

Hedge Fund Tail Risk: *An Investigation in Stressed Markets*

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The financial industry has witnessed several crises in the past, some of which have had a major global impact. Although hedge funds are expected to have the capability to at least avoid the impact of such crises by hedging against market movements, the long-term capital management crisis in 1998 and the global meltdown in 2008 showed otherwise. In both cases, most hedge funds suffered large losses; In fact, almost all Dow Jones Credit Suisse hedge fund indexes experienced huge losses. These systemic losses have been associated mostly with large, simultaneous liquidation by market participants, causing a liquidity freeze that crippled hedge funds and spilled over to the general economy (see Brown et al. [2009]; Buraschi, Kosowski, and Trojani [2014]; and Lo, Getmansky, and Lee [2015]). The experiences from these crises have motivated investors, academics, and regulators to better understand systemic risk in hedge funds and these funds' contribution to the overall risk of a portfolio (see Billio et al. [2012a, b]).

In this study, we concentrate on the risk of a portfolio of hedge funds from the investor's point of view. In this respect, our article is generally related to that of Brown, Gregoriou, and Pascalau [2012], who investigated overdiversification in funds of hedge funds that could generate tail risk, and Heuson, Hutchinson, and Kumar [2014], who

investigated hedge fund performance when returns are skewed. As funds of hedge funds primarily invest in hedge funds, it is important to develop a risk measurement technique that captures the individual hedge funds' risk exposure and the risk contribution of these hedge funds toward the overall portfolio risk, especially in light of the 1998 and 2008 hedge fund crises. As documented by several studies, hedge funds follow dynamic trading strategies. As a result, the risk exposures and thus the risk contributions of hedge funds are expected to change significantly over time. To capture the dynamic nature of hedge funds, we apply a Markov regime-switching approach, which allows us to estimate time- and state-dependent risk exposures.

There are several ways to measure the risk of a financial asset or portfolio. The common ones include volatility, semivariance, Value at Risk (VaR), and expected shortfall (ES). Different studies suggest the use of one or the other of these tools to measure the risk of financial assets and make optimal portfolio choices. The mean-variance framework of Markowitz [1952] uses volatility (or variance) to understand the risk and select an optimal portfolio. Others, like Roy [1952], have suggested a safety first approach, which accounts for the possibility of surprises (i.e., returns below a certain, acceptable threshold). These ideas of using downside risk measures were developed

further by Bawa [1978], and Fishburn [1977] suggested semivariance and other partial moments as alternative risk measures. Currently, VaR and ES are the popular measures of downside/tail risk, VaR being the most widely used. Although VaR is used to determine the capital requirements of banks and other financial institutions, it is shown to have some faults, including failing to be coherent. As a result, the academic literature seems to favor ES over VaR (see Artzner et al. [1999] and Inui and Kijima [2005]).

These measures provide different pieces of information about the risk of a portfolio and the contribution of different hedge fund strategies to a portfolio's risk. Measuring the risk using these different measures helps one to better diversify the portfolio with respect to "normal" and "extreme" events. In fact, an asset's contribution to portfolio volatility and tail risk are not necessarily the same when the asset's returns are not normally distributed.

An individual asset's contribution to the portfolio risk can be calculated in two ways. The first is by taking the difference between the portfolio risk measures computed with and without the asset. A second, and more useful, way is to compute the marginal contribution of individual assets to the portfolio risk by calculating the first derivative of a risk measure with respect to the portfolio weight of the asset. In the literature, authors who have taken this second approach include Scaillet [2004]; Fermanian and Scaillet [2005]; Hallerbach [2003]; Hans [2004]; and Litzenberger and Modest [2008]. Scaillet [2004] and Fermanian and Scaillet [2005] derived the marginal ES as a linear function of the derivatives and proposed a nonparametric estimation based on empirical averages or kernel methods. Litzenberger and Modest [2008] more specifically proposed a measure for determining the percentage marginal contribution of individual hedge funds to the ES of a fund of hedge funds. They used a mixture of normal distributions with subjectively chosen parameters, subjectively predefined types of possible crises, and fixed (subjective) probabilities of these crises to calculate marginal VaR. The idea of marginal VaR is in line with the systemic risk measure proposed by Adrian and Brunnermeier [2010] in their conditional VaR (CoVaR) measure. Our work shares this spirit and suggests a methodology for estimating all parameters based on Markov's regime-switching approach.¹

In summary, our work is perfectly in line with the spirit of Litzenberger and Modest [2008], and

rather than leaving to the risk controller the task of exogenously determining the states of nature and the probabilities of these states, we suggest the use of a Markov regime-switching approach, which allows one to estimate time- and state-dependent risk exposures, as well the state probabilities of the different risk factors.

In addition to the marginal VaR, we also calculate the marginal variance and the marginal ES. This last measure was proposed by Acharya et al. [2010] for measuring systemic risk for financial institutions. Our model has the following desired features: (1) it can accurately predict portfolio volatility during normal, non-crisis times; (2) it captures stress moves during crisis periods in a realistic way; (3) it is consistent with the empirical observation that returns in many financial markets are characterized by distributions with fat left tails; (4) it provides a unified framework for comparing trades whose risk may be composed of different proportions of typical day-to-day non-crisis volatility and crisis risk; and (5) it allows one to estimate time- and state-dependent risk exposures, as well as the state probabilities of the different risk factors, in a coherent and objective way.

We demonstrate the performance of our approach by considering an equally weighted portfolio composed of eight hedge fund strategy indexes from the Dow Jones Credit Suisse Hedge Fund Database. We study the hedge fund strategies' marginal contributions to the volatility, VaR, and ES of this portfolio through an out-of-sample analysis of an 84-month period (from January 2005 to December 2011).

The main result of our analysis is that the contribution to the tail risk of a fund of hedge funds is only partially related to the main systematic risk measure identified in the portfolio diversification literature: the market portfolio. Using the S&P 500 as a proxy for the market portfolio, we show that, even in a dynamic framework in which exposure to the market risk factor changes over time, this exposure's contribution to the tail risk of a fund of hedge fund portfolios is rather marginal and mostly appears during crisis periods. Other factors largely affect the tail risk of a fund of hedge fund portfolios, such as liquidity risk and credit risk. The effects of these other factors are so strong that another strategy, dedicated short bias, which systematically takes a negative position on the equity market, is not able to hedge (at least not fully) the tail risk of the portfolio of hedge funds during a crisis,

but it does contribute to the tail risk along with all the other strategies, showing that the dedicated short bias strategy does not work as a hedging instrument when it is most needed.

This study is important in several ways. Previous studies—including those by Tasche [1999]; Gouriéroux, Laurent, and Scaillet [2000]; Hans [2004]; Scaillet [2004]; Fermanian and Scaillet [2005]; and Litzenberger and Modest [2008]—have showed that marginal risk contribution can be used in portfolio selection models. Crucially, marginal contribution analysis shows that high risk in individual assets does not necessarily mean a high contribution toward the portfolio risk. In general, however, the asset dependence structure matters, as several assets could be negatively correlated, leading to a diversification benefit. This is even more relevant if we consider a nonlinear dependence structure, such as that implied by the Markov switching, as the asset correlation could change over time. It is thus very useful to look at the marginal risk contributions of individual hedge funds to a portfolio instead of only the risk measures for individual hedge funds. That is, the marginal volatility, marginal VaR, or marginal ES contributions of individual hedge funds tell us more than can be obtained by simply looking at the single hedge fund volatility, VaR, or ES, respectively.

Even though we concentrate on portfolios of hedge funds when discussing our methodology and results, the study can be extended to the analysis of risk contributions in any portfolio or even to the general economy. Several recent studies have showed how marginal ES can be used for measuring systemic risk. Acharya et al. [2010] showed that marginal ES is a good proxy for an institution's contribution to systemic risk. Brownlees and Engle [2010] then developed a methodology for estimating marginal ES. Although the latter authors employed a method of estimating volatility, correlation, and tails separately so as to use them later in the marginal ES estimation, our study estimates the whole distribution for each of the individual strategy indexes and the portfolio. The actual estimation in our study is performed in several steps, which allows us to minimize the estimation time. Our work shares with all the previous studies the fact that the marginal VaR and marginal ES are calculated conditional on certain risk factors.

The article is organized as follows. The first section presents the different marginal contribution risk

measures and the methodology to determine them. The second section reports the empirical analysis applied to a portfolio of hedge funds. The third section concludes.

MARGINAL RISK CONTRIBUTION MEASURES

We consider three risk measures: (1) volatility, (2) VaR, and (3) ES. Volatility is usually defined as the standard deviation conditional on past information and is a classical measure of dispersion around the mean. Volatility is, by definition, symmetric and thus not capable of capturing any asymmetric aspect of risk. This makes volatility more useful during normal market conditions, but less helpful during crises when the market is far below the mean. Common risk measures used to capture downside risk exposure are VaR and ES. VaR is theoretically defined as the maximum amount of loss that can occur over a given horizon at a certain confidence level. The alternative suggested in the literature is the ES, also known as the expected tail loss (ETL), which overcomes some of the weaknesses of VaR. ES represents the loss expected beyond the loss represented by the VaR. In other words, ES is the expected return conditional on the return being lower than the VaR. For an analytical description of these measures, see Billio, Frattarolo, and Pelizzon [2016]).²

We propose a general method for computing three marginal risk measures: marginal volatility, marginal VaR, and marginal ES. We used the regime-switching models detailed by Billio, Frattarolo, and Pelizzon [2016] to estimate and forecast these risk measures. These marginal risk measures were used to estimate the marginal effect of a component of the portfolio P . More specifically, we applied the marginal risk measures to estimate the marginal effect of single strategies in a portfolio of hedge funds.

To compute the marginal risk contribution of strategy i , we constructed a new portfolio P' , which consisted of an amount w_i invested in strategy i , and an amount $(1 - w_i)$ invested in the original portfolio P . Note that w_i is an incremental weight in a strategy that is already a component in the portfolio P . Then, by differentiating with respect to w_i at $w_i = 0$, we could easily account for all the marginal effects of strategy i , either directly through strategy i or through the resulting change in the weight of the remaining strategies in

the portfolio. The return on the new portfolio P' was obtained using:

$$r_{P',s} = w_i r_{i,s} + (1 - w_i) r_{P,s} \quad (1)$$

Clearly, the state-dependent mean and volatility of the new portfolio are given by:

$$\mu_{P',s} = w_i \mu_{i,s} + (1 - w_i) \mu_{P,s}, \quad (2)$$

$$\sigma_{P',s} = \sqrt{w_i^2 \sigma_{i,s}^2 + (1 - w_i)^2 \sigma_{P,s}^2 + 2w_i(1 - w_i) \sigma_{iP,s}} \quad (3)$$

where $\sigma_{iP,s}$ is the state-dependent covariance between strategy i and portfolio P . We know that the covariance $\sigma_{iP,s}$ can be expressed in terms of the correlation $\rho_{iP,s}$ between portfolio P and strategy i and the volatilities of the strategy and portfolio, that is, $\rho_{iP,s} \sigma_i \sigma_P$. Thus, replacing σ_{iP} with $\rho_{iP} \sigma_i \sigma_P$ and differentiating with respect to w_i at $w_i = 0$, we get the marginal volatility (labeled $M\sigma$) as follows:

$$M\sigma_{P,s} = \left[\frac{\partial \sigma_{P',s}}{\partial w_i} \right]_{w_i=0} = \rho_{iP,s} \sigma_i - \sigma_{P,s} \quad (4)$$

Moreover, the state-dependent ES of the portfolio P can be defined as:

$$\begin{aligned} ES_{P'}(\alpha, h) = \frac{1}{\alpha} \sum_{s \in S} p \left(\frac{s_{t+h}}{I_t} \right) & \left[\mu_{P',s} \Phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) \right. \\ & \left. - \sigma_{P',s} \phi \left(\frac{VaR_{P'}(\alpha, h) - \mu_{P',s}}{\sigma_{P',s}} \right) \right] \end{aligned} \quad (5)$$

Again, by differentiating this equation with respect to w_i at $w_i = 0$, we get the marginal ES (labeled MES³):

$$\begin{aligned} MES = \left[\frac{\partial ES_{P'}}{\partial w_i} \right]_{w_i=0} &= \frac{1}{\alpha} \sum_{s \in S} p \left(\frac{s_{t+h}}{I_t} \right) \left[\frac{\partial \mu_{P',s}}{\partial w_i} \Phi(z_s) \right. \\ &+ \mu_{P',s} \frac{\partial \Phi(z_s)}{\partial w_i} - \frac{\partial \sigma_{P',s}}{\partial w_i} \phi(z_s) \\ &\left. - \sigma_{P',s} \frac{\partial \phi(z_s)}{\partial w_i} \right]_{w_i=0} \end{aligned} \quad (6)$$

where $z_s = \frac{VaR_{P'} - \mu_{P',s}}{\sigma_{P',s}}$ is the standardized VaR for the portfolio P' .

With the same procedure, the marginal VaR (labeled $MVaR$) can be obtained as follows:

$$\begin{aligned} MVaR &= \left[\frac{\partial VaR_{P'}}{\partial w_i} \right]_{w_i=0} \\ &= \frac{\sum_{s \in S} p \left(\frac{s_{t+h}}{I_t} \right) \frac{\phi(z_s)}{\sigma_{P',s}} [(\mu_{i,s} - \mu_{P,s}) - z_s (\rho_{iP,s} \sigma_{i,s} - \sigma_{P,s})]}{\sum_{s \in S} p \left(\frac{s_{t+h}}{I_t} \right) \frac{\phi(z_s)}{\sigma_{P',s}}} \end{aligned} \quad (7)$$

Billio, Frattarolo, and Pelizzon [2016] describes in detail the derivation of the preceding equations and how to obtain closed-form solutions for all three risk measures ($M\sigma$, $MVaR$, and MES).

EMPIRICAL RESULTS

Data

We considered monthly aggregate hedge fund strategy index returns from the Dow Jones Credit Suisse Hedge Fund Database from January 1994 to December 2011. This database tracks approximately 8,000 hedge funds, which fall into 10 different strategies according to their investment style. The Dow Jones Credit Suisse hedge fund indexes are asset-weighted indexes of funds with a minimum of \$50 million of assets under management, a minimum one-year track record, and current audited financial statements. Currently, more than 400 funds are used to obtain these indexes. An aggregate index is computed from this universe, and 10 subindexes based on investment style are also computed using a similar method. Indexes are updated and rebalanced at a monthly frequency, and the universe of funds is redefined on a quarterly basis. The performance data used in the indexes are net of all fees. Moreover, the database accounts for survivorship bias in hedge funds (Fung and Hsieh [2000]). New funds are included on a going-forward basis only, removing instant history bias. The Dow Jones Credit Suisse indexes also include funds that are closed to new investments, which further reduces selection bias. The indexes are computed so as to get a maximal representation of their universe, with at least 85% coverage. Finally, the database tracks hedge funds from all over the world. For all of these reasons, the Dow Jones Credit Suisse indexes are a good proxy

to use in studying hedge funds. In this article, we concentrate on the eight equity-related strategy indexes.

The detailed definition of the risk factors in the factor model used, selected according to the literature,

is reported in Exhibit 1. Exhibit 2 provides summary statistics for these strategies and for the risk factors, and the second part of Exhibit 2 shows that the Gaussian assumption is also rejected for most of the risk factors.

EXHIBIT 1

Risk Factors Definitions

Risk Factor	Definition
S&P 500	Monthly return on the S&P Composite Index, including dividends
Large–Small	Monthly return differences between Russell 2000 and Russell 1000 indexes
Value–Growth	Monthly return differences between Russell 1000 Value and Growth indexes
USD	Monthly return on Bank of England Value-Weighted US\$ Index
Barclays Government Credit	Monthly return on Barclays US Aggregate Government/Credit Index
Term Spread	The difference between the yields of the 10-year Treasury bond and Three-Month Treasury Bill
Credit Spread	The difference between Moody’s U.S. Corporate Bond indexes for BAA and AAA
MSCI Em. Stock	Monthly return on MSCI Emerging Markets Stock Index
Barclays Em. Bond	Monthly return on Barclays Emerging Markets Bond Index
UMD	Momentum factor from Kenneth French’s website
Change in VIX	Monthly change in Chicago Board of Exchange’s VIX Index

Notes: Definitions of risk factors included in the multifactor regime-switching model. All data, except for UMD, were obtained from Datastream. UMD, the momentum factor, was obtained from Ken French’s website.

EXHIBIT 2

Summary Statistics

Strategy Indexes	Mean	St. Dev	Median	Skewness	Kurtosis	Minimum	Maximum
Convertible Arbitrage	0.37	2.00	0.65	–2.55	15.46	–12.63	5.80
Dedicated Short Seller	–0.46	4.90	–0.84	0.64	1.37	–11.29	22.31
Emerging Markets	0.44	4.33	1.05	–0.81	5.06	–23.43	16.04
Equity Market–Neutral	0.21	2.97	0.40	–11.99	164.07	–40.45	3.65
Long Short Equity	0.53	2.86	0.66	–0.06	3.21	–11.83	12.58
Distressed	0.58	1.91	0.84	–2.18	11.35	–12.85	4.14
Event-Driven MS	0.47	1.98	0.61	–1.75	7.80	–11.92	4.77
Risk Arbitrage	0.29	1.18	0.32	–1.09	5.27	–6.55	3.38
Risk Factors							
S&P500	0.56	4.56	0.96	–0.59	0.84	–16.69	11.25
Large–Small	0.05	3.24	–0.08	0.17	4.63	–15.71	16.78
Value–Growth	–0.06	3.19	–0.09	0.13	4.38	–12.51	14.05
USD	–0.36	2.01	–0.30	0.08	0.31	–5.88	5.67
Barclays Gov Credit	0.26	1.28	0.28	–0.10	1.16	–4.27	4.52
Term Spread	1.73	1.18	1.72	–0.06	–1.20	–0.63	3.78
Credit Spread	0.97	0.47	0.85	3.05	11.21	0.53	3.43
Barclays Emerging Bond	0.61	3.80	1.09	–2.38	13.78	–25.96	9.58
MSCI Emerging Stock	0.49	7.12	0.52	–0.69	1.71	–29.31	17.13
UMD	0.45	5.52	0.60	–1.56	9.70	–34.75	18.40
Change in VIX	0.04	4.52	–0.10	0.82	4.84	–15.28	20.50

Notes: Summary statistics for the eight hedge fund indexes considered as well as for the risk factors used to explain their systematic risk exposure. A brief definition of the risk factors can be found in Exhibit 1.

This necessitates the use of a nonlinear model, such as the Markov regime-switching model suggested here.

Hedge Fund Strategies

Our complete sample contains 216 observations for each strategy. Our estimation procedure used expanding windows to estimate the parameters. That is, we started by using the first 132 observations (January 1994 to December 2004) to estimate the parameters for December 2004 and, when necessary, to compute the out-of-sample forecasts for the subsequent month. We repeated the process, each time including one additional observation, until we reached the end of the sample, thus using all the available data. Through such a process, we were able to estimate and forecast for 84 months (January 2005 to December 2011). Later in the article, we refer to this period as the estimation period.

For each strategy index, we estimated two different models. The first, which we call S&P-RSM (S&P regime-switching model), uses the S&P 500 Composite Index as the only risk factor on the right-hand side of the equation. The second model, called MF-RSM (multi-factor regime-switching model), includes several additional risk factors. Details of the estimation procedure are available upon request.

Using the S&P-RSM model, we examined the possibility of estimating the marginal risk contributions of individual strategies by considering only their conditional distributions with respect to the equity market risk factor. An example of the estimated parameters is presented in Exhibit 3, where we use the whole sample. For this specific case, we observed that the S&P 500 coefficients are significant for all strategies except the convertible arbitrage strategy in both regimes and the equity market-neutral strategy in the normal market regime.⁴

The results in Exhibit 3 seem to suggest that the only hedge fund strategy that could contribute to a reduction of risk in the portfolio is the dedicated short bias since it has a negative exposure with respect to the market and therefore potentially a negative correlation with the other strategies. A similar argument could be applied for the convertible arbitrage strategy and the equity market-neutral strategy. Therefore, by looking to the exposure to the S&P500, we can identify strategies that are natural hedgers to the risk of the portfolio, at least for its variance risk.

The picture changes when we include additional risk factors in the model, as reported in Exhibit 4. All hedge fund strategies have significant exposure to the S&P 500 in both normal and down market regimes, except for the emerging market and equity market-neutral strategies.⁵ The results reported in Exhibit 4 are extremely important because they highlight the fact that the different hedge fund strategies do present common exposures to the different risk factors. In particular, the dedicated short bias strategy has a negative exposure to the credit risk factor when the S&P 500 is in the down market regime. This negative exposure is similar in sign and significance to that of the emerging market strategy and the distressed strategy. Therefore, the ability of the dedicated short bias strategy to hedge the portfolio's risk seems to be debatable. In our analysis, detailed in the next section, we show that this common exposure to credit and liquidity risk factors largely reduces the natural ability of some hedge fund strategies to be hedgers to the total portfolio risk, at least during crisis periods.

Tail Risk of the Portfolio of Hedge Funds

In this section, we present the estimation of the strategies' marginal risk contributions to an equally weighted portfolio using regime-switching models.⁶ We considered a portfolio P that is composed of eight strategies with equal weights. Combining two states for each strategy and two for the market, the set of all possible states for this portfolio has dimension equal to $2^9 = 512$. The state-dependent means and volatilities of P were computed in the standard way using the state-dependent means and volatilities of the individual strategies. The $M\sigma$, $MVaR$, and MES of the strategies and the portfolio were computed as described by Billio, Frattarolo, and Pelizzon [2016].

Before proceeding with presenting the dynamics of the marginal contributions over time, we provide a general picture by presenting the average marginal risk contributions of each strategy over three periods: the whole estimation period (January 2005 to December 2011), the period before the 2008 crisis (January 2005 to July 2008), and the period during and after the 2008 crisis (August 2008 to December 2011).

The results presented in Panel A of Exhibit 5 show that over the whole estimation period the emerging market strategy makes the highest contribution to each of the three risk measures. This is in line with the risk

EXHIBIT 3
Parameter Estimates (S&P-RSM)

Strategy/ Variable	Convertible Arbitrage		Dedicated Short Bias		Emerging Markets		Equity Market-Neutral		Long-Short Equity		Distressed		Event- Driven MS		Risk Arbitrage	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
α_0	0.66	6.22	0.41	1.65	0.89	4.12	0.50	8.07	0.18	1.21	0.65	6.41	0.66	6.52	0.25	3.19
α_i	-1.54	-2.49	-0.60	-0.22	-0.66	-1.13	0.24	1.10	0.77	0.37	-3.04	-2.41	-0.94	-1.84	0.01	0.02
β_0 (S&P 500)	0.07	1.22	-1.37	-7.87	0.49	3.46	0.02	0.57	0.66	6.55	0.36	5.11	0.34	4.43	0.20	4.11
β_1 (S&P 500)	0.03	1.21	-0.74	-11.40	0.42	7.06	0.07	4.19	0.34	5.79	0.13	4.21	0.15	5.44	0.07	3.20
ω_0	0.75	6.77	2.77	14.95	1.86	11.37	0.58	13.04	1.61	11.16	1.13	17.01	0.99	9.80	0.63	8.78
ω_1	3.05	7.53	6.85	2.78	5.18	11.96	1.01	7.38	6.02	3.67	2.88	4.43	2.39	8.34	1.65	7.47
π_{00}^Z	0.93	25.29	0.94	15.23	0.98	57.73	0.98	48.67	0.98	56.07	0.98	69.71	0.97	38.12	0.87	14.14
π_{11}^Z	0.78	8.15	0.00	0.00	0.99	63.28	0.97	19.01	0.77	5.19	0.73	4.48	0.90	12.41	0.72	5.94

Notes: Quasi-maximum likelihood estimates (QMLE) of the S&P-RSM model, described by Billio, Frattarolo, and Pelizzon [2016], with only the S&P 500 Composite Index as a risk factor, are calculated as follows:

$$r_{it} = \alpha_i(Z_t^i) + \beta_i(S_t)M_t + \omega_i(Z_t^i)\varepsilon_t$$

where i denotes individual strategies; r_{it} is the return on strategy i ; M_t is the return on the market index; and α_i , β_i , and ω_i are, respectively, the state-dependent constant term, market beta, and idiosyncratic volatility of strategy i . We consider two Markov chains. The first, S_t , is estimated using the S&P 500 Composite Index, and it captures the high volatility (down market) and low volatility (up/normal market) regimes of the equity market. The second Markov chain, Z_t^i , captures two idiosyncratic regimes characterized by low and high volatility. Parameters π_{00}^Z and π_{11}^Z are the transition probabilities for the idiosyncratic Markov chain Z_t^i . The sample period is January 1994 to December 2011. Parameters in bold are statistically significant at the 5% level.

EXHIBIT 4

Parameter Estimates (MF-RSM)

Strategy/Variable	Convertible Arbitrage		Dedicated Short Bias		Emerging Markets		Equity Market-Neutral		Long-Short Equity		Distressed		Event-Driven MS		Risk Arbitrage	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
α_0	1.36	6.25	1.23	2.20	0.72	2.26	0.93	5.94	-0.04	-0.11	1.97	6.26	0.67	2.29	0.23	1.09
α_1	0.61	1.35	1.05	1.89	0.64	1.41	0.66	3.08	0.42	0.92	0.74	2.05	-0.74	-1.34	0.49	2.01
β_0 (S&P 500)	0.13	2.75	-1.10	-8.74	-0.04	-0.30	0.05	0.92	0.64	8.30	0.28	4.70	0.34	3.99	0.23	4.48
β_1 (S&P 500)	-0.06	-2.66	-0.80	-20.71	-0.06	-1.10	0.04	1.97	0.37	7.43	0.14	3.82	0.20	6.14	0.06	1.94
θ_{10} (Large Small)			-0.80	-6.15			0.03	0.88	0.26	3.85	0.14	2.74	0.09	1.49	0.07	1.71
θ_{11} (Large Small)			-0.42	-5.47			-0.02	-1.12	0.29	6.29	0.14	3.47	0.16	5.20	0.11	4.09
θ_{20} (Value Growth)									0.19	2.11	-0.02	-0.26			-0.03	-0.50
θ_{21} (Value-Growth)									-0.13	-3.06	0.06	1.27			0.07	2.83
θ_{30} (USD)					-0.02	-0.19	-0.11	-2.00					0.01	0.13		
θ_{31} (USD)					0.15	2.03	-0.10	-2.40					-0.04	-0.64		
θ_{40} (Barclays Gov. Credit)	-0.16	-1.89									0.04	0.36				
θ_{41} (Barclays Gov. Credit)	0.21	2.78									0.03	0.29				
θ_{50} (Term Spread)	-0.28	-3.21					-0.04	-0.61	-0.21	-1.82						
θ_{51} (Term Spread)	-0.13	-1.20					-0.06	-0.69	-0.21	-1.25						
θ_{60} (Credit Spread)	-0.99	-3.05	-0.83	-0.99	-0.72	-1.58	-0.73	-3.11	0.50	1.00	-1.49	-3.45	0.09	0.21	-0.22	-0.72
θ_{61} (Credit Spread)	-0.19	-0.88	-1.06	-2.58	-0.54	-2.65	-0.23	-1.40	0.44	1.11	-0.51	-2.13	-0.06	-0.19	-0.10	-0.61
θ_{70} (Barclays Em. Bond)	0.05	1.36			0.29	3.03										
θ_{71} (Barclays Em. Bond)	0.10	2.64			0.25	4.46										
θ_{80} (MSCI Em. Stock)					0.39	7.68										
θ_{81} (MSCI Em. Stock)					0.38	9.39										
θ_{90} (Momentum Factor)	-0.01	-0.30			0.09	1.53			0.01	0.19						
θ_{91} (Momentum Factor)	-0.04	-2.70			0.05	2.02			0.20	7.15						
θ_{10} (Change in VIX)	0.04	0.81			0.20	2.37	0.03	0.56	0.16	2.35	0.15	2.70	0.11	1.43	0.08	1.78
θ_{11} (Change in VIX)	-0.05	-1.47			-0.01	-0.16	-0.03	-1.58	-0.03	-0.70	-0.11	-2.64	0.00	0.03	-0.10	-3.03
ω_0	0.52	9.06	1.21	5.74	0.99	15.90	0.54	13.21	1.01	15.29	0.60	6.69	0.89	14.68	0.63	9.73
ω_1	2.84	11.07	2.92	10.77	3.58	12.06	1.26	10.03	2.16	9.79	1.67	13.01	2.25	9.62	1.17	8.97
π_{00}^Z	0.86	21.36	0.91	11.16	1.00	167.49	0.98	72.74	0.98	82.23	0.90	20.81	0.96	51.95	0.97	43.12
π_{11}^Z	0.76	11.85	0.95	17.10	0.99	115.79	0.98	44.42	0.96	32.49	0.92	23.29	0.90	17.92	0.96	32.35

Notes: Quasi-maximum likelihood estimates of the MF-RSM model, described by Billio, Frattarolo, and Pelizzon [2016], for each of the eight strategies, are calculated as follows:

$$r_{it} = \alpha_i(Z_t') + \sum_{j=1}^K \beta_{ij}(S_t')F_{jt} + \omega_i(Z_t')\varepsilon_{it}$$

where i denotes individual strategies; r_{it} is the return on strategy i ; F_{jt} are the risk factors defined in Exhibit 1; α_i and ω_i are, respectively, the constant term and idiosyncratic volatility of strategy i ; and for each j , β_{ij} is the exposure of strategy i to the j th risk factor. We consider two Markov chains. The first, S_t^i is estimated using the S&P 500 Composite Index and captures the high volatility (down market) and low volatility (up/normal market) regimes of the equity market. The second Markov chain, Z_t^i , captures two idiosyncratic regimes characterized by low and high volatility. Parameters π_{00}^Z and π_{11}^Z are the transition probabilities for the idiosyncratic Markov chain Z_t^i . The sample period is January 1994 to December 2011. Parameters in bold are statistically significant at the 5% level.

EXHIBIT 5

Average Marginal Risk Contributions (S&P-RSM)

Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Panel A: The Whole Sample (January 2005 to December 2011)						
Convertible Arbitrage	1.12	-0.28	-1.27	-0.67	-1.75	0.36
Dedicated Short Seller	2.69	-16.31	-10.53	-64.03	-13.63	-67.62
Emerging Markets	5.29	89.20	-11.89	104.56	-15.14	98.21
Equity Market-Neutral	1.02	0.29	-1.51	1.17	-2.01	1.87
Long Short Equity	1.61	14.55	-4.80	34.05	-6.55	36.13
Distressed	1.18	6.66	-2.44	10.27	-3.49	13.05
Event-Driven MS	1.23	4.85	-2.19	9.70	-3.06	11.78
Risk Arbitrage	0.80	1.04	-1.29	4.96	-1.82	6.22
Panel B: The Pre-Lehman Subsample (January 2005 to November 2008)						
Convertible Arbitrage	1.16	-0.44	-1.63	1.25	-2.20	2.01
Dedicated Short Seller	2.58	-21.33	-9.19	-70.80	-12.16	-74.62
Emerging Markets	5.25	95.69	-11.59	108.89	-14.83	101.54
Equity Market-Neutral	0.92	-0.80	-1.26	-0.58	-1.70	0.30
Long Short Equity	1.58	14.41	-4.74	36.01	-6.56	38.42
Distressed	1.19	7.05	-2.48	11.13	-3.62	14.42
Event-Driven MS	1.20	4.67	-2.21	8.81	-3.12	11.16
Risk Arbitrage	0.78	0.75	-1.30	5.28	-1.86	6.78
Panel C: The Lehman Meltdown Subsample (December 2008 to June 2009)						
Convertible Arbitrage	1.03	-1.16	-0.58	-5.30	-0.85	-4.06
Dedicated Short Seller	2.87	-7.83	-12.14	-51.78	-15.09	-54.53
Emerging Markets	5.38	83.46	-12.69	105.06	-15.91	100.56
Equity Market-Neutral	1.08	1.45	-1.73	3.62	-2.29	4.19
Long Short Equity	1.68	14.18	-4.82	29.54	-6.28	30.97
Distressed	1.17	5.24	-2.23	6.80	-3.03	8.66
Event-Driven MS	1.32	3.85	-2.03	8.96	-2.69	10.31
Risk Arbitrage	0.82	0.80	-1.17	3.10	-1.58	3.90
Panel D: The Post-Lehman Subsample (July 2009 to December 2011)						
Convertible Arbitrage	1.09	2.56	-0.88	-2.51	-1.26	-1.10
Dedicated Short Seller	2.80	-4.70	-12.26	-56.44	-15.60	-59.89
Emerging Markets	5.33	90.84	-12.16	97.38	-15.44	92.17
Equity Market-Neutral	1.17	3.76	-1.85	3.32	-2.44	3.79
Long Short Equity	1.63	17.36	-4.88	32.11	-6.60	33.83
Distressed	1.17	7.39	-2.42	9.84	-3.41	12.04
Event-Driven MS	1.26	7.28	-2.19	11.34	-3.08	13.19
Risk Arbitrage	0.82	2.80	-1.30	4.96	-1.81	5.97

Notes: Average marginal contributions of the eight strategy indexes to the volatility, VaR, and ES of a fund of hedge funds, represented by an equally weighted portfolio of the eight strategies, estimated using the S&P-RSM model. Panel A presents the average marginal risk contributions for the whole test period, January 2005 to December 2011. Panels B, C, and D present the results for three subperiods before, during, and after the Lehman meltdown.

measures in terms of single strategies: The emerging market strategy has the highest volatility, VaR, and ES. The strategy with the next highest volatility, VaR and ES is the dedicated short bias strategy. However, this

strategy makes the lowest marginal contribution in terms of volatility (-20.41), VaR (-51.08), and ES (-53.36). This result can be explained by the fact that the dedicated short bias strategy is designed to be negatively

related to the other strategies through its negative exposure to the market.

Exhibit 5 shows that the dedicated short bias strategy makes negative contributions to all three portfolio risk measures. The negative contributions of those strategies are interpreted as actually decreasing the portfolio risk. For instance, a -20% volatility contribution by strategy X can be interpreted as meaning that the volatility of the portfolio is 20% less than it would be if strategy X were not included. That is, strategies with negative contributions generate great hedging benefits, which indicated that, despite having high idiosyncratic risk, such strategies help to reduce risk when included in a portfolio with other strategies. From Panels B and C of Exhibit 5, we observe that this feature is shared by the convertible arbitrage, dedicated short bias, and equity market-neutral strategies. However, if we look at the three subsamples (pre-Lehman, Lehman meltdown, and post-Lehman), this feature characterizes the equity market-neutral strategy only for the pre-Lehman period. In the Lehman meltdown and post-Lehman period, we find that the equity market-neutral strategy makes a positive marginal contribution to the variance of the portfolio of funds. The same applies, with a lower intensity, for the convertible bond arbitrage strategy. On the other hand, there is a striking difference between the $M\sigma$ and both the $MVaR$ and the MES over the whole estimation period: The convertible arbitrage and equity market-neutral strategies make negative contributions to the portfolio volatility and positive contributions to the portfolio tail risk ($MVaR$ and MES).

The complete results for the three sample periods are reported in Panels B, C, and D of Exhibit 5. The general picture of which strategy makes the greatest or smallest contribution is the same here as it is for the whole period given in Panel A. However, the actual percentage contributions of each strategy differ among the three periods. Comparing the marginal contributions to portfolio volatility, we observe that, although the marginal contributions of the strategies that make positive contributions in the first period decrease in the second, those of the strategies that make negative contributions in the first period show significant (in terms of percentage change) increases in the second period, with some contributions even switching sign. That is, those strategies that could be used as hedging instruments in the first period lose their hedging benefits

and in some cases become risk contributors. This result suggests that, during and after the Lehman meltdown, all strategies systematically become greater risk contributors, with almost no exception: Even the dedicated short bias strategy largely reduces its hedging role when considering tail risk (from -49.41 to -14.97) for the MES measure. It would be difficult to capture this effect using the individual strategy risk measures.

In summary, this first analysis shows that some strategies, like dedicated short bias, contribute negatively to the risk of the overall portfolio of hedge funds. Some strategies, like emerging market, contribute the most to the total portfolio risk. As a result, some strategies are natural hedgers, and some strategies tend to amplify portfolio risk. The effect is qualitatively the same for all three measure of marginal risk contribution but quantitatively largely different.

The second analysis we performed considers the other risk factors that may systematically affect hedge fund strategies. This analysis provides results that are rather different. As shown in Exhibit 4, factors other than the S&P 500 affect hedge fund strategies. The inclusion of these factors in the picture greatly changes the different hedge fund strategies' tail risk contributions to the portfolio. In particular, we observe that the exposure to these other risk factors affects the tail risk of the portfolio more than it does the volatility. In fact, the contribution does not change significantly in terms of marginal standard deviation but does change greatly for the $MVaR$ and the MES . Looking at the dedicated short bias strategy, for example, we can see that the ability to hedge portfolio risk reduces by one half and the equity market-neutral and convertible arbitrage strategies now positively contribute to the $MVaR$ and MES without providing any hedging benefit.

More specifically, the results presented in Panel A of Exhibit 6 show that, over the whole estimation period, it is again the emerging market strategy that makes the highest contribution to all three risk measures. It is interesting to observe that, looking at the individual strategies, the emerging market strategy has the highest volatility (3.42) but, in terms of VaR and ES , the dedicated short bias strategy has the higher tail risk (-7.03 and -9.45 respectively). Although the dedicated short bias strategy has the second-highest estimated volatility and the highest VaR and ES (-15.02 and -18.27 , respectively), this strategy again makes lower marginal risk contributions to the portfolio over all the samples

EXHIBIT 6

Average Marginal Risk Contributions (MF-RSM)

Strategy	Strategy Volatility	Strategy MC to Portfolio Volatility	Strategy VaR (95%)	Strategy MC to Portfolio VaR	Strategy ES (95%)	Strategy MC to Portfolio ES
Panel A: The Whole Sample (January 2005 to December 2011)						
Convertible Arbitrage	0.54	-3.08	-4.65	1.23	-6.01	1.20
Dedicated Short Seller	1.72	-12.93	-15.02	-30.30	-18.27	-41.93
Emerging Markets	3.42	92.79	-7.03	52.46	-9.45	59.57
Equity Market-Neutral	1.00	-1.33	-1.69	1.76	-2.17	1.21
Long-Short Equity	1.31	10.21	-9.62	32.96	-12.67	37.05
Distressed	1.71	15.82	-4.92	22.75	-6.49	22.10
Event-Driven MS	1.09	0.57	-4.59	9.97	-6.33	11.60
Risk Arbitrage	0.77	-2.05	-3.21	9.17	-4.24	9.19
Panel B: The Pre-Lehman Subsample (January 2005 to November 2008)						
Convertible Arbitrage	0.51	-4.50	-3.48	-0.14	-4.64	0.39
Dedicated Short Seller	1.63	-16.10	-14.65	-35.58	-17.90	-49.41
Emerging Markets	3.51	99.31	-6.07	50.59	-8.37	59.33
Equity Market-Neutral	0.91	-2.41	-0.88	0.63	-1.29	0.35
Long-Short Equity	1.28	9.81	-8.15	35.26	-11.06	39.58
Distressed	1.69	16.47	-3.98	28.58	-5.69	27.43
Event-Driven MS	1.01	0.56	-4.10	9.31	-5.76	11.22
Risk Arbitrage	0.75	-3.13	-3.05	11.36	-4.09	11.10
Panel C: The Lehman Meltdown Subsample (December 2008 to June 2009)						
Convertible Arbitrage	0.54	0.49	-7.16	1.66	-8.93	1.30
Dedicated Short Seller	1.82	1.43	-17.10	-7.87	-19.96	-14.97
Emerging Markets	3.32	73.69	-11.58	46.58	-14.65	51.30
Equity Market-Neutral	1.10	0.35	-2.63	3.05	-2.88	2.20
Long-Short Equity	1.39	9.95	-14.60	26.70	-17.91	29.40
Distressed	1.75	12.88	-7.18	14.04	-8.50	13.86
Event-Driven MS	1.26	0.14	-6.69	9.80	-8.43	10.66
Risk Arbitrage	0.76	1.07	-4.48	6.06	-5.42	6.24
Panel D: The Post-Lehman Subsample (July 2009 to December 2011)						
Convertible Arbitrage	0.58	-1.74	-5.83	3.33	-7.43	2.48
Dedicated Short Seller	1.83	-11.77	-15.01	-27.95	-18.41	-37.24
Emerging Markets	3.30	87.47	-7.36	57.12	-9.76	62.24
Equity Market-Neutral	1.13	-0.04	-2.79	3.24	-3.39	2.34
Long-Short Equity	1.33	10.95	-10.71	30.96	-13.82	35.07
Distressed	1.72	15.57	-5.79	15.72	-7.23	15.72
Event-Driven MS	1.19	0.71	-4.85	11.09	-6.69	12.50
Risk Arbitrage	0.83	-1.15	-3.15	6.49	-4.15	6.91

Notes: Average marginal contributions of the eight strategy indexes to the volatility, VaR, and ES of a fund of hedge funds, represented by an equally weighted portfolio of the eight strategies, estimated using the MF-RSM model. Panel A presents the average marginal risk contributions for the whole test period, January 2005 to December 2011. Panels B, C, and D present the results for three subperiods before, during, and after the Lehman meltdown.

considered. In fact, Exhibit 6 shows that the dedicated short bias strategy makes negative contributions to all three portfolio risk measures, which means that, for large sample periods, this strategy is able to reduce the

tail risk of the portfolio of hedge funds even if exposed to various risk factors.

In summary, our analysis evidences that other risk factors and financial crisis times show that all hedge fund

strategies tend to be positively exposed to liquidity and credit risk, and as such, all positively contribute to the total tail risk of the portfolio of hedge fund strategies. As a result, the natural ability of some hedge fund strategies to be hedgers to the total portfolio risk disappears during crisis periods. This is important especially during crisis periods, as investors seek diversification and hedging benefits from hedge funds.

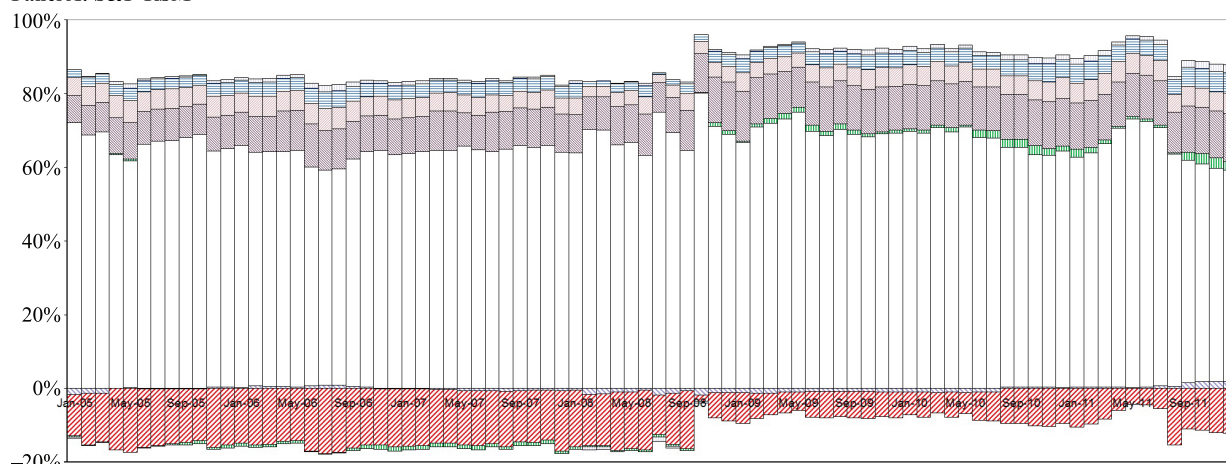
To complete the picture, we present the dynamics of the marginal risk contribution of each strategy over the period from January 2005 to December 2011. Exhibit 7

shows the percentage marginal contribution of each strategy to the portfolio volatility, estimated using the S&P-RSM and MF-RSM models in Panels A and B, respectively. These results indicate that the emerging markets strategy is the greatest contributor to the equally weighted portfolio risk throughout the whole period. This is not surprising given that this strategy has the highest estimated volatility among the group of strategies. Moreover, the dedicated short bias strategy contributes toward reducing the volatility of the portfolio for almost all months in the estimation period, with the exception of a few months in

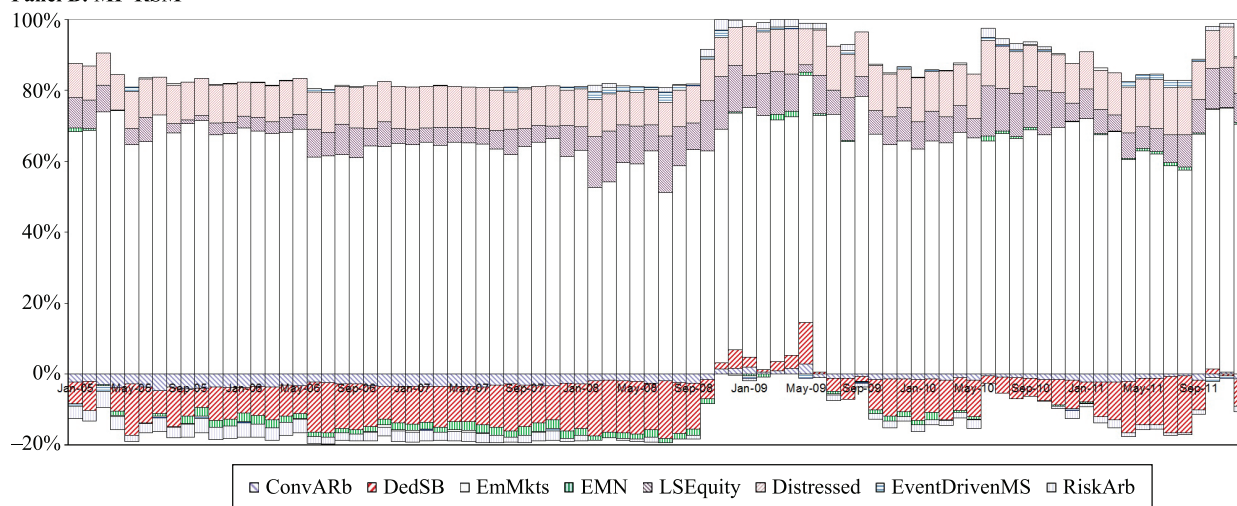
EXHIBIT 7

Dynamics of Marginal Volatility Contribution

Panel A: S&P-RSM



Panel B: MF-RSM



Note: The graphs show the dynamics of each strategy marginal contribution to the portfolio volatility, estimated using the S&P-RSM model (Panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

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2009. Other strategies also make slightly negative contributions for most of the sample period. Figures show that these negative contributions are minimal for the one-year period from September 2008 to September 2009. This is an important result, indicating that, since most hedge funds (independently of their strategy) were affected by the liquidity freeze, they lost their ability to hedge tail risk. This in turn suggests that, to some extent, the models are able to capture the systemic risk in the portfolio, especially during major crises such as the 2008 liquidity crisis. We observe that the strategies that help to reduce the volatility

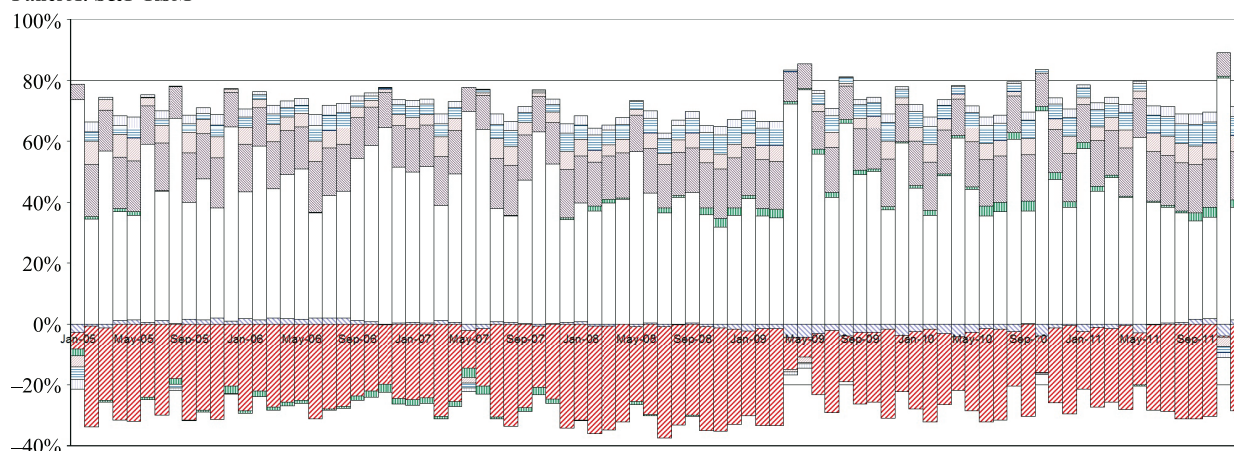
of the portfolio in other months lose this characteristic during the Lehman meltdown. In particular, the dedicated short bias strategy, which has the largest effect in terms of reducing portfolio volatility in normal periods, has the smallest capability of reducing volatility—with very small negative contributions and even positive contributions in some months—during the crisis periods.

Although largely different in terms of the magnitude of individual strategy contributions, we find similar results for the marginal contributions to portfolio tail risk, as measured by VaR and ES (see Exhibits 8 and 9).

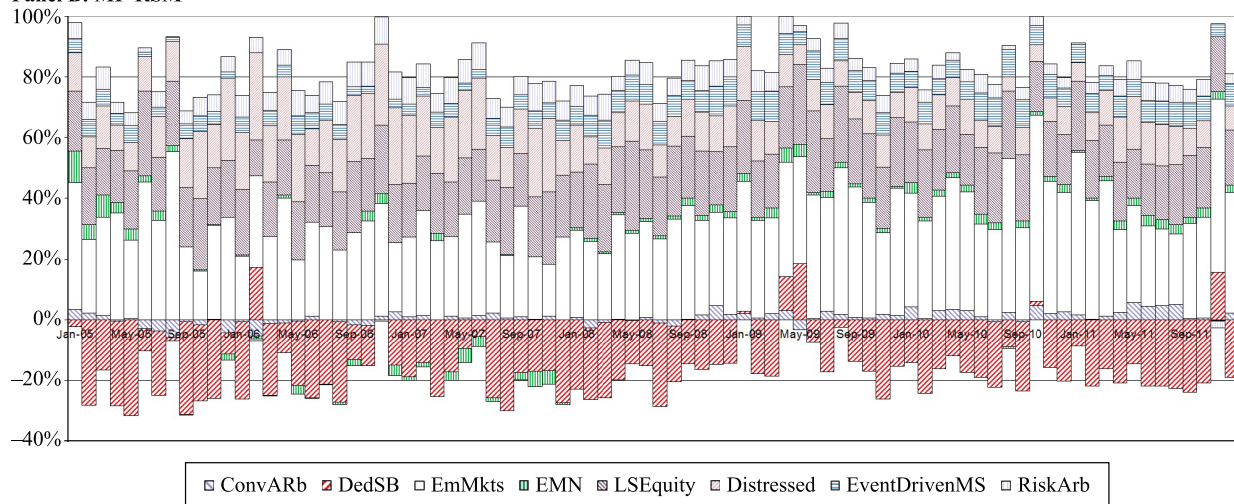
EXHIBIT 8

Dynamics of Marginal Value at Risk Contribution

Panel A: S&P-RSM



Panel B: MF-RSM



Note: The graphs show the dynamics of each strategy marginal contribution to the portfolio VaR, estimated using the S&P-RSM model (Panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

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The most significant difference between the tail risk and volatility contributions is due to the equity long-short and distressed strategies. Although they both make positive contributions to portfolio risk in general, they contribute higher percentages of the portfolio tail risk throughout the estimation period.

CONCLUSION

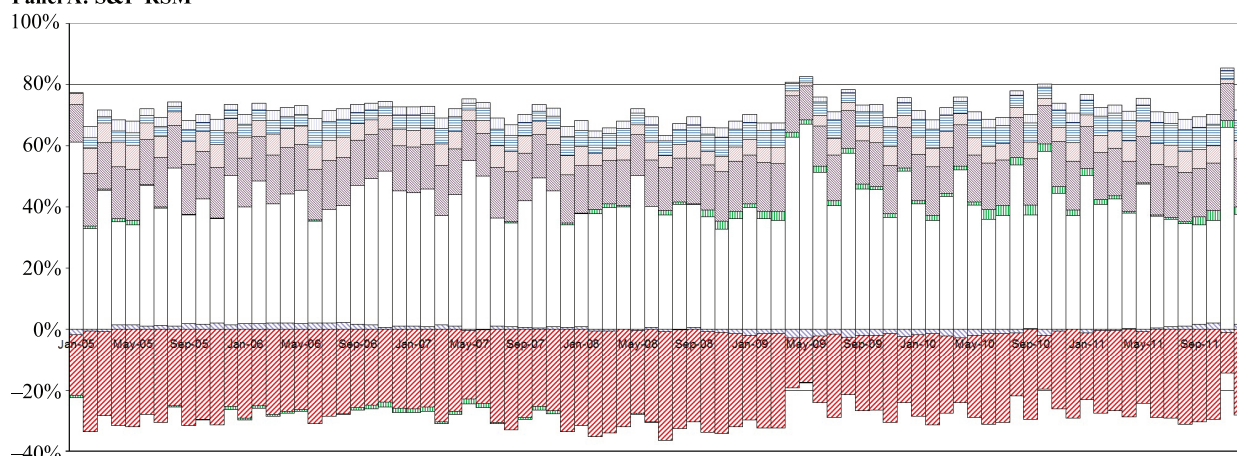
This article provided a new risk management tool for measuring tail risk for portfolio of hedge funds, based

on regime-switching models. Our empirical results showed that the contribution to the tail risk of a portfolio of hedge funds is only partially related to the main systematic risk measure identified in portfolio diversification literature: the market portfolio. By using the S&P 500 as a proxy for the market portfolio, we showed that, even in a dynamic framework in which exposure to the market risk factor changes over time, the contribution of this exposure to the tail risk of a hedge funds portfolio is largely lower than expected and mostly occurs during crisis periods. Other factors greatly affect the tail risk of a

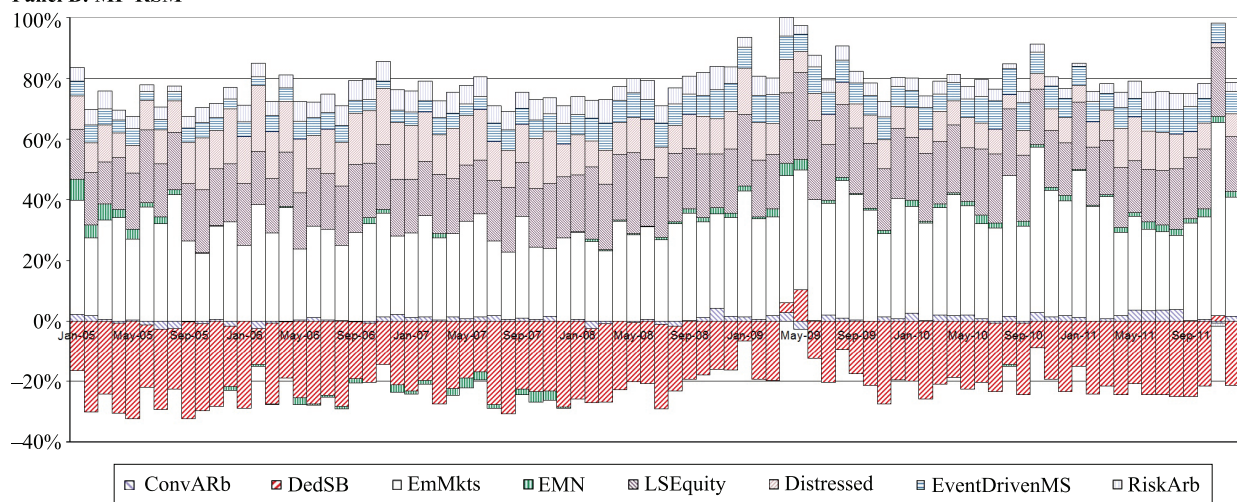
EXHIBIT 9

Dynamics of Marginal Expected Shortfall Contribution

Panel A: S&P-RSM



Panel B: MF-RSM



Note: The graphs show the dynamics of each strategy's marginal contribution to the portfolio ES, estimated using the S&P-RSM model (Panel A) and the MF-RSM model (Panel B) over the period of January 2005 to December 2011.

For a color version of this exhibit please visit The Journal of Alternative Investments at www.ijai.com.

fund of hedge funds, such as liquidity risk or credit risk. The effects of these other factors are so strong that even a strategy such as dedicated short bias, which is characterized by a negative position on the equity market and therefore is a natural hedging strategy, is not able to reduce, even partially, the tail risk during a crisis. In contrast, it contributes to the tail risk like all the other strategies and does not act as a hedging instrument when the need for it to do so is high.

Therefore, our results show that, mostly during a crisis period, all of the hedge fund strategies contribute to tail risk, largely because of their exposure to the liquidity and credit risks that affect the entire hedge fund industry, and there is no great distinction between strategies.

The results of the article have natural extensions to the growing literature on funds of funds. These funds invest in different hedge fund strategies, thus providing diversification benefits to investors. However, this benefit was nonexistent during the recent systemic crisis when all strategies became correlated. Consequently, investors who thought they were hedged lost money due to the fact that all strategies positively contributed to the tail risk of the total portfolio. This may explain the decline in the number of funds of funds and total investments allocated to them that we have observed in recent years.

ENDNOTES

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¹For further discussion of regime-switching models characterized by Markov chains, please see Hamilton [1994]; Billio and Pelizzon [2000]; Billio, Getmansky, and Pelizzon [2012a]; and Billio, Frattarolo, and Pelizzon [2014].

²Billio, Frattarolo, and Pellizon [2016] serves as an online appendix to this article.

³This derivation is similar to Litzenberger and Modest's [2008] derivation of the marginal contribution to the ETL.

⁴Although the second case is in line with the strategy of equity market-neutral funds, convertible arbitrage gives the wrong impression that the strategy is independent from

the equity market. However, the estimated market exposure for this strategy varies over time and is mostly significant in at least one of the regimes—specifically, the normal market regime in the expanding window estimation.

⁵Although the case of the equity market-neutral strategy is perfectly in line with our a priori understanding of the strategy, the nonsignificance of the emerging market strategy exposure could seem mind-boggling at first glance. However, this can be explained by the fact that we are now including the Emerging Markets Stock Index.

⁶We used this portfolio of hedge fund strategies to represent funds of hedge funds. It is obvious that it is not straightforward to extend our findings directly to structured portfolios such as those managed by a fund of funds, mostly because funds of funds invest in individual hedge funds instead of indexes and have a different structure, with layers of fees among other things. However, to the extent that strategy indexes represent the risk exposure of funds in the specific strategy, we can expect the portfolio to reflect the risk exposure of a fund of funds investing in the eight strategies we consider here. Moreover, estimating such a model for each individual hedge fund would be very computationally intensive.

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