ABSTRACT

The freshness of information is of the utmost importance in many contexts, including V2X networks and applications. One measure of this metric is the Age of Information (AoI), a notion recently introduced and explored by several authors, often with specific reference to vehicular networks. With this work, we explore the possibility of reducing the AoI of multi-hop information flooding in V2X networks exploiting the properties of the Eigenvector Centrality (EvC) of nodes in the topology, and the possibility that each node computes it exploiting only local information and very easy computations, so that each node can autonomously adapt its own networking parameters to redistribute information more efficiently. Starting from theoretical bounds and results, we explore how they hold in urban-constrained topologies and compare the AoI achieved exploiting EvC with the AoI achievable without this optimization of the nodes’ behavior. Simulation results show a meaningful improvement without using additional resources and without the need of any global coordination.

CCS CONCEPTS
• Networks → Network performance analysis; Routing protocols; Packet scheduling;

ACM Reference Format:
Density Function (ECDF) measured with our policy is a stochastic inferior of the ECDF measured without the use of the receiver equal policy.

2 BACKGROUND AND MODEL

We briefly recall here key results and notation from the literature, defining AoI and REf.

2.1 Age of Information on Networks

We mainly refer to [8] for the definitions, and we consider a flow of generated packets labeled 1, 2, ..., z, ... Without loss of generality, but differently from [8], we consider a discrete time system\(^1\) with time index \(k\), and we focus on the case of periodic information generation with inter-arrival time \(D\), leaving the analysis of stochastic inter-arrivals for future work. The age of information at node \(d\) at time \(k\) with respect to a given source of information \(s\) is:

\[ \Delta_{sd}(k) = k - u_{sd}(k) \]

where \(u_{sd}(k)\) is the generation time of the last packet received by \(d\) from \(s\). We call \(k_{sd}\) the reception time of packet \(z\) from node \(s\) by node \(d\), and we call \(F_{sd}\) the random variable (RV) that describes the dissemination delay of a message from a source \(s\) to node \(d\), hence,

\[ k_{sd}^z = zD + F_{sd} \]

and

\[ u_{sd}(k) = \max(zD, \forall z \mid k_{sd}^z \leq k) \]

Interpreting \(F_{sd}\) as the service time of a D/G/1/\(\infty\)/FIFO queuing station, with the same reasoning of AoI [8], the steady state AoI between nodes \(s\) and \(d\) is

\[ \Delta_{sd} = \lim_{k \to \infty} \Delta_{sd}(k) = \frac{DE[F_{sd}] + E[D^2]/2}{D} \]

Given a source \(s\), mediating over all destinations \(d\) we get

\[ \bar{\Delta}_s = \frac{1}{N-1} \sum_{d \neq s} \Delta_{sd} \]

and mediating over all \(s\)

\[ \bar{\Delta} = \frac{1}{N} \sum_s \bar{\Delta}_s = E[F_c] + \frac{D}{2} \]

where \(N\) is the number of nodes in the network, and \(F_c\) is the RV obtained by the superposition of all \(F_{sd}\). Unfortunately \(F_{sd}\) are not i.i.d RV so we cannot in principle make approximations based on the central limit theorem; however, it can be interesting in the future to explore how such an approximation remains close to measured results as a function of the network topology. In the derivation of Eq. (3), there is the implicit assumption that the network maintains the ordering of packets, which is not necessarily true in reality. Simulation results, where this assumption is not verified, show that the approximation in the model does not hamper its validation.

2.2 A Bound on Receiver Equal flooding

In [1] and [2] we have derived the conditions that guarantee an optimal performance in streaming/flooding given that the overall resources allocated to the process remains constant and minimal, i.e., the the total number of times a packet is propagated by any node is not changed from one flooding strategy to another. We briefly summarize here these results and a stochastic upper bound for the flooding delay that is valid independently from the network topology given some basic properties are guaranteed. A cycle \(T\) in the following can be interpreted as the elementary time step of a discrete time model.

Let \(A'\) be a stochastic transition matrix for an undirected graph \(G(V, E), |V| = N,\) so that the element \(A'_{ij} \in [0, 1] \iff (i, j) \in E.\) \(A'_{ij}\) represents the probability for node \(j\) to send a packet to node \(i\) during a cycle \(T\) and \(\bar{\Theta} = \bar{\Theta}^T.\) Let \(\Theta_i\) be the throughput (in terms of packets sent per cycle) that node \(j\) sustains on average in a cycle and \(\Theta\) the resulting column vector. We start from a condition in which flooding in a network is obtained by having each node sending one packet per cycle (\(\Theta = \bar{\Theta}\)) to one of its neighbors at random (represented by \(A'\)). We can call this strategy Sender Equal flooding (SEf), and we improve it introducing \(\Theta\) and \(A\) as follows, (from Theorem 1 in [1]):

\[ \Theta_j = x_j \sum_{l=1}^N \frac{A'_{lj}}{x_l}, \quad A_{ij} = \frac{\lambda'_{ij}}{\sum_{l=1}^N \lambda'_{lj} x_l} \]

such that:

\[ \bar{\Theta} = \Theta \]

\[ |\Theta| = |\bar{\Theta}| = N \]

\[ \bar{\Theta}^T A = \bar{\Theta}^T \]

\[ A'_{ij} = 0 \iff A_{ij} = 0 \]

where \(x_i \in \mathbb{R}\) is the PageRank centrality of node \(i\). The theorem states that the new stochastic transition matrix \(A\) describes the same links as \(A'\) but with different probabilistic weights (Eqs. (7) and (8)), and \(\Theta_j\) represents the number of packets node \(j\) sends during a cycle \(T\). In other words starting from a condition in which flooding in a network is obtained by having each node sending one packet to one of its neighbors at random we obtain a condition where nodes send more or less packets per cycle depending on their centrality, and they send them to neighbors with a

\(^1\)The reason of the discrete time modeling is due to the mathematical framework that allows the derivation of REf, but we are confident that results holds in continuous time too.
probability that depends, again, on centrality. As a result we equalize the probability of reception while the overall amount of resources used remains the same. It is important to stress that this optimization can be performed locally, through neighbour gossiping [1].

As reported in [2], it is possible to compute a stochastic upper bound $\Omega(k)$ for the probability of reception under RE that holds independently from the network topology, namely:

$$
\begin{align*}
\Omega(k+1) &= 2\Omega(k) - \frac{3}{2}\Omega^2(k) + \frac{\Omega(k)}{2} \\
\Omega(0) &= \frac{1}{N}
\end{align*}
$$

Eq. (9) express the Cumulative Density Function (CDF) of the upper bound of the flooding delay. As we work in discrete time this is a step function, and its derivative is a Probability Mass Function (PMF). By way of example, Fig. 1 reports the PMF of this upper bound for any network with 100 nodes.

Let $F_\Omega$ be the RV that describes the stochastic bound in Eq. (9), then by substituting $F_c = F_\Omega$ in Eq. (3) we obtain an upper bound $\gamma$ of the average AoI independent from the network topology:

$$
\gamma = E[F_\Omega] + \frac{D}{2}
$$

### 3 V2X NETWORK MODEL

The theoretical framework derived in Section 2 is valid for (almost) any topology, but clearly introduces approximations and assumption that need some form of validation. In particular, RE optimizes the average AoI, but does not guarantee that the average is not reduced penalizing marginal nodes, thus we need to verify distributions and different sources and topologies. Another interesting analysis regards the bound $\gamma$ defined in Eq. (10): again it is a bound on the average, so how often it is violated by outliers? Or, in other words, can we use this bound safely for dimensioning V2X multi-hop communication systems?

Furthermore, as we already noted the theory assumes that packets are delivered in order, while in a network there might be mis-ordering due to packets that follow different routes. This probability may be minimized if $D >> 1$, however to test how much reordering may affect our results we set, in simulations, $D = 1$. Finally, we want to test our results in topologies that are somewhat realistic in terms of Vehicular Communications. Being our department based in Trento, Italy, we focus on the local street map, which can be easily obtained through OpenStreetMap and is represented in Fig. 2.

The street map is centered at the GPS coordinate (46.0585, 11.1228) and has a radius of 1 km. On this map we randomly place 100 nodes (vehicles or road side units is irrelevant) uniformly with respect the street lengths and the number of street lanes in an area of radius 500 m. Different placements results in different topologies, all characterized by the same "urban" constraint. With 100 nodes and $D = 1$ Eq. (10) yields $\gamma \approx 6.93 + \frac{1}{2} = 7.43$, and this will be our base reference for analysis.

To fix ideas we can imagine that this number refers to some standard message inter-arrival time, like the 100 ms of CAMs, and the multi-hop distribution system piggy-backs information to be disseminated on the same periodic message. Other scenarios include TDMA-based technologies, 5G CV2X [5] where resources are allocated to nodes (vehicles) by the base station, or networks of fixed V2X nodes (Road

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2https://www.openstreetmap.org
3We are aware that sticking to standards CAMs do not work like described, but these are just examples to help mapping the theoretic framework on real scenarios.
Side Unit (RSU), traffic lights, etc.). Also Visible Light Communications (VLC) networking, where clearly information need multi-hop to disseminate [10, 11].

To keep the scenario simple, once the vehicles or RSUs are placed randomly on the street topology respecting street lengths and width to modulate density, the simulator create bidirectional links among all nodes that are less than 300 m apart. An example of graph generated through our model is presented in Fig. 3. For the time being we do not consider buildings, so the network we derive is not representative of technolgies that require strict line of sight, an analysis left for future work.

During simulations, we consider each node as a data producer with rate $D = 1$, and all nodes are interested in receiving all information, thus requiring flooding in the network. Data multiplexing into aggregated packets and creation of node clusters with a backbone to orchestrate the distribution can be considered, but they can only improve the basic performance gains we discuss here, obviously reducing the bound in Eq. (10).

4 RESULTS

We use an event driven simulator\(^4\) computing $\bar{\Delta}_{sd}$ for all the receiving vehicles $d \in V$ given a sender $s$ and we iterate over all nodes as senders as well as repeating the distribution for 100 messages, so that every simulation yields $9.9 \times 10^5$ message delivery delays. As already mentioned we set $D = 1$ which is the worst case scenario, sample simulations with different $D$ confirm the results.

\(^4\)The same we already used in [1, 2] and that will be made available to the community as soon as it is stable and documented.

We report data with boxplots, with minimum and maximum at the whiskers and 10th and 90th percentiles for the boxes. We are interested to compare the RE\(_f\) flooding strategy we have proposed with the theoretical bound $\gamma$ that we have found and with the “traditional” flooding strategy, where every node sends the same amount of information and that we call SE\(_f\). The two strategies use, for the entire network, exactly the same amount of transmission resources, so that the comparison is fair, and they also use similarly “blind” scheduling policies based only on a weighted random selection of the neighbors based on the stochastic adjacency matrix. Heuristic scheduling optimizations are easy for both schemes (e.g., do not send the message to the node that has sent you the information and blacklist nodes that you have already sent the message to), but would make the results not coherent with the bound $\gamma$. For this reason we do not explore heuristics here, though we are confident that they leave the relative performance of RE\(_f\) and SE\(_f\) unaltered, or even improve the relative RE\(_f\) performance because of the additional knowledge on the topology that is inherent to the Receiver Equal strategy.

Out goal is to answer these questions:

1. How do RE\(_f\) and SE\(_f\) strategies perform in terms of mean AoI?
2. What are the advantages of the RE\(_f\)?
3. How does the performance of RE\(_f\) compare with the stochastic bound of Eq. (10)?

For the same map area of Trento, we consider three possible vehicular networks, one of them is represented in Fig. 3. Fig. 4 reports AoI $\bar{\Delta}_s$ mediated over all destinations given a sender $s$. Simulations iterate over all the vehicles as senders generating 100 different messages per sender, so each boxplot includes 9900 points.

As can be easily noted, RE\(_f\) performs consistently better than SE\(_f\). In fact, the RE\(_f\) mean AoI $\bar{\Delta}_s$ is reduced both...
in terms of the absolute maxima and in terms of distribution mass, while the absolute minima remains comparable as expected. These results show that, keeping constant the overall amount of allocated resources, RE\textsubscript{f} grants a distribution boost and a consequent improvement in the AoI. Indeed, these initial measures hints to the possibility that RE\textsubscript{f} stochastically dominates SE\textsubscript{f}, an interesting possibility that might be proven in the future.

The stochastic bound of Eq. (10) states that, on average, the AoI \(\bar{\Delta}\) achieved using RE\textsubscript{f} on a network of 100 nodes is lower than 7.43. The results of Fig. 4 not only corroborates this theoretical result, but indicate that, at least for these networks, more than 90% of the values of \(\bar{\Delta}\) using RE\textsubscript{f} remain below the threshold, indicating that bound \(f\) in Eq. (10) is an extremely powerful means to dimension with great simplicity complex dynamic distribution systems.

In other words, these results indicates that not only RE\textsubscript{f} performs consistently better than SE\textsubscript{f}, but it also comes with a theoretical framework that yields a stochastic bound that is not available for SE\textsubscript{f} and that can be of paramount importance for designing and realizing real-time soft-constrained flooding systems in vehicular networks.

Although the aggregated mean AoI \(\bar{\Delta}\) allows a good insight in the overall performance on the entire network, we are also interested in studying if any receiver or any sender may be penalized by RE\textsubscript{f}. To do this we aggregate the simulation measures for one of the scenarios in sender or receiver specific boxplots. Namely, we collect \(\bar{\Delta}\)\textsubscript{sd} (see Eq. (1)) so as to build a boxplot of all \(\bar{\Delta}\)\textsubscript{s} (see Eq. (2)) as a function of the receiver, or all \(\bar{\Delta}\)\textsubscript{d} as defined in Eq. (11).

\[
\bar{\Delta}_d = \frac{1}{(N-1)} \sum_{s \neq d} \bar{\Delta}_{sd} \tag{11}
\]

We start analyzing \(\bar{\Delta}\), Figure 5 reports the boxplots of the AoI \(\bar{\Delta}\) as a function of the receiver vehicle. The x axis reports 50 out of 100 vehicles ordered by increasing AoI to improve readability, results for the other vehicles are stochastically identical to those reported, and we were careful to avoid “cutting” possible outliers. Again RE\textsubscript{f} performs consistently better than SE\textsubscript{f} and we can see that the stochastic bound \(f\) of Eq. (10) is valid for all vehicles, and for most vehicles the 90th percentile is below \(f\). This result is due to the Reception-Equal property which optimizes the network resources so to grant a propagation of data across the network as uniform as possible. This results are fundamentally the same presented in Fig. 4, but here it is possible to appreciate that no receiver is penalized in any way, while in Fig. 4 there is no way to see if all the outliers of the boxplot belong to the same vehicle or not.

Now we want to analyze if RE\textsubscript{f} penalizes any sender. To this end Fig. 6 reports the boxplots of the AoI \(\bar{\Delta}\)\textsubscript{d} as a function of the sender vehicle. Also in this case RE\textsubscript{f} performs consistently better than SE\textsubscript{f} both in terms of mean AoI maxima and mass. The stochastic bound of Eq. (10) still holds for all the nodes, although for the nodes with worst performance, probably located in a peripheral fringe of the network, the 90th percentile is above \(f\), which is perfectly consistent with the theory, albeit maybe a bit annoying from a dimensioning point of view. Recall, however, that no optimization heuristic is applied here, and we deem that a properly crafted heuristic will not only reduce \(\bar{\Delta}\)\textsubscript{sd} on average, but also “compact” the distribution, improving the performance for outliers (topologically peripheral nodes) more than that of central nodes.

5 DISCUSSION AND FUTURE WORK

AoI has been introduced to capture the concept of information freshness from the receiver point of view. In the context of vehicular networks this metric can have significant importance due to the existing communication applications that require periodic updates. Our proposed flooding strategy, called Receiver Equal flooding, optimizes the network resources for data flooding and it can grant performance improvements on the AoI for vehicular networks compared to traditional strategies that do not exploit topological properties of the network. We remark that RE\textsubscript{f} achieves this with zero signalling and without building a specific distribution overlay (e.g., a tree) on top of the network, so it is perfectly suited for a highly dynamic and time-varying environment as V2X communications. In this work, we derived and validated through simulations a stochastic bound \(f\) on the mean AoI \(\bar{\Delta}\), a bound that depends only on the number of nodes in the network and is entirely independent from the topology. Moreover, we have shown experimentally that RE\textsubscript{f} is superior to the more wide-spread SE\textsubscript{f} in terms of performance.

This contribution is just a first step in exploiting the Eigenvector Centrality in the management of V2X communications, and much work lies ahead to improve the bound, possibly finding ways of fine-tuning it either to some network properties (e.g., average number of links per node) or to scheduling heuristics, for instance finding a deterministic worst-case bound that can be used for hard real-time applications and not a stochastic one that discounts the lack of a worst case dissemination delay inherent to a blind random choice of the nodes’ neighbors. We stress that this limitation is due to the theoretical analysis to obtain the bound and is by no way inherent to RE\textsubscript{f} strategy, which can be computed and used with any scheduling strategy.

REFERENCES

Figure 5: Boxplots of $\bar{\Delta}_s$ as a function of the receiver vehicle.

Figure 6: Boxplots of $\bar{\Delta}_d$ distribution as a function of sender vehicle.


