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# Revenue and attendance simultaneous optimization in performing arts organizations

**Andrea Baldin**[1] · **Trine Bille**[1] · **Andrea Ellero**[2] · **Daniela Favaretto**[2]

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**Abstract** Performing arts organizations are characterized by different objectives other than revenue. Even if, on the one hand, theaters aim to increase revenue from box office as a consequence of the systematic reduction in public funds; on the other hand, they pursue the objective to increase its attendance. A common practice by theaters is to provide incentives to customers to discriminate among themselves according to their reservation price, offering a schedule of different prices corresponding to different seats in the venue. In this context, price and allocation of the theater seating area is decision variables that allow theater managers to manage their two conflicting goals to be pursued. In this paper, we introduce a multi-objective optimization model that jointly considers pricing and seat allocation. The framework proposed integrates a choice model estimated by multinomial logit model and the demand forecast, taking into account the impact of heterogeneity among customer categories in both choice and demand. The proposed model is validated with booking data referring to the Royal Danish theater during the period 2010–2015.

✉ Andrea Baldin
ab.mpp@cbs.dk

Trine Bille
tbi.mpp@cbs.dk

Andrea Ellero
ellero@unive.it

Daniela Favaretto
favaret@unive.it

[1] Department of Management, Politics and Philosophy, Copenhagen Business School, Porcelænshaven 18B, 3.Floor, 2000 Frederiksberg, Denmark

[2] Department of Management, Ca' Foscari University of Venice, Cannaregio 873, 30121 Venice, Italy

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## 1 Introduction

In the seminal article by Baumol and Bowen (1966), the authors claim how theaters will be more and more dependent on subsidies, due to their productivity lag. However, the last decades' tendency shows that public funds allocated to nonprofit performing arts organizations in Western countries (Marco-Serrano 2006) are decreasing.[1] This fact has forced theaters to increase other sources of revenue, including box office revenue. In addition, such organizations pursue the aim to increase the attendance, for a couple of reasons: first, they feel the mission is to spread culture as broad as possible (Hansmann 1981) legitimizing their social value; second, they prefer to avoid empty seats in the venue that can have a negative effect on the reputation of the theater.

In this context, managers of the performing arts organizations can implement revenue management (RM) techniques (see, e.g., Talluri and Van Ryzin 2006) in order to balance between the rate of occupancy and the profitability of theater. The most common among these techniques is realized through market segmentation based on the price leverage that leads to different pricing schemes. For instance, price reductions are offered to customers' segments, such as students and senior citizens, who are supposed to look for more affordable prices. Discounts are offered also to those customers—subscribers—who buy in advance a bundle of tickets, assuring a long-term commitment toward the theater. Due to heterogeneity in price sensibility within the same customer segment, one usual practice applied by theaters is to use a nonlinear tariff system offering a schedule of different prices according to the quality of the product. In this case, different prices are charged according to the seat location in the venue in order to better capture consumers' willingness to pay. Indeed, this mechanism incentivizes customers to discriminate by themselves in choosing the seating area they prefer. So, beside the pricing strategy, also the seat allocation across these fare classes (i.e., seating area) represents a decision that may encourage an orientation of the theater toward either the maximization of the total attendance or the maximization of revenue. In the first case, we expect that theater would increase the accessibility of the most expensive seating areas for all the customers: to do this, it is convenient to propose a scheme in which the prices of the different seating areas is reduced and less varied. This scheme will lead to an increase in the size of the expensive seating area and, in addition, can favor a customer buy-up behavior (i.e., buying a ticket for a more expensive fare class when the ticket

[1] This framework holds also for our case study: the Royal Danish theater. According to the National Danish Statistics (http://www.statbank.dk), the public subsidy to the Royal Danish theater decreases from 608,675 Danish crowns in the 2011/2012 season, to 573,900 Danish crowns in the 2014/2015 season.

for the required seating area is not available). In the second case, we expect that theater would strengthen the self-discrimination exhibited by customers. Thus, the allocation policy will strongly depend on the type of customer attending the performance: if the performance attracts an audience group (as young customers) that is supposed to be highly price sensitive, the theater would enlarge the cheapest seating area in order to prevent a loss in revenue. In the opposite case, the theater would take advantage of the inelastic demand by enlarging the expensive seating area.

Considering this pricing and allocation strategy, not only the demand forecasting becomes essential, but also the understanding of the customers' behavior with respect to price discrimination by seating area. Since the paper by Talluri and Van Ryzin (2004), discrete choice models have emerged as a standard approach to incorporate the buy-up and buy-down behavior.

This paper proposes an optimization model that considers the pricing and allocation problem in the performing arts context. To this end, the demand forecasting is integrated with a customer choice model. In order to accommodate for heterogeneity in preference over seating areas, we adopt a multinomial logit model (MNL) using customer's characteristics and performance-production attributes as variables to be interacted with the characteristics of the choice alternatives.

Our model has been implemented to a data set provided by the Royal Danish theater which refers to the period 2010–2015. A simulation is conducted considering three performances that differ from each other by characteristics affecting the demand.

The remainder of the paper is organized as follows: Sect. 2 presents the relevant literature on demand-management decisions in the performing arts context; Sect. 3 describes the research framework, whereas Sects. 4 and 5 present, respectively, the demand estimation and the choice model. Section 6 describes the optimization model, whereas Sect. 7 presents the results of our simulation. Finally, Sect. 8 provides some conclusions.

## 2 Literature review

The literature of cultural economics has been dealing with the objectives of performing arts institutions. Since most performing arts institutions are nonprofit firms, this taps into a more general literature on the objectives of nonprofit firms (e.g., Hansmann 1980; Steinberg 1986). Steinberg (1986) suggests that nonprofit firms are either service maximizers or budget maximizers or something in between. However, in the performing arts, the concept of service is not straightforward. Several authors (e.g., Throsby et al. 1993; Throsby 1994; Hansmann 1981) have suggested three different measures of output: (1) quality, (2) audience size and (3) budget. Several empirical studies have shown that the performing arts are primarily output maximizers (either quality or quantity), and less budget maximizers (see, e.g., Luksetich and Lange 1995; Gapinski et al. 1985). For an overview of the literature, see Brooks (2006). To our knowledge, no studies have been made dealing with the optimization decisions in the performing arts when the repertoire is planned (based on quality decisions), while the theater wants to make the optimal decision on how to maximize attendance as well as revenue, basing this decision on prices and seat categories.

Most of the research related to the demand-management decision in the theater sector has focused on the price discrimination practice. Hansmann (1981) claims that in the nonprofit performing arts sector, price discrimination is not effective due to the difficulty of identifying customers with inelastic demand. Therefore, according to the author, the only form of discrimination that nonprofit enterprises can apply is by asking for a voluntary donation, in order to extract a part of consumer's surplus. Seaman (1985) raises some doubts about Hansmann (1981) hypotheses: the author measures the degree of price discrimination (such as the number of different prices charged and the standard deviation of the prices charged) to a set of nonprofit performing art organizations. He concludes that price discrimination varies significantly across art forms (opera, ballet, theater, symphony concert) and that the organizations that discriminate more are characterized by a high ratio between fixed cost and attendance. Huntington (1993) justifies the adoption of price discrimination by seating area, by referring to Rosen's utility model [i.e., the hedonic price model, see Rosen (1974)], as there are observable differences between different seats. Moreover, the author compares the box office revenue between theaters operating a single price policy and those operating a discrimination pricing policy: he finds that the price range policy is statistically significant and positively correlated with the revenue of the theater, controlling for seat capacity and the number of performances per year. Rosen and Rosenfield (1997) describe a model in which theater venue has two types of seats: (high and low quality), and the theater manager knows the distribution of reservation price for both seat categories. First, the authors solve the pricing problem, given the quantity of seats for each category. Second, the authors solve the allocation problem, given the optimal pricing policy. Leslie (2004) considers the Broadway show "Seven Guitars" and estimates a structural econometric model of price discrimination based on an individual consumer behavior model that incorporates all the types of price discrimination (by seating area and social category). The model allows him to perform different experiments using alternative pricing policies. Tereyagoglu et al. (2012) use the data from the ticket purchase transactions of the shows of a symphony orchestra in the northeast region of the USA, in order to employ a proportional hazard framework to analyze how pricing and discount actions affect the timing of customers purchase over time.

## 3 Research framework

### 3.1 The Royal Danish theater

The Royal Danish theater was founded in 1748 and is the Danish national theater. It has three main Stages in Copenhagen. The Old Stage from 1874, a new Royal Opera House from 2005 and a new Royal Playhouse from 2008. The Opera House and the Playhouse have a main stage and smaller stages for experimental productions. It is one of the few theaters in the world offering both opera, ballet and theater performances as well as classical concerts. Today, The Old Stage is the house where ballet is performed.

The law of the Royal Danish theater states that it is the national theater for the whole country and the entire population. Besides, it has an obligation to produce a broad repertoire of high artistic quality among ballet, opera and plays. It is required to continue the classical traditions as well as developing the performing arts in new and contemporary ways, with a special attention on productions of Danish origin.

The Royal Danish theater is on the state budget under the Ministry of Culture and has a number of more specific obligations in agreement with the current Minister of Culture. Included in these obligations, there are general cultural policy goals, such as having special productions for children and youth, and keeping prices to a level that makes the theater accessible for all socioeconomic groups.

In 2015, the theater had a total budget of 705.4 million DKK (94 million Euros), of which 76% were public support from the Government. The theater had 165.8 million DKK (22 million Euros) in own earnings, of which 69% (15 million Euros) was from ticket sales, the rest was income from sponsors, etc.

Due to its obligations as a national theater, it has to decide its repertoire based on a number of parameters, namely quality and variety, understood as a fairly large number of different productions from the classical repertoire as well as new productions, developing the performing arts, of Danish as well as international productions.

In addition, it has to decide the number of performances of each production during the season and how they are scheduled on weekdays and weekends. It should be noticed that when a given production is played less than demanded by the audience as well as if a performance is played more times than demanded, it will create a loss in earnings. Moreover, there are high fixed costs in taking a new producing on stage (due to rehearsal time, designing the staging, etc.), while the costs of prolonging a production with extra performances are small, and the marginal costs are lower than the marginal revenue (Hansen 1991).

Finally, the theater has also to decide its price policy, including price differentiation based on different audience groups (like young, senior people and subscribers) as well as seat categories, time of the performance, the type of the performance, the production costs, etc.

### 3.2 Problem description

In this paper, we assume that the repertoire decisions are already determined by the theater, both with regard to the variety of productions and the number of performances of each production during a season. With this assumption, the theater has to decide on the price and the allocation of seat categories for the individual performances. It is assumed that the theater wants to optimize both attendance and revenue, where the former finds an upper limit in the theater capacity. Thus, we will consider a bi-objective optimization model that incorporates the demand forecast and the customers' seat choice model. We will adopt a two-step procedure: firstly, we estimate a demand function, in order to predict the total attendance of each performance. One of the independent variables of the demand function is the average price across seats, which reflects the level of price charged in terms of expensiveness. It is true, on the one hand that the average price depends also on the non-purchased seating area; however, on the other hand, the relationship among the price of the different tiers follows a regular pattern in terms of ratio. We can then suppose that the general level of price affects the decision to attend/not attend the performance. In the second step, after the decision to attend a production, the consumer decides the seating area. This decision is modeled by means of a multinomial logit (MNL) model that predicts the probability to choose a particular seating area as a function of price and performance characteristics. Hence, it is in the second step that the price of each seating area are considered, as it affects the customer's choice The methodological procedure in this paper follows the study by Hetrakul and Cirillo (2014) that proposes, in a railway setting, an optimization model in which discrete choice models and demand function are integrated, in order to calculate the price and fraction of the demand to be accepted for each origin–destination pair.

## 4 Demand forecast

### 4.1 Sample selection

The demand estimation is based on booking data from the sale system of the Royal Danish theater for the period 2010/2011 to 2014/2015. The sample consists of 401 opera performances which took place during that period. We estimate a demand function for two customers' categories identified: standard ticket buyers and young/student customers. Indeed from Felton (1994) and Baldin and Bille (2018), we know that some audience groups (especially young people) are quite price sensitive, while other groups are less price sensitive (e.g., standard ticket buyers and subscribers). Among the numerous existing price types in the price discrimination process across buyers, the standard ticket buyers (i.e., the ones who pay the full price ticket) represent a large portion of the total attendance (46% in our sample). Another important customer segment is identified as subscribers, which account for 26% of the total attendance. However, they represent a different kind of differentiation, as the theaters can decide which performance subscribers will attend. Thus, the theater already knows how many subscribers will attend a given performance in advance of the date of the performance. The third customer category in terms of size is represented by young (under 25 years)/student customers (6% of the total tickets sold) for which tickets are discounted by 50%.

For the purpose of model simplicity, there are some remarkable categories that, given their low number of attendees per performance, are not considered. For example, tickets for senior customers, who are entitled to a discount of 50%, represent only 2.5% of the tickets since this discount is made available only for some performances decided by theater management. Indeed, as many senior customers are subscribers, it is not convenient to offer this discount for all the performances. We exclude also other price types as the customers with a loyalty card, employees, group sales, disabled and so on. Hence, for the reasons exposed above, we will consider two customers' categories: standard ticket buyers and young/students customers.

### 4.2 Demand estimation

Following the literature, we adopt a double-log specification, which is the most popular functional form adopted in estimating theater-attendance demand (Seaman 2006). For each category $j$, the following demand function is estimated[2]:

$$\ln(D_j) = \alpha_j + \beta_j \ln(p_j) + \gamma_j' z + \epsilon_j \tag{1}$$

so as:

$$D_j = \exp\left(\alpha_j + \beta_j ln(p_j) + \gamma_j' z + \epsilon_j\right) \tag{2}$$

where, for a given performance, $D_j$ is the number of tickets sold to category $j$, $p_j$ is the ticket average price of deflated by CPI[3] charged to category $j$: as anticipated in Sect. 3.2, we take the advertised average price of the different seat categories offered by the theaters. $z$ is a vector of performance and production characteristics, while $\epsilon_j$ is an error term.

Concerning the performances scheduling, we include three dummy variables to take into account the weekly seasonal effect: WKDAY denotes performances run during weekdays (from Monday morning to Friday morning); WKEND indicates performances run during Friday and Saturday evenings or during the evening before a public holiday. Finally, SUNDAY denotes performances that take place on Sunday or in a public holiday. This latter group of performances is “matinée” as no evening performances take place on Sunday. Besides Sundays, in the other days of the week, performances can take place either on Monday afternoon or during the evening. We denote with EVE performances that take place during the evening.

In order to capture the seasonality effect, we construct monthly dummy variables for each month of the year, except for July and August when the theater is closed.

In addition, following Corning and Levy (2002) we also include REMAIN and TOTPERF denoting, respectively, the number of remaining and total performances of a given production. We also find a significant interaction between these two variables: indeed, this interaction term allows to weigh the amount of remaining performance with respect to the total number of performances.

We also control for the production characteristics: to capture the popularity of an opera show, we introduce the variable POP measured as the number of times the production is performed worldwide during the same year it has been performed at the Royal Danish theater.[4] However, it should be considered that some Danish

[2] We are aware that a potential problem of partial endogeneity may exist, as the variation of price also reflects different quality factors not explained by the model. However, the main sources of price variation are represented by factors included in the demand estimation, such as time and day of the performance, if the performance is run for the first time at the Royal Danish theater, and so on.

[3] CPI data are collected by *Statistics Denmark*: http://www.dst.dk/en.

[4] We collect these data through “Operabase,” a website designed to collect statistics about operatic activity worldwide: http://operabase.com.

**Table 1** Descriptive statistics of OLS variables

| Variable | Mean | SD | Min | Max |
|---|---|---|---|---|
| Price (standard ticket) | 456.06 | 74.93 | 208.96 | 661.13 |
| Standard tickets sold | 562.34 | 252.11 | 62 | 1117 |
| Young tickets sold | 73.46 | 63.65 | 0 | 576 |
| REMAIN | 7.49 | 5.38 | 1 | 30 |
| TOTPERF | 14 | 6.07 | 6 | 30 |
| CAPACITY | 1482.89 | 45.51 | 1297 | 1529 |
| POP | 213.17 | 186.00 | 1 | 507 |
| SUNDAY | 0.174 | 0.380 | 0 | 1 |
| WKEND | 0.257 | 0.437 | 0 | 1 |
| WKDAY | 0.568 | 0.496 | 0 | 1 |
| EVE | 0.733 | 0.443 | 0 | 1 |
| JANUARY | 0.157 | 0.364 | 0 | 1 |
| FEBRUARY | 0.117 | 0.322 | 0 | 1 |
| MARCH | 0.149 | 0.357 | 0 | 1 |
| APRIL | 0.115 | 0.319 | 0 | 1 |
| MAY | 0.147 | 0.355 | 0 | 1 |
| JUNE | 0.047 | 0.2127 | 0 | 1 |
| SEPTEMBER | 0.027 | 0.163 | 0 | 1 |
| OCTOBER | 0.085 | 0.279 | 0 | 1 |
| NOVEMBER | 0.125 | 0.331 | 0 | 1 |
| DECEMBER | 0.030 | 0.171 | 0 | 1 |
| 1920–2015 | 0.160 | 0.366 | 0 | 1 |
| 1850–1919 | 0.486 | 0.500 | 0 | 1 |
| BEFORE 1850 | 0.354 | 0.479 | 0 | 1 |
| DANISH | 0.027 | 0.163 | 0 | 1 |
| NEW DKT | 0.651 | 0.477 | 0 | 1 |
| *t* | 3.06 | 1.295 | 1 | 5 |

401 observations

productions (e.g., *Maskarade*, *Livlægens besøg*) are popular in Denmark but not worldwide. To control for this aspect, we include the dummy variable DANISH, denoting Danish productions. Moreover, the dummy variable NEWDKT controls for productions that take place for the first time at Royal Danish theater.

In addition, we control for the year in which the production was created by introducing three dummies: 1920–2015, 1850–1919, BEFORE 1850.

As our analysis is based on performances running throughout 5 years, we include a time trend variable *t*. Finally, considering that the total capacity of the theater can change due to production requirements and fire code regulations, we add the variable CAPACITY indicating the number of the available seats for a specific show.

Table 1 provides a descriptive statistics of the data.

We estimate (1) by OLS with robust standard error. Although more sophisticated models are available for a forecast analysis (Ainslie et al. 2005), such techniques do not necessarily provide a significant improvement (Andrews et al. 2008; Eliashberg et al. 2009).

We also have checked for multicollinearity issues that do not seem to arise.

Table 2 shows the estimation results of the demand function for all the categories considered.

Results of the demand estimation reveal that price elasticity differs across the two customer categories. Young customers are the most price sensitive audience group: a 1% increase in ticket price results in approximately 1.84% decline in quantity demanded. Standard ticket buyers are less price sensitive as the price elasticity is less than unity: a 1% increase in ticket price results in approximately 0.49% decline in quantity demanded.

The results for the single ticket buyers show a strong explanatory power ($R^2 = 0.75$), and almost all variables are statistically significant. In particular, Table 2 shows that, for this type of customers, the demand is higher for Friday/Saturday evening performances. The number of times a title is rerun (TOTPERF), which is supposed to be an indicator of the total expected demand for that production, has a positive impact on the demand for a single performance. Moreover, given the same production, each performance has a 5.75% higher demand than the previous, keeping fixed the number of times a performance is rerun. This is probably due to a word-of-mouth effect (Laamanen 2013). Furthermore, we can deduce that single ticket buyers prefer traditional and less risky productions than the more experimental ones: indeed the productions that take place for the first time at Royal Danish theater have a negative impact on demand, whereas popularity score has a positive impact, as well as those productions composed before 1919.

Results for young customers have a lower explanatory power ($R^2 = 0.42$ ). For this kind of customers, there is a positive word-of-mouth and time trend effect. Furthermore, the Danish productions have a strong positive effect on demand, as well as the popularity of the production worldwide, but also the productions that take place for the first time at Royal Danish theater seem to be appealing to young customers. Table 3 compares the actual attendance with the values predicted by the demand functions. The prediction capability of the model is measured with different indicators, such as root-mean-squared error, mean absolute error, average error and Pearson correlation between predicted and actual. In addition, we perform the out of sample validation. We consider 74 performances run during season 2015/2016 that is not included in our sample. The demand functions for such performances are estimated using the coefficients obtained for our initial sample, and their final estimations are compared with the actual attendance.

Whereas the average errors are decidedly higher for the out of sample performances than the sample performances, the other measures are similar among the two groups of performances.

**Table 2** Estimation results of demand functions

| Variable | Single tickets | Young |
|---|---|---|
| Intercept | 2.3538** | 6.7917**** |
| | (*1.017*) | (*1.986*) |
| Log price | − 0.4904*** | − 1.8440**** |
| | (*0.1811*) | (*0.3994*) |
| SUNDAY | 0.22741**** | − 0.0459 |
| | (*0.0654*) | (*0.1321*) |
| WKEND | 0.4620**** | 0.0083 |
| | (*0.0357*) | (*0.0644*) |
| EVE | − 0.1205** | − 0.0220 |
| | (*0.0596*) | (*0.1106*) |
| REMAIN | − 0.0575**** | − 0.0483*** |
| | (*0.0089*) | (*0.0173*) |
| TOTPERF | 0.0365**** | − 0.0081 |
| | (*0.0049*) | (*0.0097*) |
| REMAIN × TOTPERF | 0.0020**** | 0.0018** |
| | (*0.0004*) | (*0.0007*) |
| JANUARY | 0.2743* | − 0.0458 |
| | (*0.1603*) | (*0.2493*) |
| FEBRUARY | 0.3370** | 0.1374 |
| | (*0.1590*) | (*0.2415*) |
| MARCH | 0.3383** | − 0.0316 |
| | (*0.1568*) | (*0.2350*) |
| APRIL | 0.4957*** | − 0.0492 |
| | (*0.1580*) | (*0.2384*) |
| MAY | 0.5726**** | − 0.1028 |
| | (*0.1542*) | (*0.2318*) |
| JUNE | 0.5192*** | − 0.2148 |
| | (*0.1631*) | (*0.2764*) |
| SEPTEMBER | − 0.2083 | − 0.9979*** |
| | (*0.1949*) | (*0.3619*) |
| OCTOBER | 0.0237 | − 0.3690 |
| | (*0.1597*) | (*0.2449*) |
| NOVEMBER | 0.0575 | − 0.2394 |
| | (*0.1554*) | (*0.2315*) |
| POP | 0.0007**** | 0.0022**** |
| | (*0.0001*) | (*0.0002*) |
| 1850–1919 | 0.6935**** | 0.1236 |
| | (*0.0758*) | (*0.1903*) |
| BEFORE 1850 | 0.6385**** | 0.1108 |
| | (*0.0743*) | (*0.1851*) |
| DANISH | − 0.1132 | 0.8734**** |
| | (*0.0858*) | (*0.1247*) |

**Table 2** (continued)

| Variable | Single tickets | Young |
|---|---|---|
| NEWDKT | −0.0648* | 0.1970*** |
| | (*0.0376*) | (*0.0771*) |
| CAPACITY | 0.0037**** | 0.0045**** |
| | (*0.0004*) | (*0.0009*) |
| *t* | −0.0101 | 0.0606** |
| | (*0.0164*) | (*0.0303*) |
| $R^2$ | 0.7512 | 0.4213 |
| Model *F*-value | 51.64**** | 13.22**** |
| No. of observations | 401 | 401 |

Values in italics (listed under the estimated coefficients) are the robust standard errors

****$p < 0.001$; ***$p < 0.01$; **$p < 0.05$; *$p < 0.10$

**Table 3** Predictive performance of the demand functions

| | Root-mean-squared errors | Mean absolute errors | Pearson correlation | Average errors |
|---|---|---|---|---|
| 2010/2011–2014/2015 | | | | |
| Single tickets | 148.33 | 114.55 | 0.78 | 12.76 |
| Young | 52.09 | 28.36 | 0.68 | 9.92 |
| 2015/2016 | | | | |
| Single tickets | 155.31 | 133.06 | 0.83 | −86.11 |
| Young | 53.52 | 30.69 | 0.56 | 12.11 |

# 5 Customer choice model

## 5.1 Sample selection

The choice model concerns the price discrimination across seating areas. The theater policy has been refined in the last years. In 2010, the *OperaHouse* offered 5 different price zones, 6 price zones in 2011 and 8 seating areas from 2012 onwards (Fig. 1).

The subdivision is not physically evident: for example, zone called "price A" includes both stall seats and first balcony seats, whereas zone called "price B" includes stall seats as well as first and second balcony seats, and so forth. This allows the theater manager to be quite flexible in the subdivision of the venue.

Since the number of price zones changed during the period under examination, we aggregated productions with more than five price zones into five seat

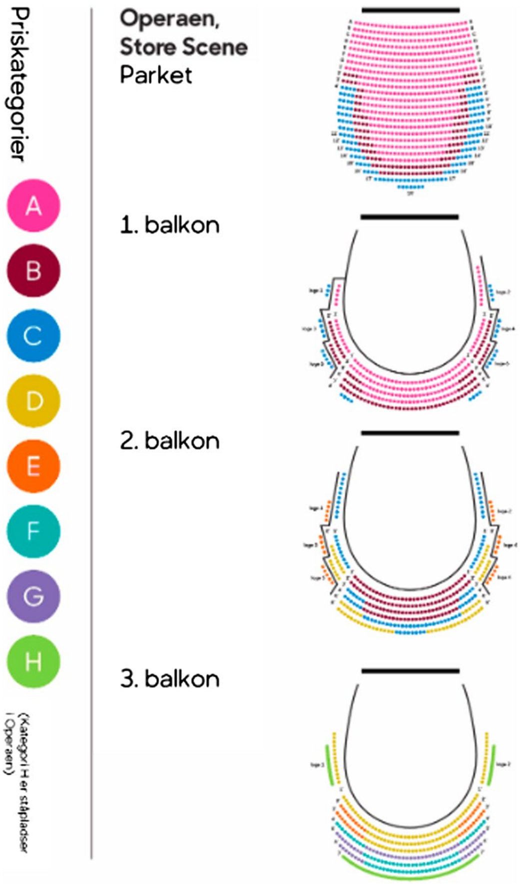


**Fig. 1** Price zones at the Opera House. *Source*: https://kglteater.dk/en/

categories, where the first seating area is the cheapest one and the fifth seating area is the most expensive one. The procedure adopted follows Baldin and Bille (2018), to which we refer for details.

Clearly, inside the same tier some seats are more valuable than other seats, raising the importance of queuing among consumers aiming to get the best seats in a given tier (Leslie and Sorensen 2013). Following Huntington (1993), we can consider each seat as a distinct good with its quality, according to its position in the venue. We suppose that it is possible to rank all the seats in the venue, such that the worst seat is assigned to the first seating area, and the best seat is assigned to the fifth seating area. Given the unrealistic possibility to apply a different price for each seats individually, in practice the reallocation of the number of seats considers the different quality of seats in a given tier. For example, suppose the model suggests the enlargement of the fourth seating area together with the reduction in the third seating area. This result is achieved by assigning the best seats of the third seating area to the fourth tier: such seats will become the worst seats of the fourth tier, but in any case all of them are better than any other seats of the third tier.

For logistic reasons, it has not been possible to collect data for the choice model estimation for the whole sample considered in the demand function. Our sample consists in 70,513 bookings which involve 11 opera productions and 122 performances.

### 5.2 Estimation of seat choice

After estimating the demand for each performance, in this section we propose a choice model for the seating area decision. To this aim, we adopt a multinomial logit (MNL) approach. Hence, we assume that each customer chooses the seat that maximizes her utility. The independent variables that enter in the model as the attributes of each choice are: *price* and a dummy variable for each seat category. These variables aim to capture the trade-off behavior between cheap seats with low visibility and/or acoustics, and more expensive high-quality seats. Moreover, in order to address heterogeneity, we allow the price sensitivity and the marginal utility of the seating areas to vary across customer categories. The price coefficient also interacts with variables related to the performance characteristics.

The utility of a customer that buys a ticket which refers to the seating area $s$, for the performance $i$, can be formulated as:

$$U_{sj} = V_{sj} + \epsilon_{sj} \tag{3}$$

with

$$V_{sj} = p_{sj} \cdot (\beta_1 + \beta_2 \cdot \text{young} + \gamma' z) + \text{seat}_s \cdot (\delta_1 + \delta_2 \cdot \text{young}) \tag{4}$$

where *young* is a dummy variable denoting whether the customer is a young customer. This implies that single ticket buyers are treated as the base category. $z$ is a vector of performance and production characteristics. In our estimation, such characteristics are represented by the dummy variables SUNDAY and WEEKEND, already defined in the demand function. Moreover, we used the number of times the production is performed worldwide during the same year, to define three dummy variables denoting the degree of popularity of the production: *Low popularity* (for productions run less than 50 times worldwide) treated as base variable; *Medium popularity* (for productions run between 50 and 150 times worldwide) and *High popularity* (for productions run more than 150 times worldwide). Finally, $\text{seat}_s$ is a dummy variable denoting whether the seat belongs to area $s$ or not. *Seat*1 is used as baseline in order to guarantee the identification of the model.

In (4), the price coefficient has a different interpretation than in (2). Whereas in the latter case it indicates the price elasticity, in the MNL it represents the effect of price on the odds of making a given choice. Notice that the customer category and the performance/production characteristics are variables that do not vary over alternatives. As only differences in utility matter in the estimation of the MNL model, one possible way to introduce choice invariant variables is to include them in the model specification only as interaction terms with the alternatives attributes (see Hensher et al. 2005; Train 2009) Assuming that the error components in (3) are independent and identically distributed according to a Gumbel distribution,

**Table 4** Estimation of multinomial logit model

| | Coefficient | $t$-stat |
|---|---|---|
| Price | −0.00109**** | −9.26 |
| Price—young | −0.00975**** | −23.82 |
| Price—popularity medium | −0.000309**** | −5.47 |
| Price—popularity high | 0.000143**** | −2.58 |
| Price—WKEND | 0.00001 | −0.04 |
| Price-Sunday | 0.000389**** | 8.21 |
| Seat 2 | 0.765**** | 27.92 |
| Seat 2—young | 0.340**** | 6.09 |
| Seat 3 | 1.28**** | 31.06 |
| Seat 3—young | 0.470**** | 5.70 |
| Seat 4 | 1.83**** | 33.31 |
| Seat 4—young | 0.845**** | 7.73 |
| Seat 5 | 1.89**** | 26.60 |
| Seat 5—young | 1.13**** | 8.29 |
| No. of observations | | 70513 |
| Adjusted $\rho^2$ | | 0.053 |
| Null log-likelihood | | −113,486.296 |
| Final log-likelihood | | −107,500.784 |

****$p \leq 0.001$

the probability of a customer belonging to category $j$ purchasing a ticket of seating area $s$ (among the 5 seating areas) is given by:

$$Pr(s \mid j) = \frac{\exp[V_{sj}]}{\sum_{t=1}^{5} \exp[V_{tj}]} \tag{5}$$

Estimation results for the MNL model are displayed in Table 4.

As expected, young customers are more price sensitive than standard ticket buyers. In addition, the price coefficient increases significantly when we consider popular productions as well as, surprisingly, performances that take place on Sunday. Notice that for low popular shows, the price coefficient is higher than for medium-popular performance

With regard to the seat quality, the coefficients reflect an expected pattern: keeping the price fixed, an increase in the quality of the seat leads to a greater utility. This pattern holds for all the customer categories considered. Contrary to Baldin and Bille (2018), we cannot compare the marginal utility of the seat categories across customers categories because each category has its own price coefficient. However, in terms of willingness to pay (WTP), i.e., the ratio between the coefficient of the attribute and the price coefficient, it results that this value is greater for standard ticket buyers.

## 6 Bi-objective optimization of revenue and attendance

The optimization model we propose considers the two objectives of the theater, i.e., to maximize revenue and attendance, in a constrained bi-objective maximization framework. It incorporates both the demand function and the customers' seat choices described in Sect. 5. The decision variables are the prices $p_{sj}$, for each seating area $s$ and each customer category $j$. As these prices affect the demand and the customers' seat choice, the optimal prices determine the optimal splitting into fare classes of the seats in the theater.

The expected revenue and attendance can be written as, respectively,

$$\text{Revenue} = \sum_{j=1}^{2} D_j(p_j) \cdot \left[ \sum_{s=1}^{5} Pr(s \mid j) \cdot p_{sj} \right] \tag{6}$$

and

$$\text{Attendance} = \sum_{j=1}^{2} D_j(p_j) \cdot \left[ \sum_{s=1}^{5} Pr(s \mid j) \right], \tag{7}$$

where $D_j$ is the number of tickets sold to category $j$, defined by the estimated demand function (2); $p_j$ is the average price for a customer belonging to category $j$; $Pr(s \mid j)$ is the probability of buying a ticket of seating area $s$, given the customer category $j$, for the considered performance, as defined by (5). The maximum number of seats that can be sold is bounded by the capacity of the theater $C$:

$$\sum_{j=1}^{2} D_j(p_j) \cdot \left[ \sum_{s=1}^{5} Pr(s \mid j) \right] \leq C. \tag{8}$$

Moreover, we have to consider a set of constraints that are required by the theater policy:

$$p_{(s-1)j} < p_{sj} < p_{(s+1)j}, \quad \text{for each } j \text{ and } s \tag{9}$$

As seen in Sect. 4.1, the ticket price for a young customer is obtained discounting the standard ticket price, given a seating area $s$. This is a normal practice by the theater manager that we should take into account. However, we allow for a more flexible relationship:

$$0.4 \cdot p_{\text{standard ticket}} < p_{\text{young}} < 0.6 \cdot p_{\text{standard ticket}}, \tag{10}$$

Finally, we have the constraint that defines the relation between $p_{sj}$ and $p_j$

$$p_j = \frac{1}{5} \sum_{s=1}^{5} p_{sj}. \tag{11}$$

## 7 Optimization results

The bi-objective optimization model we defined consists in maximizing the two objectives, *Revenue* and *Attendance*, under the above defined constraints: the solution of such a problem is the set of Pareto optimal points, the so-called Pareto frontier of the problem. We observe that we are facing a nonlinear bi-objective problem, due to the exponential term both in the demand function and in the formulation of the probability in the multinomial logit model. As usual in multi-objective optimization, in particular in the nonlinear case, it is convenient to look for some points of the Pareto frontier; those points should be interesting from the point of view of the decision maker, in our case the direction of the theater.

We solved the problem by means of the Synchronous Approach adopted by Miettinen and Mäkelä (2006). Their model, called NIMBUS (nondifferentiable interactive multi-objective bundle-based optimization system), allows us to deal with non-differentiable and nonconvex multi-objective optimization problems. The approach is based on the interaction between the decision maker and the solution algorithm, and is realized via the Internet based system WWW-NIMBUS (https://wwwnimbus.it.jyu.fi). The single steps of the solution approach consist in the solution of single objective (sub)problems via classical subgradients methods (see, e.g., Clarke 1990). Successive single optimization subproblems are then solved under the guidance of the decision maker: each successive solution is a Pareto optimal solution of the multi-objective problem. At each iteration, the decision maker can indicate the preferred way to navigate the set of Pareto optimal solutions, choosing the objectives whose value should be improved and, at the same time, which objectives should pay the cost of such improvement. In this way, the most appropriate solutions from the decision maker's point of view are selected from the Pareto optimal solutions set.

The software is free for the academic community and is operated directly on the Internet site,[5] requiring neither the download of any software nor huge computing capabilities of the client computer.

As case studies we consider three performances that differ by characteristics affecting the demand, to verify how different levels of theater occupancy require different pricing and allocation policies, in particular considering the peak-load pricing issue (i.e., differentiating prices charged depending on peak and off-peak periods). For the purpose of a better comparison between the actual pricing and allocation policies, and those resulting from the optimization model, we have chosen three performances that show a fitted value of the demand, which is very close to the real demand. The first performance is a high-demand performance, namely a Saturday evening performance of *La Tosca* that fills up to 91.05% capacity. The second

[5] https://wwwnimbus.it.jyu.fi.

**Table 5** Revenue and attendance comparison. Case study: *La Tosca*, season 2014–2015

| Seat | Actual | | Bi-objective | | Revenue max. | | Attendance max. | |
|---|---|---|---|---|---|---|---|---|
| | Price | No. of seats | Price | No. of seats | Price | No. of seats | Price | No. of seats |
| Seat1—standard | 160 | 46 | 177 | 74 | 320 | 78 | 80 | 79 |
| Seat2—standard | 345 | 79 | 344 | 136 | 690 | 117 | 205 | 150 |
| Seat3—standard | 525 | 144 | 689 | 165 | 1050 | 140 | 363 | 217 |
| Seat4—standard | 720 | 366 | 939 | 225 | 1432 | 169 | 700 | 273 |
| Seat5—standard | 895 | 258 | 1186 | 189 | 1790 | 128 | 875 | 246 |
| Seat1—young | 80 | 13 | 89 | 18 | 160 | 15 | 40 | 23 |
| Seat2—young | 173 | 23 | 151 | 28 | 303 | 10 | 91 | 41 |
| Seat3—young | 263 | 14 | 276 | 14 | 420 | 5 | 208 | 22 |
| Seat4—young | 360 | 14 | 376 | 12 | 573 | 3 | 341 | 14 |
| Seat5—young | 448 | 8 | 475 | 6 | 717 | 1 | 473 | 5 |
| Total | 621,970 | 965 | 626,643 | 867 | 731,697 | 666 | 537,894 | 1070 |
| % improve (revenue and attendance) | | | +0.75 | −10.15 | +17.64 | −31.29 | −13.51 | +10.88 |

performance analyzed is a low-demand performance, *Djævlene fra Loudun*, run in a weekday: this performance fills less than half of the total capacity (41.98%). The third performance is a medium-popular production, namely *Rusalka*, with 67.86% of the total capacity filled.

We are therefore able to compare the price[6] and seat allocation results of the bi-objective optimization model with the actual results and also with those resulting from other two bi-objective optimization models in which the decision maker wants to find the points in the Pareto frontier that provide the highest value, respectively, of the total revenue and total attendance (see Tables 5, 7, 8).

Some remarks about the implementation of the models: first, for the purpose of realism we have established a lower and an upper bound to the 10 decision variables, respectively, equal to the half and the double value of the actual price. Second, we subtract from the value of the capacity $C$ the number of tickets sold to other

[6] Price is expressed in Danish crown (DKK): 1 DKK ≈ 0.13€.

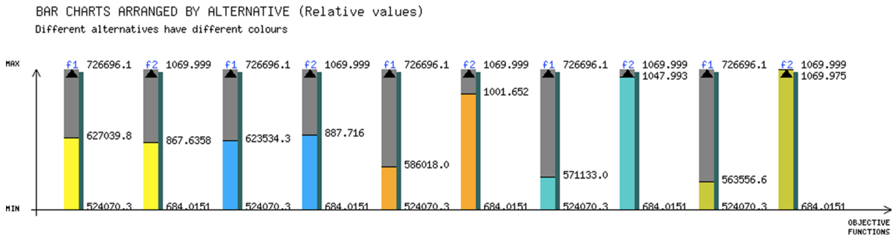


**Fig. 2** Some alternative optimal values of the bi-objective model. *Source*: https://wwwnimbus.it.jyu.fi/

categories which were not considered, including subscribers, assuming it is already known by the theater manager.

Table 5 considers the results obtained for a Saturday evening performance of *La Tosca*. It is a high-demand event almost (but not completely) sold-out.

The bi-objective optimization model solutions shown in Table 5 (as well as all the other solutions shown in Tables 7, 8) represent one of the points of the Pareto frontier. Hence, there are other possible solutions. Figure 2 shows some alternative solutions of the bi-objective model. The solution proposed in Table 5 leads to an increment in revenue of 0.75% and a decrease in the total attendance of 10.15%.

From Fig. 2, the existence of a trade-off among the two objectives becomes evident: an increase in revenue is associated with a lower value of the attendance, and viceversa.

It is interesting to observe how price and seat allocation can change according to the orientation of the theater manager toward the two objectives. From Table 5, we can deduce that when the only objective is the maximization of the revenue, the theater exploits the inelasticity that characterizes standard tickets buyers by increasing the price to the upper bound. As young customers are price sensitive, the corresponding price is increased until the loss of young customers is not more balanced by a higher revenue per seat. In the attendance maximization perspective, since the performance almost reaches the capacity constraint, the objective is achieved by lowering only the prices of the most expensive seat category.

In relation to the allocation policy, we notice that when the theater is "attendance maximizer" customers are more likely to shift to a higher seat quality (buy-up behavior) as a consequence of a generalized price reduction. Viceversa, if the theater is "revenue maximizer," customers are more likely to buy a ticket for a cheap seat because they are not willing to pay more. This behavior is evident when we refer to price sensitive customers. On the contrary, price insensitive customers are not influenced by the theater policy in their choice of the seating area, which is confirmed in Figs. 3 and 4—respectively, for young customers and standard ticket buyers—showing how the probability of buying a ticket of certain seating areas changes according to the theater policy. In particular, we can see that young customers still prefer the less expensive seats; however, this preference is more accentuated when theater is revenue maximizer. Thus, the optimal pricing and allocation policy depends on the type of customers the theater expects to accommodate.

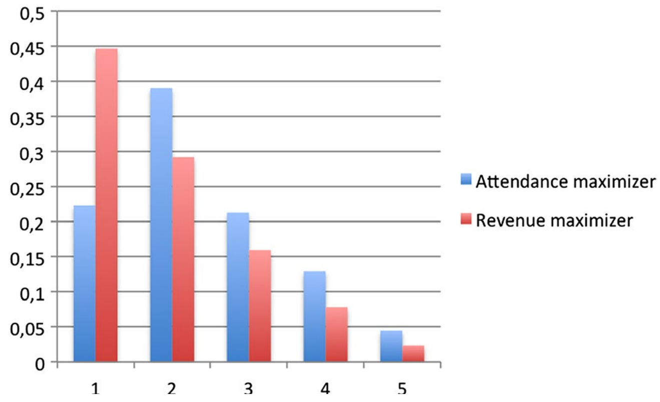


**Fig. 3** Young customers' choice probabilities

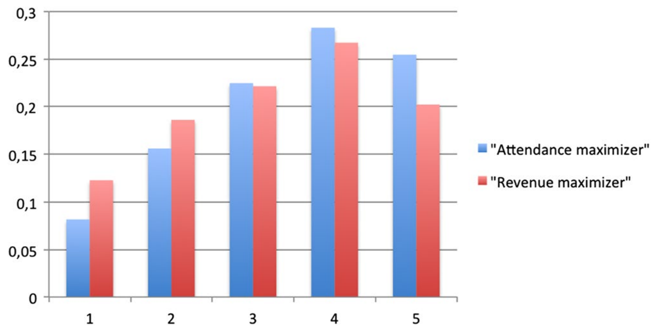


**Fig. 4** Standard ticket buyers' choice probabilities

**Table 6** Marginal effect of price on choice probability for young customers

| | Seat1 | Seat2 | Seat3 | Seat4 | Seat5 |
|---|---|---|---|---|---|
| Seat1 | −0.133 | 0.033 | 0.031 | 0.042 | 0.026 |
| Seat2 | 0.033 | −0.179 | 0.046 | 0.061 | 0.039 |
| Seat3 | 0.031 | 0.046 | −0.173 | 0.058 | 0.037 |
| Seat4 | 0.042 | 0.061 | 0.058 | −0.210 | 0.049 |
| Seat5 | 0.026 | 0.039 | 0.037 | 0.049 | −0.151 |

It should be highlighted that the increase/decrease in the size of each seating area, respect to actual conformation, depends not only on how much its optimal price differs from the actual one, but also on how much the prices of the alternative seating area differ and on the value attributed by customers on each seating area. To clarify this point, Table 6 shows the marginal effect on the probability choice for young customers due to the change in price, with respect to the actual situation.

**Table 7** Revenue and attendance comparison. Case study: *Djævlene fra Loudun*, season 2012–2013

| Seat | Actual | | Bi-objective | | Revenue max. | | Attendance max. | |
|---|---|---|---|---|---|---|---|---|
| | Price | No. of seats | Price | No. of seats | Price | No. of seats | Price | No. of seats |
| Seat1—standard | 160 | 41 | 80 | 13 | 320 | 12 | 80 | 14 |
| Seat2—standard | 295 | 23 | 201 | 26 | 590 | 19 | 148 | 28 |
| Seat3—standard | 425 | 29 | 377 | 39 | 850 | 24 | 213 | 43 |
| Seat4—standard | 545 | 50 | 591 | 61 | 1090 | 32 | 273 | 70 |
| Seat5—standard | 695 | 42 | 755 | 59 | 1390 | 25 | 348 | 68 |
| Seat1—young | 80 | 16 | 40 | 11 | 160 | 5 | 40 | 11 |
| Seat2—young | 148 | 11 | 85 | 20 | 272 | 5 | 74 | 23 |
| Seat3—young | 213 | 1 | 151 | 23 | 341 | 4 | 107 | 31 |
| Seat4—young | 273 | 9 | 237 | 34 | 436 | 4 | 137 | 57 |
| Seat5—young | 348 | 3 | 302 | 30 | 546 | 2 | 174 | 54 |
| Total | 88,732 | 225 | 92,204 | 242 | 111,611 | 132 | 79,671 | 398 |
| % improve (revenue and attendance) | | | +3.91 | +7.55 | +25.78 | −41.33 | −10.21 | +76.89 |

The marginal effect $\mathrm{ME}_{ss}$ on the probability to choose the seating area s when its price increases by 1 DKK is given by:

$$\mathrm{ME}_{ss} = \pi_s(1 - \pi_s) \cdot \beta_p \tag{12}$$

where $\pi_s$ is the probability of choosing the seating area $s$ as defined in (5), and $\beta_p = \beta_1 + \beta_2 \cdot \text{young} + \beta_3 \cdot \text{sub} + \gamma' z$ with reference to the formulation (4). Instead, the marginal effect $\mathrm{ME}_{sj}$ on the probability to choose the seating area $s$ when the price of the seating area $j$ increases by 1 DKK is given by:

$$\mathrm{ME}_{sj} = -\beta_p \cdot \pi_s \cdot \pi_j \tag{13}$$

Given that 1 DKK is a very low value, we consider in Table 6 the change in probability when the price increases by 100 DKK.

For instance, if the price of *Seat 1* increases by 100 DKK, the probability for a young customer to choose that seating area decreases by 13.3%, whereas the probability to purchase a ticket for *Seat 2* increases by 3.3%, for *Seat 3* increases by 3.1% and so on.

**Table 8** Revenue and attendance comparison. Case study: *Rusalka*, season 2013–2014

| Seat | Actual | | Bi-objective | | Revenue max. | | Attendance max. | |
|---|---|---|---|---|---|---|---|---|
| | Price | No. of seats | Price | No. of seats | Price | No. of seats | Price | No. of seats |
| Seat1—standard | 160 | 43 | 80 | 37 | 320 | 37 | 80 | 37 |
| Seat2—standard | 345 | 70 | 217 | 68 | 690 | 55 | 173 | 73 |
| Seat3—standard | 525 | 72 | 352 | 100 | 1050 | 64 | 263 | 111 |
| Seat4—standard | 720 | 150 | 635 | 130 | 1432 | 75 | 360 | 174 |
| Seat5—standard | 895 | 109 | 800 | 117 | 1790 | 56 | 454 | 168 |
| Seat1—young | 80 | 6 | 40 | 14 | 160 | 8 | 46 | 13 |
| Seat2—young | 173 | 18 | 87 | 25 | 303 | 5 | 102 | 21 |
| Seat3—young | 263 | 7 | 141 | 27 | 420 | 3 | 135 | 28 |
| Seat4—young | 360 | 8 | 254 | 20 | 573 | 1 | 180 | 44 |
| Seat5—young | 448 | 2 | 320 | 14 | 717 | 0 | 227 | 37 |
| Total | 283,596 | 510 | 245,139 | 551 | 328,757 | 303 | 206,860 | 707 |
| % improve (revenue and attendance) | | | −13.56 | +8.04 | +15.92 | −40.59 | −27.06 | +38.63 |

Table 7 considers the results obtained for a weekday performance of *Djævlene fra Loudun*. It is a low-demand event in which the theater is usually occupied approximately only a little bit more than a third of its capacity after subtracting to it the number of tickets sold to the other customers' categories.

In this case, the bi-optimization model provides a solution that dominates the current value of the objectives. Indeed, the solution proposed allows an increase in revenue of 3.91% and, at the same time, an increase in attendance of 7.55%. This indicates that, regardless the existence of a trade-off between the two objectives, the actual prices are not optimal. Compared to the previous case, here the theater is forced to reduce prices to the lower bound when it aims to maximize attendance. As this performance is supposed to attract a low share of theatergoers, the theater could potentially further lower the price to zero. However, this can be done at the end of the sale period. As our model does not include forms of dynamic pricing, the optimal prices can be interpreted as optimal advertised prices, which are set when the ticket sale period starts.

Concerning the allocation policy, the pattern previously described is more accentuated in the revenue maximization case: as the price coefficient of the MNL model has decreased with respect to the previous case, it is suggested to enlarge the cheapest seating area more. On the contrary, when the attendance is the main goal, it is always suggested to increase the size of the most expensive seating areas but, with respect to a high-demand event, this indication must be taken more cautiously, for two reasons: firstly, because price has a greater negative effect on the choice of seating area when the performance is not popular. Secondly, whereas for the high-demand events that almost reach the capacity constraint it is suggested to decrease only the price of the most expensive seating areas (when the main objective is the attendance maximization), for the less popular events it is suggested to reduce the price of all the seating areas. Thus, the most expensive seats will still be preferred over the cheapest ones, as a consequence of a generalized price reduction, but this preference will be weaker compared to the context of high-demand events.

Table 8 considers the results obtained for a Sunday performance of *Rusalka*. This is an intermediate situation compared to the previous two. In this case, the bi-objective optimization model provides a solution which allows an increase in attendance of 8.04% and a decrease in revenue of 13.56%.

It is worth pointing out that the optimization model proposed in this paper does not pretend to provide exact and precise values of prices to be taken automatically, especially considering the margin of errors of the forecast estimation. Nevertheless, the model can provide some guidance to the theater manager in pricing and allocation policies. Given the trade-off between revenue and attendance, the model gives some indications on how to adjust both prices (by either increasing or decreasing them) and the size of each seating area (by either enlarging or reducing it) according to the preference of the theater manager along a continuum from only maximization of revenue to only maximization of attendance.

Such indications are summarized in Table 9.

## 8 Conclusions

This paper has proposed a model that simultaneously optimizes the pricing and seating-allocation policy of a theater. In particular, we present a bi-objective optimization model that integrates the demand forecast and a choice model, where the customer chooses one among different seating areas which differ in price and quality. The multi-objective nature of our model reflects the multi-dimensional nature of nonprofit performing arts organizations. In our case, the objectives we assume to be maximized are revenue and attendance. The approach adopted also allows to take into account heterogeneity in price sensitivity and choice behavior across different customer segments. The proposed model is applied to booking data provided by the Royal Danish theater referring to the period 2010–2015. More precisely, we consider three different performances in order to explore the potentialities of the model.

From a managerial perspective, the model can provide theater managers with insightful policy implications in terms of demand-management decisions. The results obtained confirm the existence of a trade-off between the two theater

**Table 9** Guidance to theater manager

| | Revenue max. | Attendance max. |
|---|---|---|
| Pricing policy | Raise the price to standard tickets holders and subscribers. Raise the price to young customers until the loss of customers is not balanced by a higher revenue per seat | Reduce the price. For high-demand event, reduce only the price of high-quality seats |
| Allocation policy | Enlarge the cheapest seating area in order to prevent a loss of revenue, especially in the case of low-demand events | Increase the number of seats allocated to the most expensive area, in order to encourage a buy-up behavior. This action is more effective in case of high-demand events |

objectives. When the theater is "revenue maximizer," prices charged to price insensitive customers are raised, and the cheapest seating area is enlarged to prevent a loss of revenue, in particular when a performance is expected to attract customers with an elastic demand, since they are more sensitive to price changes in their seat choice; and also when the performance will probably not attract a large audience. Viceversa, when the theater is audience maximizer, prices are set at lower levels, in particular the ones associated with the most expensive seating area. As a consequence, it is recommended to increase the number of seats allocated to the most expensive area, in order to encourage a shift of customer choices to higher quality seats: this is particularly effective when a performance is supposed to be a high-demand show Moreover, in one case the bi-objective model provides a solution that causes an improvement in both revenue and attendance from the current situation, denoting how, regardless the existence of a trade-off between revenue and attendance, the actual prices set by the theater for that performance were not optimal.

Overall, our examples clarify that both price and capacity allocation are leverages with which a theater can calibrate its objectives, even when revenue is not considered as the main goal to pursue.

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