using data from the Panel Study of Income Dynamics.

reduces to 0.30 for the latter one.

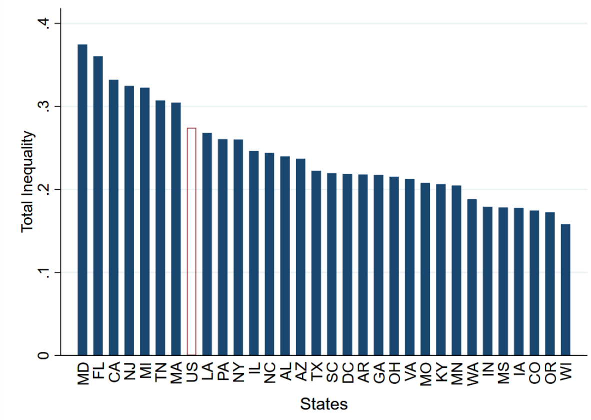
5.1.2 Innovation

My measure of innovation builds on patent data. A patent is an exclusionary right conferred for a set period to the patent holder, in exchange for sharing the details of the invention. In the US, the USPTO is the agency that issues patents. Since 1976, it has provided information on the state of residence of the inventors and citation links between individual patents.

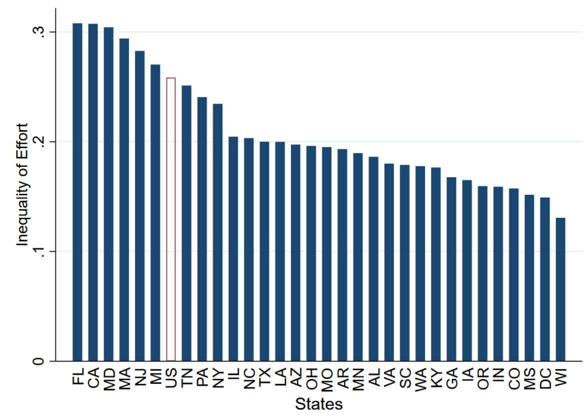
From the great amount of information available from the USPTO, Aghion *et al.* (2019, Supplementary Data) provide a ready-to-use dataset containing information on utility patents granted between 1976 and 2009 (up to 2006 when using quality-adjusted measures). In particular, for each state and year, they provide the flow numbers of patents, both as is and weighted for various proxies for a patent's quality, like the number of citations received.

As a measure of innovation, I use the number of patents granted, weighted by the number of citations received within 5 years of the application date, and corrected for the different propensity to cite in different sectors and time periods (Hall *et al.*, 2001). A patent is associated with the state of residence of the patent inventor; a patent is split proportionally across states if co-inventors live in different states.[26] The total number of patents in a state is then weighted by the number of residents. Figures 7 provides some visual representations.

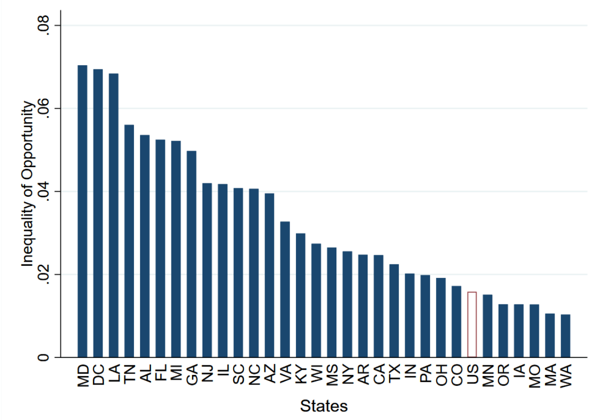
[26]See Aghion *et al.* (2019, Section 3.1.2) for more information. Results are similar when I use only the number of patents and when I adjust for different quality-measures: these are available on request.



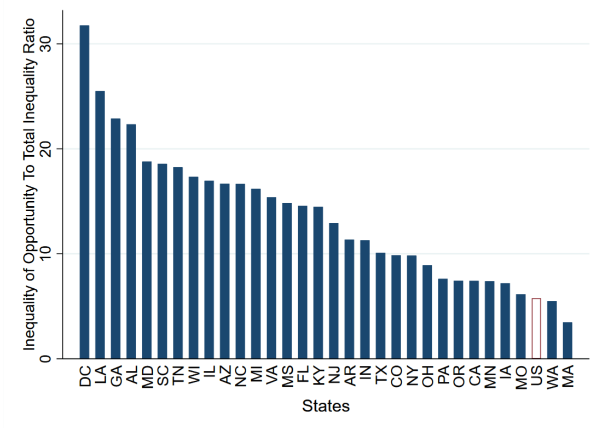
(a) Total Inequality



(b) Inequality of Effort



(c) Inequality of Opportunity



(d) Inequality of Opportunity Ratio (in %)

Figure 4: Inequality Rankings

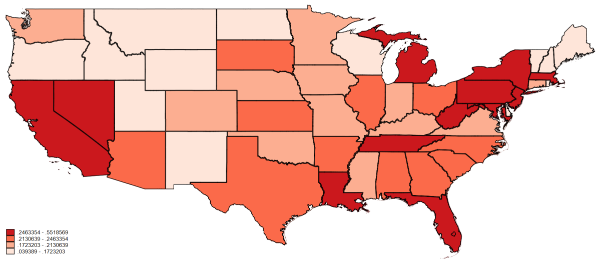
Notes. Inequality indexes are 1968 - 2017 averages of the decompositions of the Theil's T index calculated by the author using data from the Panel Study of Income Dynamics. Only years with more than 50 observations are used.

5.1.3 Results

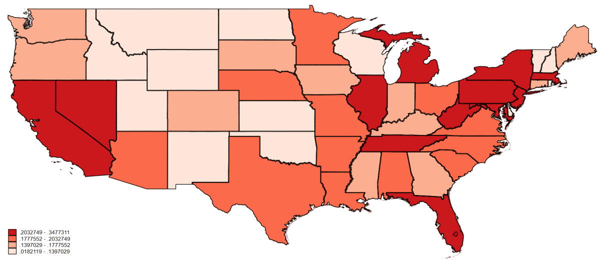
In this section, I look at the effect of inequality on innovation. My estimated equation is

$$log\left(innov_{i,t}\right) = \beta_1 log\left(ineq_{i,t-10}\right) + \beta_2 \boldsymbol{x}_{i,t-10} + \alpha_i + \epsilon_{i,t}, \tag{23}$$

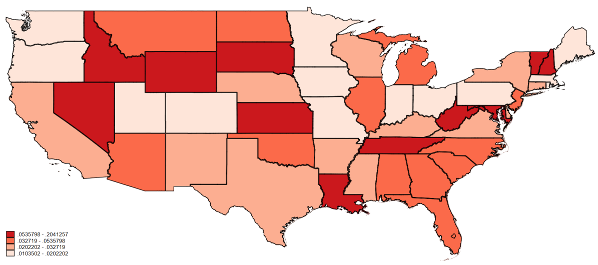
where $innov_{i,t}$ is the flow of (quality-adjusted) patents per capita in state i in year t, $ineq_{i,t-10}$ is a vector of inequality indexes in year $t-10$, and $\boldsymbol{x}_{i,t-10}$ is a vector of control variables. The error term has two components: $\epsilon_{i,t}$ is an idiosyncratic error, whereas α_i captures unobservable heterogeneity across states that is invariant across times. The vector of controls is parsimonious and includes only the unemployment rate (to control for the business cycle), lagged GDP per capita (in logs), the growth rate of total population, population density, the share of the manufacturing sector, and the size of the government sector: whereas the resulting estimation may suffer from omitted-variable bias problems,



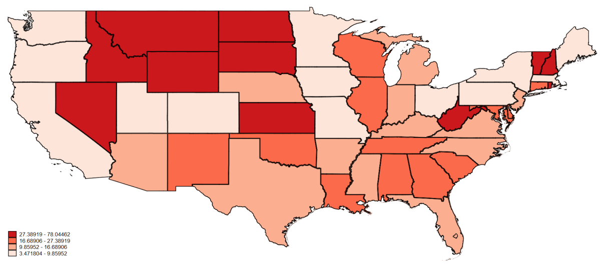
(a) Total Inequality



(b) Inequality of Effort



(c) Inequality of Opportunity



(d) Inequality of Opportunity Ratio

Figure 5: Geography of Inequality

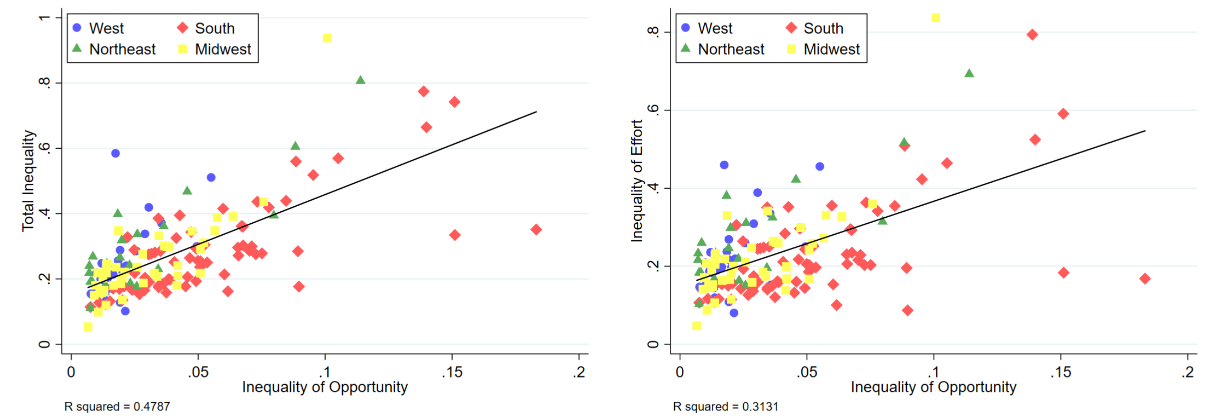
Notes. Inequality indexes are 1968 - 2017 averages of the decompositions of the Theil's T index calculated by the author using data from the Panel Study of Income Dynamics. Only years with more than 50 observations are used. For all these measures, darker colours represent relatively higher indexes.

I avoid introducing significant collinearity problems.

I first estimate equation (23) using Pooled OLS, and thus estimate the relationship between inequality and innovation across states. I then implement a within regression, and thus estimate the correlation between changes in innovation and changes in inequality within a given state.

Results are presented in Table 1, where the first two columns refer to the OLS estimation, whereas the second two columns refer to the FE estimation. In columns 2 and 4, I break down total inequality into the between-group component (the inequality of opportunity term) and the within-group component (the inequality of effort term). By including the inequality of opportunity term, I control for the observed circumstances, i.e. father's education and race. Whereas most terms are insignificant when I employ FE,[27] in the OLS framework the coefficient of the between component is strongly significant and negative, whereas the within-group term is associated with a significantly positive effect. In particular, a one standard deviation increase in the inequality of opportunity measure is associated with a 43 point decrease in the measure of innovation; conversely, a one standard deviation increase in the within component is associated with a 21 point

[27] Whereas OLS ignore the error structure, the fixed effect technique is problematic because it relies mostly on within-state variability. Panizza (2002) suggests regressing inequality on time and state dummies, and use the resulting R-squared measure as a proxy of within-state and within-period variability. R-squareds around to 0.30 indicate that indeed the inequality measures mostly vary cross-sectionally. This low within-state variability exacerbates the measurement error of the within regression (Panizza, 2002).



(a) Values (b) Ratio

Figure 6: Inequality by US Regions

Notes. Inequality indexes are averages for the 1970s, 1980s, 1990s, 2000s, and 2010s of the decompositions of the Theil's T index calculated by the author using data from the Panel Study of Income Dynamics (from 1968 to 2017). The last panel is the ratio of inequality of opportunity to total inequality (in %). Only years with more than 50 observations are used.

increase.[28] These effects are hidden behind a positive, but smaller in magnitude and insignificant, coefficient when I consider only total inequality.

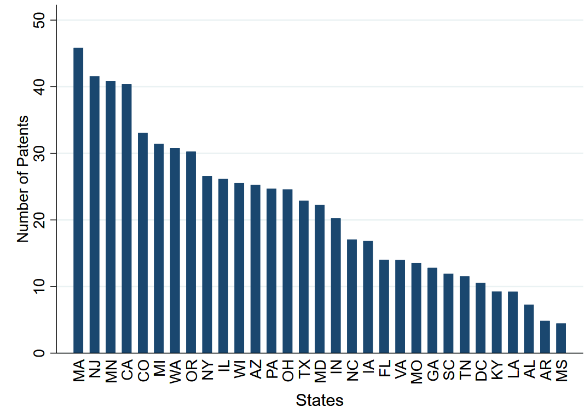
5.2 An Event Study Using State Death Taxes

In this section, I study the dynamic effects of a change in inequality on innovation. In particular, I use an event study to measure the effect of the elimination of estate, inheritance, and gift (henceforth, EIG) taxes at the US state level on the number of patents granted in that state.[29]

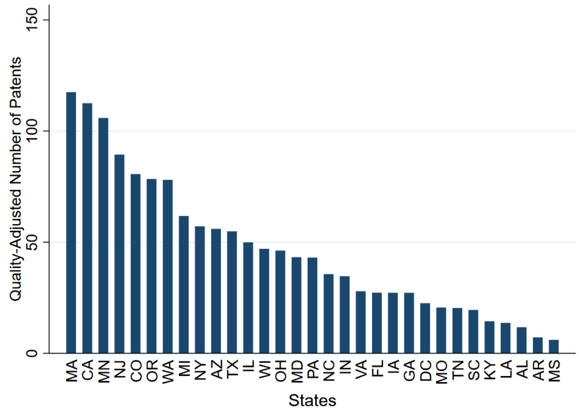
I take the elimination of EIG taxes as an imperfect proxy for a state level change in the political attitudes to economic inequality. Indeed, the legal literature in favour of inheritance and estate taxes proposes equality of opportunity as a guiding principle (see e.g. Gross *et al.*, 2017), whereas critics of the "death" taxes argue that they have disincentive effects toward risky activities (see e.g. Fleenor and Foster, 1994). As a consequence, I assume that the elimination of EIG taxes entails a short-term increase in inequality of effort and a long-term increase in inequality of opportunity: I thus hypothesise, following the elimination of EIG taxes, a short-term increase but a long-term decrease in innovation.

[28] These results include only those states that have at least 50 observations when calculating the Theil's indexes, but the results are robust to selection criteria equal to 20, 30, or 100 observations. Moreover, signs and significance are robust to changes in the lags of the regressors to 5, 15, or 20 years.

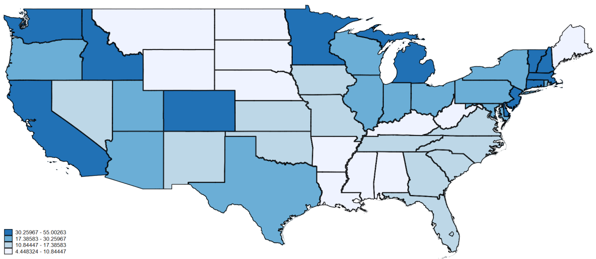
[29] There is a growing empirical literature that exploits variation in estate, inheritance, and gift taxes treatment to study the effect on e.g. entrepreneurial activity (Bruce and Mohsin, 2006), charitable donations (Bakija *et al.*, 2003, Conway and Rork, 2006), and migration (Bakija and Slemrod, 2004).



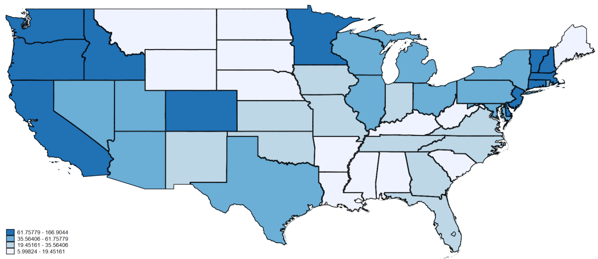
(a) Number of Patents



(b) Quality-Adjusted Number of Patents



(c) Number of Patents



(d) Quality-Adjusted Number of Patents

Figure 7: Innovation by US State

Notes. The left panels present 1976 - 2009 averages of the flow number of utility patents granted per 100k residents. The right panels present 1976 - 2005 averages of the flow number of patents per 100k residents, weighted by the number of citations received within 5 years of the application date, and corrected for the different propensity to cite in different sectors and time periods. A patent is associated with the state of residence of the patent inventor; a patent is split proportionally across states if co-inventors live in different states. Data elaborated by the authors from Aghion *et al.* (2019).

5.2.1 The Death of State Death Taxes

As explained by Conway and Rork (2004), five states (Alabama, Arkansas, Florida, Georgia, and Nevada) eliminated EIG taxes prior to 1960 or never had them, thus deciding to rely only on the so-called “pick-up” tax whereby individual states capture a fraction of the federal estate tax revenue without increasing the total tax liability of the estate. Since 1976, most of the remaining contiguous states have eliminated their EIG taxes in favour of only the pick-up tax.[30] Table 2 summarises the chronology of these events, which shows a high degree of time-series variation and a very limited geographical pattern. Because of limited data availability on patents for the most recent years, I focus my analysis on states that abolished EIG taxes before the year 1993.

[30]Nevada only took advantage of the pick-up tax in the late 1980s. Arizona effectively only had EIG taxes in place between 1977 and 1979.

Table 1: Innovation and Different Types of Inequality

	OLS		FE	
Total Inequality	0.23		0.05	
	(0.29)		(0.13)	
Inequality of Opportunity		-8.06***		0.35
		(0.92)		(0.45)
Inequality of Effort		2.12***		-0.01
		(0.32)		(0.15)
GDP per capita	3.49***	3.28***	-0.11	-0.13
	(0.18)	(0.18)	(0.19)	(0.18)
Unemployment	-0.08***	-0.07***	0.05***	0.05***
	(0.02)	(0.02)	(0.01)	(0.01)
Population Growth	1.88***	2.24***	-0.45*	-0.48
	(0.26)	(0.25)	(0.32)	(0.34)
Population Density	-0.00***	-0.00***	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Manufacturing Sector	2.82***	2.99***	2.10***	2.09***
	(0.41)	(0.38)	(0.62)	(0.63)
Government Sector	0.12***	0.16***	-0.07	-0.07
	(0.03)	(0.03)	(0.05)	(0.04)
N	819	819	818	818
R^2	0.64	0.68	0.90	0.90

The dependent variable is the log of (citations-adjusted) per capita number of patents
Cluster robust standard errors are reported in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
A constant and time dummies are included

5.2.2 Strategy

I take the event as being the effective elimination of EIG taxes at the state level. Recycling previously used notation, let t represents the year of the event. For each state i, I calculate the growth rate of the flow number of patents granted in t with respect to the previous year, $g_{i,t}$. To appraise the event's impact, I measure the difference between the actual growth rate in state i and year t and the annual growth rate in utility patent grants at the US level, $g_{US,t}$. I use data from Aghion *et al.* (2019) for the former, and from United States Patent and Trademark Office (2016) for the latter. For state i and year t, I thus define the abnormal growth rate as

$$ag_{i,t} = g_{i,t} - g_{US,t}. \tag{24}$$

Abnormal growth rates are then indexed in event time using τ, where $\tau = 0$ indicates the event time. I aggregate abnormal growth rates through times but within states.

Table 2: Year of Elimination of State EIG Taxes

Date	States	Date	States
Prior to 1976	Alabama, Arkansas, Florida, Georgia, Nevada	1988	Idaho
1976	New Mexico	1991	Rhode Island
1977	Utah	1992	South Carolina, Wisconsin
1979	North Dakota	1993	Michigan
1980	Arizona, Colorado, Vermont, Virginia	1997	Massachusetts
1981	Missouri	1998	Kansas
1982	California, Washington	1999	Delaware, North Carolina
1983	Illinois, Texas, Wyoming	2000	Mississippi, New York
1985	West Virginia	2001	Montana, South Dakota
1986	Maine, Minnesota	2003	New Hampshire
1987	Oregon	2004	Louisiana
		2005	Connecticut

Source: Conway and Rork (2004).

Define $cag_i(\tau_1, \tau_2)$ as the cumulative abnormal growth rate from τ_1 to τ_2 in state i, i.e. the sum of the included abnormal growth rate,

$$cag_i(\tau_1, \tau_2) = \sum_{\tau=\tau_1}^{\tau_2} ag_{i,\tau}. \tag{25}$$

I study these cumulative abnormal growth rate over different periods, or event windows: I consider a short-term event window around and immediately after the event, i.e. $\tau_1 = 0$ and $\tau_2 = 5$; and a long-term window, which covers the ten-year interval from $\tau_1 = 10$ to $\tau_2 = 19$. I construct a standard test statistic by dividing the cumulative absolute growth rate in state i by an estimate of its standard deviation (i.e. the sample standard deviation): this statistic is assumed to be unit normal in the absence of abnormal performances (MacKinlay, 1997).

5.2.3 Results and Limitations

Results are summarised in Table 3. The results of this analysis support the hypothesis that rising inequality, here proxied by the elimination of the EIG taxes, has short-term positive effects on innovation, but long-term negative effects. Out of our sample of 19 states, almost all of them experienced a faster than average increase in the growth rate of patents granted in the five-year windows following the event (with a significant positive change for more than 53% of them). However, ten years after the event, patents were growing at a slower rate than the national average for almost 75% of the states in the sample (37% of all states in the sample experienced a significant slow-down).

Table 3: Cumulative Abnormal Growth Rates

State	Short-Term		Long-Term	
	cag_i	sd_i	cag_i	sd_i
California	0.510*	(0.133)	-0.657*	(0.149)
Colorado	0.066	(0.226)	-0.545	(0.183)
Idaho	2.810***	(0.489)	-1.054***	(0.137)
Illinois	0.440***	(0.072)	-0.822*	(0.160)
Maine	1.273*	(0.340)	-0.600	(0.283)
Minnesota	0.612*	(0.181)	-0.820***	(0.092)
New Mexico	-0.005	(0.561)	-0.210	(0.208)
North Dakota	0.246	(0.392)	1.488	(0.566)
Oregon	0.709*	(0.225)	-0.756*	(0.185)
Rhode Island	1.257**	(0.310)	-0.661**	(0.232)
South Carolina	0.417**	(0.090)	0.009	(0.000)
Texas	0.645***	(0.055)	-0.984**	(0.194)
Utah	-0.242	(0.183)	-0.153	(0.213)
Vermont	1.031	(0.443)	0.451	(0.318)
Virginia	-0.036	(0.116)	-0.652	(0.176)
Washington	0.616	(0.222)	-0.238	(0.190)
West Virginia	1.043**	(0.313)	-0.266	(0.447)
Wisconsin	0.250	(0.133)	-0.377	(0.000)
Wyoming	2.165	(0.929)	0.028	(0.516)

cag_i stands for cumulative abnormal growth rate in state i.
sd_i stands for standard deviation in state i.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ based on one-side tests.

This analysis is only suggestive, in part because the elimination of EIG taxes is a very imprecise proxy for changes in the level of inequality in a state. Moreover, the date of the effective elimination may not be the best indicator: first, as noted by Conway and Rork (2004), many states reduced their EIG taxes during the period analysed here even if they did not eliminate them; second, it is likely that there was a significant lag between the time at which the decision was made and when the change become effective; finally, many of these changes were clustered around major changes in the federal tax law.

Whereas we have analysed the cumulative abnormal growth rate at the state level, one would usually calculate an average of those to see if the so-obtained abnormal growth rate is statistical different from zero. Here, the resulting p-value is 0.001 for the short-term effect and 0.016 for the long-term one, clearly indicating that the cumulative abnormal growth rate were significantly different from zero. Unfortunately, such test is based on the assumption that there is no overlaps in the event windows of the included states, which is clearly not satisfied here (but see MacKinlay, 1997, for some possibile workarounds).

6 Conclusions

Is inequality good or bad for innovation? In this paper, I have argued, both theoretically and empirically, that to study the relationship between inequality and innovation is important to distinguish between inequality stemming from circumstances beyond the individual's responsibility, like socio-economic background, and inequality caused by individual responsible choices, like the level of effort exerted. These two types of inequality, indeed, are likely to influence innovation in opposite ways. Moreover, I have underlined that a dynamic perspective is needed, as these two types of inequality influence each other over time through intergenerational linkages.

I have offered an endogenous growth model, with the novelty that agents differ in observable wealth and unobservable ability. Due to credit market frictions, inequalities in wealth translate into unequal opportunities that lead to a misallocation of talent: poor talented agents are displaced by relatively wealthier but untalented inventors. I have shown that the number of innovations in any given period positively depends on the relative reward to a successful inventor, but may negatively depend on the degree of inequality in the distribution of opportunity. Dynamically, an intertemporal trade-off emerges: inequality in the reward scheme incentivises innovation in any given period, but in the presence of bequests this translates into a more unequal distribution of opportunity, which hampers innovation in the future.

I have also provided indicative evidence in support of these theoretical predictions. I have found that whereas the quality-adjusted number of patents in a given year and US state is uncorrelated with total income inequality, this is the result of a negative correlation with inequality of opportunity and a positive correlation with inequality of effort. Moreover, I have shown that, following the elimination of state estate, inheritance, and gift taxes, most US states experienced a positive abnormal growth rate in the number of patents granted in the short-run (perhaps due to an increase in the incentives), but a negative one in the long-run (perhaps due to more unequal opportunities).

I have made many simplifying assumptions to keep the model tractable, e.g. (i) I assumed that abilities and wealth are uncorrelated, and thus that abilities are also intergenerationally uncorrelated; (ii) I have taken the riskless interest rate and the wage rate as exogenously given, thus abstracting from important sources of general equilibrium effects (see e.g. Grüner, 2003, Inci, 2013, Spiganti, 2018); (iii) I considered only two types of agent, whereas introducing more types may lead to different policy implications. Moreover, the empirical analysis lacks a number of important features: (i) I have not controlled for conditional convergence across states; (ii) due to data limitations, I have only controlled for two circumstances (parental education and race) and thus nothing can be said about other sources of inequality of opportunity and, at the same time, the measure of inequality of effort still contains a certain amount of inequality of opportunity; (iii) results may suffer from omitted variable bias,[31] and, in general, endogeneity problems may be present; (iv) standard errors are not robust to autocorrelation. I leave these interesting extensions to future research.

[31]For example Akcigit *et al.* (2017) argue that financial development is an important determinant of innovation; as explained by Panizza (2002), one should compare the results obtained here with those resulting from a model with more covariates, which may suffer from collinearity problems.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hémous. (2012), The environment and directed technical change, *American Economic Review 102*, 131 – 166.

Acemoglu, D., J. A. Robinson, and T. Verdier. (2017), Asymmetric growth and institutions in an interdependent world, *Journal of Political Economy 125*, 1245 – 1305.

Aghion, P., U. Akcigit, A. Hyytinen, and O. Toivanen. (2018), The social origins and IQ of inventors, Mimeo.

Aghion, P., and P. Howitt. (2009), *The Economics of Growth*, MIT Press, Cambridge.

Aghion, P., and J. Tirole. (1994), The management of innovation, *Quarterly Journal of Economics 109*, 1185 – 1209.

Aghion, P., U. Akcigit, A. Bergeaud, R. Blundell, and D. Hémous. (2019), Innovation and top income inequality, *Review of Economic Studies 86*, 1 – 45.

Aghion, P., and P. Howitt. (1992), A model of growth through creative destruction, *Econometrica 60*, 323 – 351.

Akcigit, U., J. Grigsby, and T. Nicholas. (2017), The rise of American ingenuity: Innovation and inventors of the golden age, Mimeo.

Andreoni, J. (1989), Giving with impure altruism: Applications to charity and Ricardian equivalence, *Journal of Political Economy 97*, 1447 – 1458.

Andreoni, J. (1990), Impure altruism and donations to public goods: A theory of warm-glow giving, *The Economic Journal 100*, 464 – 477.

Atkinson, A. B. (1970), On the measurement of inequality, *Journal of Economic Theory 2*, 244 – 263.

Bakija, J. M., W. G. Gale, and J. B. Slemrod. (2003), Charitable bequests and taxes on inheritances and estates: Aggregate evidence from across states and time, *The American Economic Review 93*, 366 – 370.

Bakija, J., and J. Slemrod. (2004), Do the rich flee from high state taxes? Evidence from federal estate tax returns, NBER Working Paper No. 10645. Cambridge, MA: National Bureau of Economic Research.

Banerjee, A. V., and A. F. Newman. (1993), Occupational choice and the process of development, *Journal of Political Economy 101*, 274 – 298.

Barro, R. (2000), Inequality and growth in a panel of countries, *Journal of Economic Growth 5*, 5 – 32.

Barro, R., and X. Sala-I-Martin. (1995), *Economic Growth*, McGraw Hill, New York.

Becker, G. (1993), *Human Capital*, The University of Chicago Press, Chicago.

Bell, A., R. Chetty, X. Jaravel, N. Petkova, and J. Van Reenen. (2019), Who becomes an inventor in America? The importance of exposure to innovation, *Quarterly Journal of Economics 134*, 647 – 713.

Bruce, D., and M. Mohsin. (2006), Tax policy and entrepreneurship: New time series evidence, *Small Business Economics 25*, 409 – 425.

Brunori, P., F. H. G. Ferreira, and V. Peragine. (2013), Inequality of opportunity, income inequality, and economic mobility: Some international comparisons, In *Getting Development Right: Structural Transformation, Inclusion, and Sustainability in the Post-Crisis Era.* Ed. by E. Paus. Palgrave Macmillan, New York.

Caselli, F., and N. Gennaioli. (2013), Dynastic management, *Economic Inquiry 51*, 971 – 996.

Celik, M. (2018), Does the cream always rise to the top? The misallocation of talent in innovation, Mimeo.

Chetty, R., J. Friedman, N. Hendren, M. R. Jones, and S. R. Porter. (2018), The Opportunity Atlas: Mapping the childhood roots of social mobility, NBER Working Paper No. 25147. Cambridge, MA: National Bureau of Economic Research. `https://www.opportunityatlas.org/`.

Clementi, F., and M. Gallegati. (2016), *The Distribution of Income and Wealth. Parametric Modeling with the κ-Generalized Family*, Springer International Publishing, New York.

Conway, K. S., and J. C. Rork. (2004), Diagnosis murder: The death of state death taxes, *Economic Inquiry 42*, 537 – 559.

Conway, K. S., and J. C. Rork. (2006), State "Death" taxes and elderly migration: The chicken or the egg?, *National Tax Journal 59*, 97 – 128.

DeMeza, D., and D. Webb. (1987), Too much investment: A problem of asymmetric information, *Quaterly Journal of Economics 102*, 281 – 292.

Ferreira, F., and J. Gignoux. (2011), The measurement of inequality of opportunity: Theory and an application to Latin America, *Review of Income and Wealth 57*, 622 – 657.

Fleenor, P., and J. D. Foster. (1994), An analysis of the disincentive effects of the estate tax on entrepreneurship, Tax Foundation Background Paper No. 9, Washington, DC: Tax Foundation.

Foellmi, R., and J. Zweimüller. (2006), Income distribution and demand-induced innovations, *Review of Economic Studies 73*, 941 – 960.

Foellmi, R., and J. Zweimüller. (2017), Is inequality harmful for innovation and growth? Price versus market size effects, *Journal of Evolutionary Economics 27*, 359 – 378.

Galor, O., and J. Zeira. (1993), Income distribution and macroeconomics, *Review of Economic Studies 60*, 35–52.

Ghatak, M., M. Morelli, and T. Sjöström. (2007), Entrepreneurial talent, occupational choice, and trickle up policies, *Journal of Economic Theory 137*, 27 – 48.

Gross, C., K. Lorek, and F. Richter. (2017), Attitudes towards inheritance taxation: Results from a survey experiment, *Journal of Economic Inequality 15*, 93 – 112.

Grüner, H. P. (2003), Redistribution as a selection device, *Journal of Economic Theory*

108, 194 – 216.

Hall, B. H., A. Jaffe, and M. Trajtenberg. (2001), The NBER patent citation data file: Lessons, insights and methodological tools, NBER Working Paper No. 8498, Cambridge, MA: National Bureau of Economic Research.

Hatipoğlu, O. (2012), The relationship between inequality and innovative activity: A Schumpeterian theory and evidence from cross-country data, *Scottish Journal of Political Economy 59*, 224 – 248.

Hellwig, M. (1987), Some recent developments in the theory of competition in markets with adverse selection, *European Economic Review 31*, 319 – 325.

Holmström, B. (1989), Agency costs and innovation, *Journal of Economic Behavior and Organization 12*, 305 – 327.

Hunter, J. D. (2007), Matplotlib: A 2D graphics environment, *Computing in Science & Engineering 9*, 90 – 95.

Inci, E. (2013), Occupational choice and the quality of entrepreneurs, *Journal of Economic Behavior & Organization 92*, 1 – 21.

Ingram, D. D., and S. J. Franco. (2014), NCHS urbanrural classification scheme for counties, *National Center for Health Statistics: Vital and Health Statistics 2*, 1 – 73.

Jaimovich, E. (2011), Sectoral differentiation, allocation of talent, and financial development, *Journal of Development Economics 96*, 47 – 60.

Jones, C., and J. Kim. (2018), A Schumpeterian model of top income inequality, *Journal of Political Economy 126*, 1785 – 1826.

Kambourov, G., and I. Manovskii. (2009), Occupational mobility and wage inequality, *Review of Economic Studies 76*, 731 – 759.

Keeley, B. (2015), *Income Inequality: The Gap between Rich and Poor*, OECD Insights, OECD Publishing, Paris.

Krueger, A. (2012), The Rise and Consequences of Inequality, Presentation made to the Center for American Progress, January 12th. `http://www.americanprogress.org/events//2012/01/12/17181/the-rise-and-consequencesof-inequality/`.

Lefranc, A., N. Pistolesi, and A. Trannoy. (2009), Equality of opportunity and luck: Definitions and testable conditions, with an application to income in France, *Journal of Public Economics 93*, 1189 – 1207.

Liao, T. F. (2016a), Evaluating distributional differences in income inequality, *Socius 2*, 1 – 14.

Liao, T. F. (2016b), Evaluating distributional differences in income inequality with sampling weights, *Socius: Supplements 2*, 1 – 6.

Liao, T. F. (2016c), THEILDECO: Stata module to produce refined Theil index decomposition by group and quantile, Statistical Software Components, Boston College Department of Economics.

Lo Bello, S., and I. Morchio. (2016), Like father, like son: Occupational choice, intergenerational persistence and misallocation, Mimeo.

MacKinlay, A. C. (1997), Event studies in economics and finance, *Journal of Economic Literature 35*, 13 – 39.

Mankiw, G. (2013), Defending the one percent, *Journal of Economic Perspectives 27*, 21 – 34.

Manso, G. (2011), Motivating innovation, *The Journal of Finance 66*, 1823 – 1860.

Marrero, G. A., and J. G. Rodríguez. (2012), Inequality of opportunity in Europe, *Review of Income and Wealth 58*, 597 – 621.

Marrero, G. A., and J. G. Rodríguez. (2013), Inequality of opportunity and growth, *Journal of Development Economics 104*, 107 – 122.

Mirrlees, J. A. (1971), An Exploration in the Theory of Optimum Income Taxation12, *The Review of Economic Studies 38*, 175 – 208.

Modigliani, F. (1988), Measuring the contribution of intergenerational transfers to total wealth: Conceptual issues and empirical findings, In *Modeling the Accumulation and Distribution of Wealth*. Eds. by D. Kessler, and A. Masson. Clarendon Press, Oxford.

Murphy, K. M., A. Shleifer, and R. L. Vishny. (1989), Income distribution, market size, and industrialization, *Quarterly Journal of Economics 104*, 537 – 564.

Okun, A. (1975), *Equality and Efficiency: The Big Tradeoff*, Brookings Institution Press, Washington D.C.

Opportunity Nation and Child Trends. (2019), Opportunity Index: How Opportunity Measures Up in Your Community, `https://opportunityindex.org/`.

Panizza, U. (2002), Income inequality and economic growth: Evidence from American data, *Journal of Economic Growth 7*, 25 – 41.

Roemer, J. E. (1993), A pragmatic theory of responsibility for the egalitarian planner, *Philosophy & Public Affairs 20*, 146 – 166.

Roemer, J. E. (1998), *Equality of Opportunity*, Harvard University Press, Cambridge.

Roemer, J. E., and A. Trannoy. (2016), Equality of opportunity: Theory and measurement, *Journal of Economic Literature 54*, 1288 – 1332.

Rothschild, M., and J. Stiglitz. (1970), Increasing risk I: A definition, *Journal of Economic Theory 2*, 225 – 243.

Rothschild, M., and J. Stiglitz. (1976), Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *Quarterly Journal of Economics 90*, 629 – 649.

Saez, E. (2019), Striking it richer: The evolution of top incomes in the United States, Mimeo.

Solt, F. (2019), Measuring income inequality across countries and over time: The standardized world income inequality database, Feb, OSF. `https://osf.io/3djtq`. SWIID Version 8.0, February 2019.

Spiganti, A. (2018), Wealth inequality, innovative talent, and occupational choices, Mimeo.

Spiganti, A. (forthcoming), Can starving start-ups beat fat labs? A bandit model of innovation with endogenous financing constraint, *Scandinavian Journal of Economics.*

StataCorp. (2017), Stata Statistical Software: Release 15, College Station, TX: StataCorp LP.

Theil, H. (1967), *Economics and Information Theory*, North Holland, Amsterdam.

Tirole, J. (1988), *The Theory of Industrial Organization*, MIT Press, Cambridge.

United States Census Bureau. (2016a), County Intercensal Datasets: 2000-2010, `https://www.census.gov/data/datasets/time-series/demo/popest/intercensal-2000-2010-counties.html`.

United States Census Bureau. (2016b), Historical Income Tables: Counties, `https://www.census.gov/data/tables/time-series/dec/historical-income-counties.html`.

United States Patent and Trademark Office. (2016), U.S. Patent Statistics Chart, `https://www.uspto.gov/web/offices/ac/ido/oeip/taf/us_stat.htm`.

United States Patent and Trademark Office. (2019), U.S. Resident Inventors and Their Utility Patents: Breakout by State Regional Component, `https://www.uspto.gov/web/offices/ac/ido/oeip/taf/inv_countyall/usa_invcounty_gd.htm`.

Violante, G. L. (2008), Skill-biased technical change, In *The New Palgrave Dictionary of Economics.* Eds. by S. Durlauf, and L. Blume.

van der Walt, S., S. C. Colbert, and G. Varoquaux. (2011), The NumPy array: A structure for efficient numerical computation, *Computing in Science & Engineering 13*, 22 – 30.

Wilson, C. (1977), A model of insurance markets with incomplete information, *Journal of Economic Theory 16*, 167 – 207.

Zweimüller, J. (2000), Schumpeterian entrepreneurs meet Engel's law: The impact of inequality on innovation-driven growth, *Journal of Economic Growth 5*, 185 – 206.

A Appendix

A.1 Glossary of variables and parameters

Variables and parameters definition	
$c_{t,i}$	consumption of the final good
$b_{t,i}$	bequest
δ	budget share
λ_t	proportion of talented agent
A	wealth endowment
y_t	final good
α	output share
l_t	labour
$x_{t,m}$	quantity of machine m
$Q_{t,m}$	quality of machine m
p_t	price of the final good
w_t	wage rate
$r_{t,m}$	price of machine m
ρ_i	probability of innovating
γ	increase in quality after innovation
R_t	riskless rate of return
I_t	setup cost
D_i^s	repayment in realised state s
V_i^s	realised payoff of the innovator

A.2 Proofs and Maths

Monopoly price is a constant markup over cost. Define the point elasticity with respect to demand as

$$e = \frac{\partial Q}{\partial P} \times \frac{P}{Q} \rightarrow \frac{\partial Q}{\partial P} = e \times \frac{Q}{P},$$

where Q is quantity and P is price. Express the profit function of the monopolist as $\pi = PQ(P) - cost(Q(P))$ so that the FOC with respect to price is

$$\frac{|e|}{|e|-1} \times MC = P^{\star}.$$

Here, $MC = \psi$ and $e = -1/(1-\alpha)$ and thus $r_m = \psi/\alpha$. □

Proof of Proposition 1. Below, we derive the various contracts that banks offer to given wealth classes. The proofs are similar to any adverse selection model in financial markets (e.g. Grüner, 2003, Jaimovich, 2011, Inci, 2013).

For generality, we do not impose the repayment in the failure state equals to zero: nevertheless, given limited liability, this must be non-positive. A general contract then is

$$\boldsymbol{\sigma}_z(A,\Omega) = \begin{bmatrix} \sigma_H(A,\Omega) \\ \sigma_L(A,\Omega) \end{bmatrix} = \begin{bmatrix} D_H^S(A,\Omega) & D_H^F(A,\Omega) \\ D_L^S(A,\Omega) & D_L^F(A,\Omega) \end{bmatrix}, \tag{A.1}$$

where D_i^S and D_i^F are the repayments to the bank by the i-type agent in the success and failure state, respectively. Let the net payoff of an i-type innovator in the success state be given by $V_i^S = \pi(\Omega) - D_i^S(A,\Omega) - e_i$; limited liability implies $V_i^S \geq 0$. Let the net payoff of an i-type innovator in the failure state be given by $V_i^F = -D_i^F(A,\Omega) - e_i$; limited liability implies $V_i^F \geq -e_i$. Thus, the expected payoff of an innovator, is

$$\mathcal{V}_i(A,\Omega) \equiv \rho_i V_i^S + (1-\rho_i) V_i^F, \quad \forall i = \{H, L\}.$$

The expected payoff of an agent who becomes a worker is $\mathcal{W}_i(A,\Omega) \equiv w + RA, \forall i = \{H, L\}$: we call this payoff "the outside option" to innovation.

The zero-profit conditions from separating contracts are

$$\rho_H \left(\pi(\Omega) - V_H^S\right) - (1-\rho_H) V_H^F = R(I-A) \tag{A.2a}$$
$$\rho_L \left(\pi(\Omega) - V_L^S\right) - (1-\rho_L) V_L^F = R(I-A), \tag{A.2b}$$

where the first line refers to talented agents, and the second one to untalented agents with wealth A. For given levels of $\mathcal{V}_H$ and $\mathcal{V}_L$, respectively, the corresponding iso-profit lines of the borrowers are

$$\bar{\mathcal{V}}_H = \rho_H V_H^S + (1-\rho_H) V_H^F \tag{A.3a}$$
$$\bar{\mathcal{V}}_L = \rho_L V_L^S + (1-\rho_L) V_L^F. \tag{A.3b}$$

The zero-profit condition from a pooling contract is given by

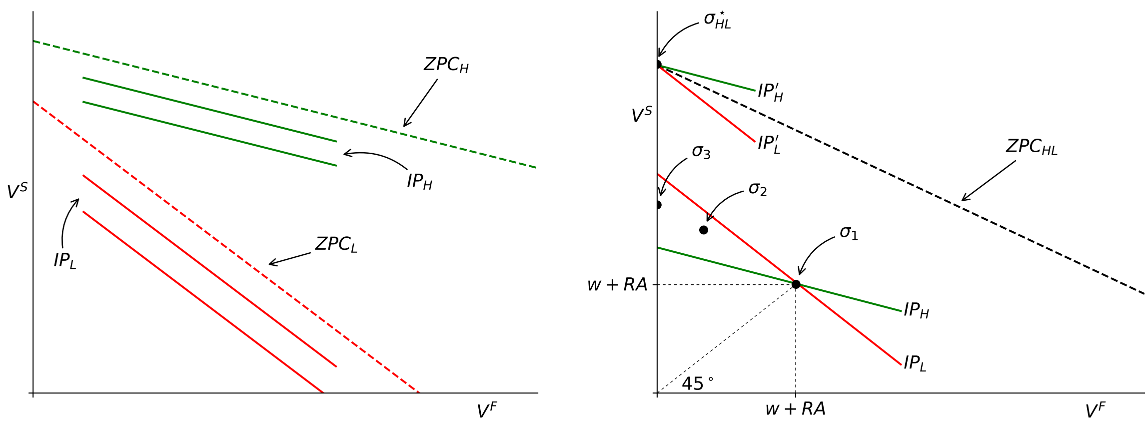
$$\rho_A D^S + (1-\rho_A) D^F = R(I-A), \tag{A.4}$$

where D^S and D^F are repayments of a random borrower with wealth A in the success and failure state, respectively, and ρ_A is her Bayesian probability of success.

The zero-profit conditions in (A.2) and groups of iso-payoffs in (A.3) are drawn in Figure A.1a. Each zero-profit condition ZPC_i has the same slop as the corresponding iso-profit IP_i; agent's expected payoff is increasing as we move north-east, bank's profits are increasing as we move south-west. Imagine a bank offering two distinct contracts each on a zero-profit condition: the untalented agent would always pretend to be talented. Indeed, it is impossible to find a menu of contracts such that the zero-profit conditions hold and the untalented agents do not prefer the contract designed for the talented innovators.

Which contract can the banks then offer? It turns out that the equilibrium contract differs depending on the wealth class of the agents. Focus on Figure A.1b, where the outside option to innovation is given by the point $\boldsymbol{\sigma}_1$. The iso-profit curve for a talented and an untalented agent passing through this point are labelled IP_H and IP_L, respectively. From the iso-profits in (A.3), we know that the iso-profit of the untalented agent is steeper. The banks could then offer any contract on the north-west of $\boldsymbol{\sigma}_1$, like $\boldsymbol{\sigma}_2$: under any reasonable belief, such a contract would attract only talented agents, and since we are below the zero-profit condition with only talented agents (not shown but it would be above ZPC_{HL}), the deviating bank would make positive profit. But any contract offered in this area can be undercut by another contract on its left. Due to limited liability, however, we cannot move further west than $V^F = 0$, like in $\boldsymbol{\sigma}_3$. But, similarly, $\boldsymbol{\sigma}_3$ can be undercut by any contract offering slightly better repayment in case of success. If such contract is below the zero-profit conditions with both types, ZPC_{HL}, a deviating bank would still make positive profits. Thus, undercutting goes on until banks make zero-profit, like in $\boldsymbol{\sigma}^{\star}_{HL}$, at which point no profitable deviation exists. Since this equilibrium pooling contract is on the vertical axis and lies on the zero-profit condition with both types given by (A.4), it follows that

$$\boldsymbol{\sigma}^{\star}_{HL}(A, \Omega) = \begin{bmatrix} R(I-A)/\rho_A & 0 \\ R(I-A)/\rho_A & 0 \end{bmatrix}. \tag{A.5}$$



(a) No Separating Contract

(b) Pooling Contact

Figure A.1: Contracts

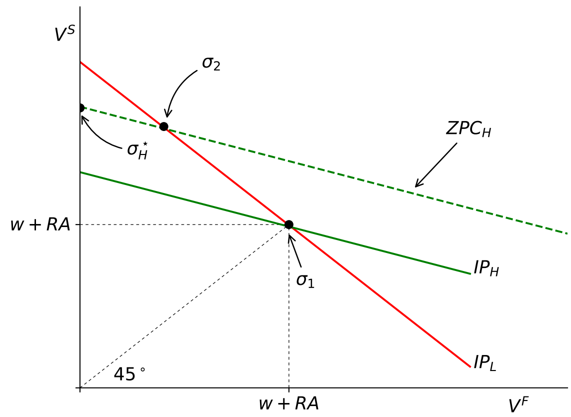
Notes. IP_i are iso-payoffs, ZPC_i are zero-profit conditions.

Now consider the situation illustrated in Figure A.2a. Agents are wealthy enough, so that their outside option is better than any pooling contract on the zero-profit condition with both types (not shown). A contract like $\boldsymbol{\sigma}_1$ cannot be an equilibrium: a deviating bank could offer any contract in the area between IP_L, IP_H, and ZPC_H: such contract would be accepted by talented agents only, and would thus entail positive profits. The equilibrium contract must thus lie on ZPC_H, whose equation is given in (A.2a). However, this time we have a continuum of equilibria in between $[\boldsymbol{\sigma}_2, \boldsymbol{\sigma}^\star_H]$. For simplicity, we choose to focus on the contract on the vertical axis,

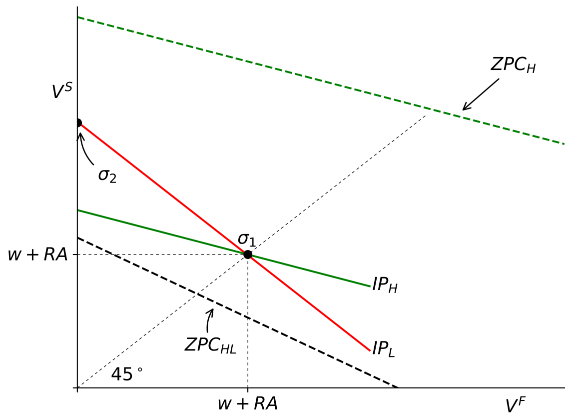
$$\boldsymbol{\sigma}^\star_H(A,\Omega) = \begin{bmatrix} R(I-A)/\rho_H & 0 \\ R(I-A)/\rho_H & 0 \end{bmatrix}, \tag{A.6}$$

but this is without loss of generality since all these contracts entail the same expected payment and the same occupational choices.

Consider now Figure A.2b, which represents a wealth class for which banks can offer neither the zero-profit pooling contract $\boldsymbol{\sigma}^\star_{HL}$ nor the separating contract $\boldsymbol{\sigma}^\star_H$. The pooling contract on the zero-profit condition cannot be offered because it yields an expected payoff that is always lower than the outside option. A separating contract on ZPC_H cannot be offered because it would also be accepted by untalented agents. Can the separating menu of contract $[\sigma_2, \sigma_1]^T$ be offered? Untalented agents are indifferent between σ_1, σ_2, and their outside option, so, by assumption, they choose to stay out of the innovation sector. Talented agents strictly prefer σ_2 to σ_1 and the outside option, and thus accept the contract. If banks can offer only loan contracts, then this menu of contract is an equilibrium in which banks make positive profits, since σ_2 is below the zero-profit condition with only talented agents, and untalented agents do not apply for loans.[32] This is the equilibrium contract we consider in the main text.



(a) Separating Contract



(b) Profitable Separating Contract

Figure A.2: Separating Contracts

Notes. IP_i are iso-payoffs, ZPC_i are zero-profit conditions.

Assume for a moment that banks are not limited to loan contracts alone. Then banks can undercut each other by offering a contract to the talented agent that is slightly above σ_2 but still below ZPC_H, and by paying lenders in both states of the world something more than the usual interest on deposits. Undercutting goes on until banks make zero-profit on these contracts, like $[\sigma_3, \sigma_4]^T$ in Figure A.3. Since banks make profits on σ_3

[32] Indeed, there is no profitable deviation. A deviation contract below IP_L is not accepted by anyone; one above IP_L but below IP'_H is only accepted by untalented agents; a contract with $V^F < 0$ would violate limited liability; any pair of contracts above IP'_H would be accepted by everyone but would incur losses because it would be above the zero-profit condition with both types.

and losses on σ_4, a Nash player would cancel σ_4: since the other banks are still offering it, the deviating bank would be better off. A solution to this non-existence problem, is to impose a Wilson's (1977) equilibrium concept, where players are non-myopic rational. In a Wilson's (1977) world, the deviating bank would take into account the effects of its action on the actions of other banks. The non-myopic player knows that other banks would react to the cancelling of σ_4 by withdrawing σ_4 as well, and thus would incur losses: as a consequence, it would not deviate in the first place. Whether I impose a Nash-Bertrand equilibrium concept with a restricted set of contracts, or this Wilson's (1977) equilibrium concept with a larger set of feasible contracts, the qualitative results are unchanged.

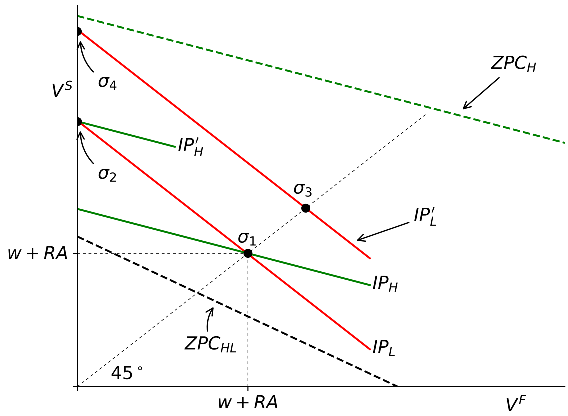


Figure A.3: Wilson's (1977) Separating Contract
Notes. IP_i are iso-payoffs, ZPC_i are zero-profit conditions.

Given the contracts derived above, and the participation and incentive compatibility constraints given in the main text, occupational choices are trivial. □

Proof of Lemma 1. Notice that

$$\Delta(A, \Omega) = \begin{cases} 0, & A \le A_e; \\ (\rho_H - \rho_L)\left(\pi - R(I-A)/\bar{\rho}\right), & A \in [A_e, A_L]; \\ (w + RA)\left(\rho_H - \rho_L\right)/\rho_L, & A \in [A_L, A_{HH}]; \\ \rho_H \pi - RI - w, & A \ge A_{HH}. \end{cases} \tag{A.7}$$

By visual inspection of (A.7), the talent premium is (i) unaffected by w for the working and lemons-class, strictly increasing in w for the riches, and strictly decreasing in w for the unconstrained-class; (ii) independent of R for the working-class, strictly decreasing in R for the lemons-class, strictly increasing in R for the rich-class, and again strictly decreasing for the unconstrained agents; (iii) weakly increasing in the average quality of the machines at the beginning of the period (since the expected profits from innovations are increasing in the quality of the machines); (iv) weakly increasing in A both within and across classes (note that the talent premium is continuous across classes, and weakly increasing within classes). □

Proof of Lemma 2. (i) From (14a), it is clear that A_e is strictly decreasing in π and A_L is strictly increasing in π: thus, since $\pi > \pi'$, $A_e < A'_e$ and $A_L > A'_L$. This means that there are more agents in the lemon-class and fewer agents in the working-class under π

than under π', since everything else is equal. This implies $n_H \geq n'_H$ and $n_L \geq n'_L$. Since the growth rate is increasing in the number of innovators, the economy with π grows faster than the economy with π', even if some of the additional innovators are of low ability. Moreover, note that π is strictly increasing in Q, and thus more technologically advanced economies grow faster. (ii) If everything but the wealth distributions is equal, the wealth thresholds are the same. If $\Phi(A)$ first-order stochastically dominates $\Phi'(A)$, then by definition $\Phi(A) \leq \Phi'(A)$ for all A, with strict equality for some A. Therefore, surely $\Phi(A_e) \leq \Phi'(A_e)$, and thus $n_H \geq n'_H$. □

Proof of Lemma 3. (i) If $\tilde{A} < A_e$, by the definition of single mean-preserving spread $\Phi(A_e) > \Phi'(A_e)$, and thus $n_H < n'_H$. However, also $\Phi(A_L) > \Phi'(A_L)$, and thus $n_L \gtreqless n'_L$ depending on the shapes of the distributions. (ii) If $\tilde{A} = A_e$, $\Phi(A_e) = \Phi'(A_e)$, and thus $n_H = n'_H$. Moreover, $\Phi(A_L) > \Phi'(A_L)$, and thus $n_L > n'_L$. (iii) If $A_e < \tilde{A} \leq A_L$, $\Phi(A_e) < \Phi'(A_e)$, and thus $n_H > n'_H$. Moreover, $\Phi(A_L) \geq \Phi'(A_L)$, and thus $n_L > n'_L$. (iv) If $A_L < \tilde{A}$, $\Phi(A_e) < \Phi'(A_e)$, and thus $n_H > n'_H$. However, also $\Phi(A_L) < \Phi'(A_L)$, and thus $n_L \gtreqless n'_L$ depending on the shapes of the distributions. □

Proof of Proposition 2. If n and ρ are constant over time, then Q_t is growing at the constant rate $g = \gamma\rho n$. Since $l_t = 1, \forall t$ by assumption, from equation (8) it follows that π_t also grows at rate g. In each period, there are ρn machines produced under monopoly with demand function given by (7), and $1 - \rho n$ machines produced under competition with demand function given by (9). Therefore, as shown in Appendix B.2,

$$y_t = l\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\rho n\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right)Q_t, \tag{A.8}$$

which implies that y_t also grows at rate g. Since w_t is a linear function of y_t (see Appendix B.2), it also grows at rate g. In equilibrium, the market for the final good must clear, which means that total consumption is equal to output. Given the utility function in (1), agents optimally consume a fraction $1 - \delta$ of their end-of-period wealth, and they bequeath the remaining fraction δ. This latter fraction accounts for total (and average) wealth of the new generation: thus, aggregate consumption, bequests, and average wealth also grow at rate g. □

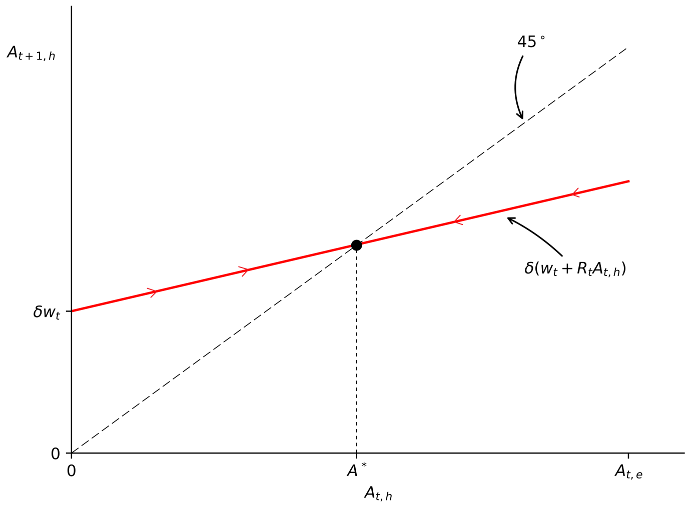


Figure A.4: No Innovation Trap

B Miscellaneous Algebra

B.1 Growth Rate

The growth rate of average quality under full information, g^{FB}, is greater than the growth rate in the constrained equilibrium, g, if and only if

$$\begin{aligned}
\gamma\rho_H\lambda &> \gamma\rho n \\
\rho_H\lambda &> \rho_H n_H + \rho_L n_L \\
\rho_H\left(\lambda - n_H\right) &> \rho_L n_L \\
\frac{\rho_H}{\rho_L} &> \frac{n_L}{\lambda - n_H} \\
\frac{\rho_H}{\rho_L} &> \frac{(1-\lambda)\left[\Phi\left(A_L\left(\Omega\right)\right) - \Phi\left(A_e\left(\Omega\right)\right)\right]}{\lambda\Phi\left(A_e\left(\Omega\right)\right)}
\end{aligned}$$

B.2 Final Good Production

In equilibrium, there are ρn machines produced under monopoly with demand function given by (7), and $1-\rho n$ machines produced under competition with demand function given by (9). Therefore

$$\begin{aligned}
&\int_0^1 Q_{t,m}^{1-\alpha} x_{t,m}^{\alpha} dm = \\
&= \int_0^1 Q_{t,m}^{1-\alpha}\left[\rho n\left(x_{t,m}^M\right)^{\alpha} + (1-\rho n)\left(x_{t,m}^C\right)^{\alpha}\right] dm = \\
&= \int_0^1 Q_{t,m}^{1-\alpha}\left[\rho n\left(\left(\frac{\alpha^2 p_t}{\psi}\right)^{\frac{1}{1-\alpha}} Q_{t,m} l_t\right)^{\alpha} + (1-\rho n)\left(\left(\frac{\alpha p_t}{\psi}\right)^{\frac{1}{1-\alpha}} Q_{t,m} l_t\right)^{\alpha}\right] dm = \\
&= \int_0^1 Q_{t,m} l_t^{\alpha}\left[\rho n\left(\frac{\alpha^2 p_t}{\psi}\right)^{\frac{\alpha}{1-\alpha}} + (1-\rho n)\left(\frac{\alpha p_t}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\right] dm = \\
&= \int_0^1 Q_{t,m} l_t^{\alpha}\left(\frac{\alpha p_t}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left[\rho n \alpha^{\frac{\alpha}{1-\alpha}} + (1-\rho n)\right] dm = \\
&= l_t^{\alpha}\left(\frac{\alpha p_t}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left[\rho n\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right]\int_0^1 Q_{t,m} dm = \\
&= l_t^{\alpha}\left(\frac{\alpha p_t}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left[\rho n\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right] Q_t
\end{aligned}$$

The production of final good in the constrained equilibrium is thus

$$y = l\left(\frac{\alpha p}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\rho n\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) Q \equiv l\left(\alpha p/\psi\right)^{\frac{\alpha}{1-\alpha}} \hat{Q},$$

where

$$\hat{Q} \equiv \left(\rho n\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) Q.$$

Note that $\hat{Q} < Q$, since

- $\left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) \in (-0.63, 0)$
- $\left(\rho n \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) < 1$

Also

$$\frac{\partial \hat{Q}_t}{\partial n_t \rho_t} = Q_{t-1} \left\{ \gamma + \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right)(1 + 2\gamma\rho_t n_t) \right\}$$

which is negative if

$$\begin{aligned}
\gamma + \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right)(1 + 2\gamma\rho_t n_t) &< 0 \\
\gamma &< \left(1 - \alpha^{\frac{\alpha}{1-\alpha}}\right)(1 + 2\gamma\rho_t n_t) \\
\frac{\gamma}{\left(1 - \alpha^{\frac{\alpha}{1-\alpha}}\right)} &< (1 + 2\gamma\rho_t n_t) \\
\frac{1}{2\left(1 - \alpha^{\frac{\alpha}{1-\alpha}}\right)} - \frac{1}{2\gamma} &< \rho_t n_t
\end{aligned}$$

$\hat{Q}$ evolves over time according to

$$\frac{\left(\rho_t n_t \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right)(1 + \gamma\rho_t n_t) Q_{t-1} - \left(\rho_{t-1} n_{t-1} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) Q_{t-1}}{\left(\rho_{t-1} n_{t-1} \left(\alpha^{\frac{\alpha}{1-\alpha}} - 1\right) + 1\right) Q_{t-1}}.$$

Obviously, if $\rho_t n_t = \rho_{t-1} n_{t-1}$, then $\hat{Q}_t$ grows at the same rate as Q_t.

Combining the FOCs in (4) with the production function in (2),

$$\begin{aligned}
(1-\alpha) p_t \frac{y_t}{l_t} &= w_t \rightarrow & (1-\alpha) p_t \frac{y_t}{w_t} &= l_t \\
\alpha p_t \frac{y_t}{k_t} &= r_t \rightarrow & \alpha p_t \frac{y_t}{r_t} &= k_t
\end{aligned}$$

where k_t is an aggregate input considering all machines and r_t is the weighted average price of these machines. Thus

$$\begin{aligned}
y_t &= \left(\frac{(1-\alpha) p_t y_t}{l_t}\right)^{1-\alpha} \left(\frac{\alpha p_t y_t}{r_t}\right)^{\alpha} \\
y_t^{1-1+\alpha-\alpha} &= \left(\frac{(1-\alpha) p_t}{l_t}\right)^{1-\alpha} \left(\frac{\alpha p_t}{r_t}\right)^{\alpha} \\
1 &= \left(\frac{(1-\alpha) p_t}{l_t}\right)^{1-\alpha} \left(\frac{\alpha p_t}{r_t}\right)^{\alpha}
\end{aligned}$$

B.3 Evolution of Wealth

B.3.1 Evolution of Average Wealth

Consider an economy with average wealth equal to $\bar{A}_t$. Given the wealth transition equations in the main text, average wealth in $t+1$ is given by[33]

$$
\begin{aligned}
\bar{A}_{t+1} = {} & \delta \left[w_t + R\bar{A}_{t,W} \right] \Phi\left(A_{t,e}\right) + \\
& + \left\{ \delta \left[\varpi(\Omega_t) - R_t(I_t - \bar{A}_{t,L})/\bar{\rho} - e \right] \lambda \rho_H + \delta \left[\varpi(\Omega_t) - R_t(I_t - \bar{A}_{t,L})/\bar{\rho} \right] (1-\lambda)\rho_L \right\} \\
& \quad \times \left[\Phi\left(A_{t,L}\right) - \Phi\left(A_{t,e}\right) \right] + \\
& + \left\{ \delta \left[\left(w_t + R_t\bar{A}_{t,R} \right)/\rho_L - e \right] \lambda \rho_H + \delta \left[w_t + R_t\bar{A}_{t,R} \right](1-\lambda) \right\} \left[\Phi\left(A_{t,HH}\right) - \Phi\left(A_{t,L}\right) \right] + \\
& + \left\{ \delta \left[\varpi(\Omega_t) - R_t(I_t - \bar{A}_{t,U}) - e \right] \rho_H + \delta \left[w_t + R_t\bar{A}_{t,U} \right](1-\lambda) \right\} \left[\Phi\left(I_t\right) - \Phi\left(A_{t,HH}\right) \right] + \\
& + \left\{ \delta \left[\varpi(\Omega_t) + R_t(\bar{A}_{t,SR} - I_t) - e \right] \lambda \rho_H + \delta \left[R_t(\bar{A}_{t,SR} - I_t) \right] \lambda (1-\rho_H) + \right. \\
& \quad \left. + \delta \left[w_t + R_t\bar{A}_{t,SR} \right](1-\lambda) \right\} \left[1 - \Phi\left(I_t\right) \right]
\end{aligned}
$$

where $\bar{A}_{t,i}$ is the average wealth of agents in the wealth class $i = \{W, L, R, U, SR\}$ (i.e. working, lemon, rich, unconstrained, and super-rich, who are the unconstrained agents who can completely self-finance themselves).

B.3.2 Evolution of Wealth

In the working class, the wealth transition equation is the same for everybody, $\delta[w_t + R_t A_t]$.

At the wealth threshold A_e, the talented agent is indifferent between exerting and non exerting effort with the pooling contract; the untalented agent is strictly better off accepting the contract than taking the outside option (because she is going to be indifferent in A_L)

B.4 Assumptions and Parametrization

Assumption 1 is satisfied in the parametrization if

$$
w \in \left(\rho_L \varpi - RI, \rho_L \varpi - RI \frac{\rho_L}{\bar{\rho}} \right).
$$

$A_L > A_e$ requires

$$
I < \frac{(\rho_H - \rho_L)(\bar{\rho}\varpi - w) - (\bar{\rho} - \rho_L)e}{R(\rho_H - \rho_L)} \qquad \longleftrightarrow \qquad e < \frac{(\rho_H - \rho_L)(\bar{\rho}\varpi - RI - w)}{(\bar{\rho} - \rho_L)},
$$

whereas $A_e > 0$ requires

$$
I > \frac{\bar{\rho}(\rho_H - \rho_L)\varpi - \bar{\rho}e}{R(\rho_H - \rho_L)} \qquad \longleftrightarrow \qquad e > \frac{(\rho_H - \rho_L)(\bar{\rho}\varpi - RI)}{\bar{\rho}}.
$$

[33] In addition, banks make positive profits on the contracts offered to rich talented agents. These are not considered here.

C Extensions

In this section, we briefly discuss how different extensions would change the main model.

C.1 The Case of a Single Wealth Class

Here, we consider the case where all individuals have the same wealth, $\bar{A}$. There are different static equilibria that can be considered: (i) if $\bar{A} < A_e$ all agents become workers, (ii) if $\bar{A} \in [A_e, A_L]$ all agents become innovators, and (iii) if $\bar{A} > A_L$, all talented individuals become innovators, whereas the untalented agents become workers.

In case (i), there is no innovation in the economy, and thus the quality of machine is constant over time, as is the wage. At the end of their life, individuals have wealth equal to $R\bar{A} + w$, of which they consume a proportion $1 - \delta$ and bequeath the remaining δ. If the bequest is smaller than $\bar{A}$, eventually wealth disappears, and all agents receive δw at their birth. If the bequest is greater than $\bar{A}$, i.e. $\bar{A} < \delta w/(1 - \delta R)$, then agents eventually accumulate enough wealth to enter (ii).

In case (ii), everyone is an innovator, and thus the average quantity of the machine increases by a factor $\gamma\bar{\rho}$ (greater than first-best). At the end of the period, there are three lineages: failed innovators, successful talented innovators, and successful untalented innovators. On average, talented agents bequeath more than untalented ones. Depending on the size of δ, an economy may accumulate wealth or not.

In case (iii), all talented agents become innovators, whereas untalented agents become workers. This is first-best. At the end of the first period, there are three classes af agents: workers (i.e. untalented individuals), failed innovators, and successful innovators. Each of this agent has an end-of-period wealth equal to, respectively, $w + R\bar{A} + y(A)$, $R(\bar{A} - I) + y(A)$, and $\varpi + R(\bar{A} - I) - x(A) + y(A)$. Thus a non degenerate distribution of wealth is created starting from the second period.

C.2 The Limited Pledgeability Case

In this section, I briefly analyse how the model is modified if adverse selection is replaced by limited pledgeability.

Agents are still heterogeneous in talent and wealth, but there is perfect information on both. However, agents can only pledge up to a fraction $\nu \in [0, 1]$ of their revenues in case of success, whereas the remaining $1 - \nu$ fraction can be diverted. This means that any equilibrium credit contract must satisfy the following borrowing constraint

$$Rb_i(A) \leq \nu\rho_i\varpi(\Omega), \tag{A.10}$$

where $b_i(A)$ is the amount borrowed by a type-i agent with wealth A.

Assume (A.10) to be binding, and assume that it is optimal for an agent to self-finance themselves as much as possible, so that $b_i(A) = (I - A)$. Thus, agent can borrow to run a risky innovative project if and only if the following borrowing constraint is satisfied,

$$R(I - A) \leq \nu\rho_i\varpi(\Omega) \rightarrow -A \leq \frac{\nu\rho_i\varpi(\Omega)}{R} - I \rightarrow A \geq I - \frac{\nu\rho_i\varpi(\Omega)}{R}. \tag{A.11}$$

This means that talented and untalented agents with wealth lower than A_H and A_L, respectively, will not be able to become innovators, where these wealth thresholds are given by

$$A_i = I - \frac{\nu \rho_i \varpi(\Omega)}{R}. \tag{A.12}$$

An agent will only invest if they are willing to do so. By becoming an inventor, a type-i agent expects an end-of-period wealth equal to $\rho_i \varpi(\Omega) - R(I - A)$; by becoming a worker, this would be equal to $w + RA$. Thus, an agent is willing to borrow and become an innovator if and only if her participation constraint is satisfied,

$$\rho_i \varpi(\Omega) - R(I - A) \geq w + RA \rightarrow \rho_i \varpi(\Omega) - RI \geq w. \tag{A.13}$$

Assume (A.13) is satisfied with a strict inequality for p_L (meaning that it is first-best efficient in a given period to have everyone innovating).[34] As a consequence, we can partition the wealth distribution into three wealth classes. First, all agents with $A < A_H$ are in the lower-class: since they cannot borrow, they all become workers. Agents with $A \in [A_H, A_L]$ are in the middle-class, where talented agents innovate whereas untalented agents work. Finally, agents with $A > A_L$ are in the rich-class, where everybody becomes an innovator.

D General Equilibrium Analysis

In this section, I first carry the model to a general equilibrium by defining the market clearing conditions for the credit and labour markets and thus finding the endogenous wage and risk-free interest rates. Due to the complexity involved in analysing the general equilibrium analytically, I instead provide several quantitative illustrations. The aim of these illustrations is not to develop a comprehensive quantitative evaluation but to better understand the mechanism of the model, to highlight the dynamic effects of different initial conditions, and to suggest potential policy interventions.[35]

D.1 Endogenous Prices

As explained in Section 4.2.2, the total number of innovators is given by equation (18a), and their average probability of success is given by equation (18b). Each of these innovators needs I units of capital for the initial investment, and thus the total demand for capital is $n(\Omega) \times I$. The total availability of funds is given by aggregate wealth, $\bar{A}$, and thus the credit market clears when

$$\bar{A} = n \times I. \tag{A.14}$$

Finally, the final good producer demands $l(\Omega)$ workers, as given by (5a). Since there are $1 - n(\Omega)$ workers, the labour market clears when

$$l(\Omega) = 1 - n(\Omega). \tag{A.15}$$

The general equilibrium of this economy is found by solving the set of four equations

[34] If it is not satisfied, all agents prefer to become workers. If it is satisfied only for ρ_H, then only talented wealthy agents become innovators, everybody else become a worker.

[35] Simulations are run using Numpy (Walt *et al.*, 2011); graphs are drawn in Matplotlib (Hunter, 2007). I used Python 2.7.

(18a), (18b), (A.14), and (A.15) in the four unknowns R, w, ρ, and n. This is done computationally in the following quantitative illustrations.

D.2 Parametrization

A period corresponds to 30 years. I normalize the price of the final good, $p_t = 1, \forall t$, and $Q_0 = 1$. I take $\alpha = 1/3$, so that the share of national income spent on machines is approximately equal to the share of capital. I set the cost of producing one machine to $\psi = 0.1\alpha$, and $\iota = 0.3$. I match the US transfer-wealth ratio of 20% documented by Modigliani (1988) by imposing $\delta = 0.2$. I set the proportional increase in productivity resulting from innovation to $\gamma = 1$, the probability of success of a talented innovator to be equal to 0.03 per annum (i.e. $\rho_H \approx 60\%$), and the fraction of talented agents to be $\lambda = 1/3$. Untalented innovators are assumed to be half as likely to succeed as talented ones over their lifetime, $\rho_L \approx 30\%$.

I assume initial wealth to be distributed according to a mixture of an atomic and a continuous distribution, $\phi_0(A) = \theta\phi_{0,1}(A) + (1-\theta)\,\phi_{0,2}(A)$, where θ is the mixture proportion. The atomic distribution concentrates its unit mass of agents at zero, and therefore accounts for those individuals with no inherited wealth, i.e. $\phi_{0,1}(0) = 1$. The continuous distribution accounts for the strictly positive values of initial wealth, and is specified as a lognormal model, i.e. $\phi_{0,2}(A)$ is such that $\ln(A) \sim \mathcal{N}(\mu, \sigma^2)$, $A > 0$. This mixture allows a comprehensive description of the overall distribution, including the spike at zero that it is observed in most sample data on wealth (see Clementi and Gallegati, 2016, for a review on empirical evidences and parametric models). The corresponding cumulative distribution function reads $\Phi_0(A) = \theta\Phi_{0,1}(A) + (1-\theta)\,\Phi_{0,2}(A)$, with

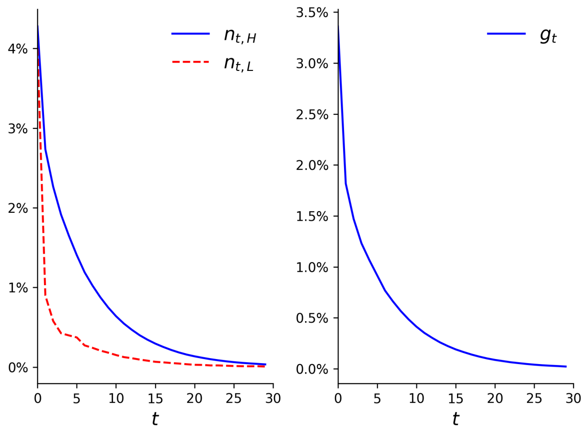
$$\Phi_{0,1}(A) = \begin{cases} 0, & \text{if } A < 0 \\ 1, & \text{if } A \geq 0 \end{cases} \qquad \Phi_{0,2}(A) = \begin{cases} 0, & \text{if } A \leq 0 \\ \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left[\frac{\ln A - \mu}{\sqrt{2\sigma^2}}\right], & \text{if } A > 0, \end{cases}$$

where erf is the Gauss error function. It follows that

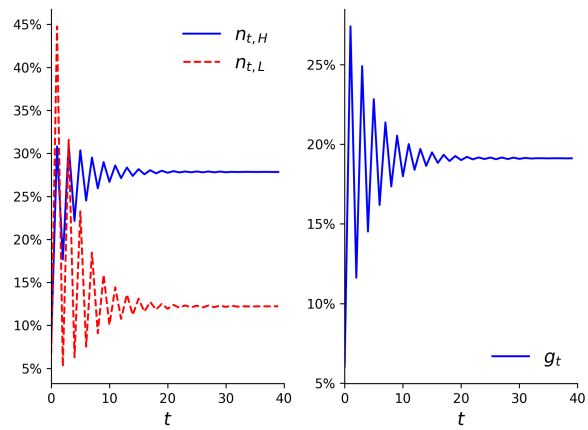
$$\Phi_0(A) = \begin{cases} 0, & \text{if } A < 0 \\ \theta, & \text{if } A = 0 \\ \theta + (1-\theta)\left\{\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left[\frac{\ln A - \mu}{\sqrt{2\sigma^2}}\right]\right\}, & \text{if } A > 0. \end{cases}$$

I set $\theta = 0.25$ and $\sigma \approx 1.80$. As a consequence, 25% of the agents at time 0 have zero wealth, and the initial wealth Gini coefficient is 0.847. These are in line with the corresponding US statistics found by Clementi and Gallegati (2016).

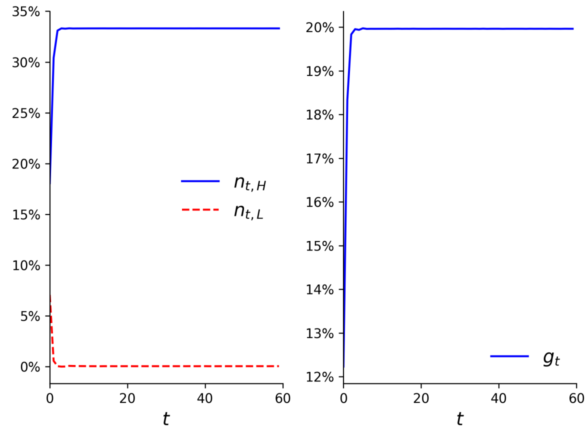
D.3 Quantitative Illustrations



(a) Stagnation



(b) Stable Growth



(c) Perfect Information

Figure A.5: Long-Run Equilibrium

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