A nonparametric measure of riskness in financial systems

Una misura nonparametrica di rischiosità nei sistemi finanziari

Francesca Parpinel and Claudio Pizzi

Abstract In the literature of risk analysis different synthetic indices are artificially built and in this work we propose to use the combination statistical procedure of the univariate indices proposed by *V-lab*. The combination technique may also be considered to perform nonparametric inference. So we propose to highlight systemic risk in a network of companies performing a nonparametric test to reveal heterogeneity behaviour; in this case one ranking is used to create different behavioural groups.

Abstract *In letteratura le diverse misure sintetiche di rischio sono costruite artificiosamente e in questo lavoro ne indichiamo uno che si basa su una procedura di combinazione statistica di indici univariati che sono proposti dalla piattaforma* V-lab. *La tecnica di combinazione può essere usata anche per procedure inferenziali nonparametriche. Con tale strumento si può evidenziare il rischio sistemico di una rete di compagnie, usando un test non parametrico per rivelare comportamenti di eterogeneità; qui l'ordinamento si usa per creare i diversi gruppi.*

Key words: Systemic risk, ranking, nonparametric combination.

1 Introduction

The recent Financial Crisis of 2007-2009 is defined by some economists as "the worst crisis after the Great Depression of the thirties", highlighting the need of new definitions and measures of the risk associated both to the financial system and to its institutions. The history of risk definition and importance of measuring it are very old going back to the days of ancient Greece, but nowadays this need is increas-

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ingly pressing (see [1]). Roughly speaking, the term risk may refer to two different "dimensions"; in fact, considering the institutions–level, it is a measure of some peculiar aspects of their riskiness whereas if we consider the system–level, it is a measure capturing the depth and the breadth of the networks linking the financial institutions. In this last case we call it systemic risk and in literature we find many different definitions of it, with consequently several measurement procedure. For example in [6], it is considered as the possibility that an insolvent financial Institution may transferred its insolvency to the whole financial system and in [3] the systemic risk is represented by network diagrams depending on Granger causality index for each institution. Other peculiar definitions compare it to Nessie, the Loch Ness Monster (see [2]), as everyone knows it *but nobody knows when and where it might strike*. Other authors (see[5]) consider the risk coming from some unusual event with strong correlations among different assets. Kaufman states that it is the consequence of a series of losses moving within a network of markets or institutions (see [8]). Further definitions can be found in literature. The importance of defining, and then measuring, systemic risk is really strong as financial surveillance is nowadays necessary for the governments policies of various contries (see [7]).

In the present work we highlight the relations of the individual institutions, proposing an index linking several variables that characterize each financial institutions, with the aim of ranking the companies in the network by riskiness. At first, the ranking obtained by this index will be compared with the ranking induced by the systemic risk measure proposed by *V-Lab*[1], the platform of the NYU Stern School of Business. The risk measure computed by *V-Lab* estimates the amount of recapitalization necessary to a company not to fail in a financial crisis, while the index proposed in this work estimates the effective level risk at a specific time. The comparison lets us to find analogies and differences in the two rankings constructed with real data, and leads us to propose some new risk measures.

2 Nonparametric combination of dependent rankings

As pointed out in the previous section, the number of variables involved in the measuring process may be high. The idea is to reduce high dimensionality in order to create only one dimension and to treat easily ranking among institutions.To this goal we may use several statistical tools. For example we may consider the ranking obtained by principal component analysis. With this tecnique, in a Gaussian framework, it is possible to reduce the number of variables keeping as much as possible the variability structure, represented by the covariance matrix, through a linear combination of them (see [11]). In such a way, if the first components were able to get most of the data variability, we could consider only them as representative of the whole system. One of the bounds in this technique is linked with the Gaussian

[1] Volatility Laboratory, {`http://vlab.stern.nyu.edu/` }

assumption, as all the multivariate variability is represented only by the variance matrix.

Here we propose to use, in alternative to a linear combination of statistical measures, a nonparametric one based on the rankings of such measures, according to Pesarin's work (see [10]), as it is satisfactory even when the rankings may be dependent. Each risk measure can capture only some feature of risk and of systemic risk too, so our idea is to use all the available variables giving some partial, even overlapping, information about it. Let's suppose to measure $K > 1$ random variables, denoted by $(X_1,\dots,X_k,\dots,X_K)$ and to transform them in some variables denoted by λ_k, $k = 1\dots,K$ each defined over $(0,1)$. The combination function ψ of these new variables λ_k may depend on some weights, denoted by $(w_1,\dots,w_K)$, according to the importance of each variable and produce a new variable Y through a function $\psi : R^{2K} \to R^1$. Following [9], the idea of combining different statistical indices, typically dependent on each other, arises from the same procedure for combination of dependent tests in multivariate analysis (see [10]). In the inferential case the combining functions are applied to p-values associated to marginal tests and is tipically a nonparametric one. We must underline that this procedure doesn't explicitly involve the whole time series.As well described in [10], the combination function ψ has to satisfy some minimal properties, that are continuity in all its arguments, non-decreasing in each λ_k, symmetry, i.e. invariant with respect to rearrangements of the variables λ_k, its supremum, $\bar{\psi}$, is attained when even one value of λ_k tends to zero, the value of ψ is strictly less than $\bar{\psi}$.

In our work we will use the Fisher combination function

$$\psi = -\sum_{k=1}^{K} w_k \cdot \log(1-\lambda_k). \tag{1}$$

In literature other important combining functions, all satisfying the above properties have been proposed, for example: the Tippett one, where $\psi_T = \max_k(w_k \cdot \lambda_k)$; Normal one, in which $\psi_N = \sum_{k=1}^{K} w_k \cdot \Phi^{-1}(\lambda_k)$, and the Logistic one, with $\psi_l = \sum_{k=1}^{K} w_k \cdot \log[\lambda_k/(1-\lambda_k)]$.

3 A further index to measure risk

In the literature, there are a lot of different measures to evaluate risk in a company, that are often used in comparisons; but, why do we compare these indices? We propose to use all of them in order to get a more complete information about risk and the obtained results will be compared with the *ranking* estimated by *V-Lab* in order to evaluate the correspondences and the main differences for European Banks. With this aims we use the variables described in the following and summarized in Table 1 and we embed them in our framework to compute $Y = \psi_1(\cdot)$, by (1). First of all X_1 is *Marginal Expected Shortfall* denoting the expected loss (per dollar invested in capital) in which a company would occur with a fall market equal to 2%. Variable X_2

Table 1 The involved variables

Variable	Description
X_1	MES: Marginal Expected Shortfall
X_2	Beta: slope between firm's stock return and market returns
X_3	Correlation: between share return and Market Value Weighted Index
X_4	Volatility: The annualized volatility of company share capital
X_5	Leverage: Indebtness
X_6	SRISK: the systemic risk measure indicated by VLab

is *Beta* that is the covariance between a firm's stock return and the market, divided by the variance of market returns; in our case, it explains the correlation between the *Eurostoxx50* and the main equity security of each institution. The *Correlation* between the share return and the *Market Value Weighted Index*, representing the movement of the market in which changes in the price of the various stocks lead to the final value of the index in proportion to its value of market capitalization, is denoted by X_3. Variable X_4 is the *Volatility*, measured by the annualized standard deviation of returns based on daily returns. At last we consider X_5 the indebtedness, *Leverage*. The new index is compared to *SRISK*, see [4], that is the measure of systemic risk of each institution over the global European risk, and here denoted with X_6; it is an estimate of the amount of recapitalization that a company needs not to fail in a financial crisis.

At last we propose to use the combination only for the variables characterizing each institution, X_1, X_2 and X_3, but not explicitely their riskiness, instead represented by X_4 and X_5, calling the combined variable as *Combined Index 3*. So two or more groups of companies may be identified using quantiles of the obtained *Combined Index 3*. Then, on these groups we test the combination of X_4, X_5 and X_6 following a permutation procedure proposed by [10].

3.1 Case study

The dataset is composed by a set of $N = 103$ financial institutions for which we observe the variables described in Section 3 and that we can get from *VLAB*, (data recorded in March, 2014). The Spearman correlation matrix, showed in Table 2 and the tests performed on each pair of variables show the cases in which we can reject the hypothesis of null correlation. We can note that in most cases the correlations are significative (here we don't consider any adjustment for multiple tests). Only variable *Volatility* may be considered incorrelated to the other ones, in particular *Correlation*, *Leverage* and *SRISK*.. This correlation structure explains the weak dependence among the considered variables, and it is positive as we may use all of them in order to gain a better comprehension of the phenomena. To compute the combined index, first of all, the institution are ranked in increasing order with re-

Table 2 Correlations and significativities (*) at $\alpha = 0.01$

	MES	Beta	Correlation	Volatility	Leverage
Beta	0.9991(*)	-	-	-	-
Cor	0.7084(*)	0.7100(*)	-	-	-
Vol	0.3734(*)	0.3739(*)	-0.1777	-	-
Lvg	-0.2923(*)	-0.2879(*)	-0.3663(*)	0.1219	-
SRISK.	0.3803(*)	0.3837(*)	0.4058(*)	0.0934	0.4865(*)

spect to each variable, then, such ranks are transformed in sample percentiles called η_k, $k = 1, \ldots, K$.

Let X_{ki} denote the value of k-th variable, with $k = 1, \ldots, K$, on unit i, with $i = 1, \ldots, N$. Function $I(A)$ is 1 if A is true and zero otherwise. Then for each variable X_k we consider the following transformation

$$\eta_{ki} = \frac{\sum_{j=1}^{N} I(X_{ki} \geq X_{kj}) + 0.5}{N+1} \tag{2}$$

where values 0.5 and 1 assure the absence of 0 and 1, for variable η_k, and so we avoid the not finiteness problems of combination function. In such a way, we obtain a $K \times N$ matrix for values η_{ki}. As each column of the matrix, ordered in decreasing way, are the partial rankings, the global one is gained applying the Fisher combination function to each row, with $w_k = 1$, $\psi_i = -\sum_{k=1}^{K} \log(1 - \eta_{ki})$. Some comparisons in term of correlations of the rankings induced by *SRISK*, by the combined index including X_1, X_2, X_3, X_4, X_5 and called *Combined.1*, by the combined index including X_1, X_2, X_3, X_4, X_5 and X_6 and called *Combined.2*, and by the first component produced by PCA) show that the correlations are not too strong but they are significatively different from zero. This confirms the idea that the problem is a complex one and cannot be reduced to only one variable.

4 To define a new measure of riskiness

The same combination strategy may be used to identify and test the heterogeneity of a set of data, considering a permutation tests for complex data. To this aim we can think to distinguish 2 groups of institutions created considering the combination only for the variables characterizing each institution, that is X_1, X_2 and X_3, defined as *Combined Index 3*, and considering the third quartile, Q_3 as the element to divide the data in two subsets. The summary statistics of this new index are reported in Table 3.

The variables, that are more explicitly explaining risk, may be represented by X_4, X_5 and X_6. So two, or in general even more than two, groups of companies may be identified using the quartiles of variable *Combined Index 3*. In our case the size of dataset is 103, so we can consider two groups separated by the third quartile, equal

Table 3 Summary statistics of Combined Index 3

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.12	0.91	1.88	2.01	2.81	5.41

to 2.807, one of size 77, the other one of size 26. Then, on these groups we test the combination of X_4, X_5 and X_6 following the permutation procedure proposed by [10]. We consider the difference of the *sample coefficients of variability* computed in each groups as statistic test: $s = cv_1 - cv_2$ where cv_i is the ratio of the sample standard deviation to the sample mean for $i = 1,2$. The combining function we use is the Fisher *omnibus* defined by (1) and the hypothesis system is

$$\begin{cases} H_0 : CV_1 = CV_2 \\ H_1 : CV_1 \neq CV_2 \end{cases}$$

If the data leads us to accept the null hypothesis, that means that the two groups are very similar in term of behaviour, so we may suggest that the two group are similar and that the riskiness is high for this network of institutions. Otherwise we refuse the idea of high risk.

With the available dataset, the Non Parametric Combination method is performed over $B = 1000$ randomized permutation of the two groups, and we find that the p-value associated to the observation is 0.23776. This value does not allow to reject the hypothesis of high risk for this network of institutions to significance level $\alpha = 0.05$.

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