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Dynamic Patterns in Similarity-based cooperation

An Agent-based investigation

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Abstract Understanding what motivates and fosters collective actions has major implications in the governance and management of organizations, in the regulation and design of public policies, and has long attracted the interests of scholars and practitioners in business and economics. This paper deals with how groups of agents emerge in a dynamic contest characterized by lack of formal structure and uncertainty regarding the possible individual outcomes, focusing on the features of the cooperators and on the dynamics emerging among them. Through the development of a stylized agent-based model we start by showing how similarity in

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1 Introduction

The emergence of cooperation among utility-maximizing individuals is a long-standing puzzle for scholars. Economic and organizational research dedicated significant attention to the study of cooperative dynamics, addressing its drivers through a plurality of methods and tools. Among others, Agent-based literature and Game Theory contributed to this field, modeling possible explanations for the emergence and evolution of cooperation among utility-maximizing agents resorting to *reciprocation* -direct or indirect [Axelrod, 1984,Nowak and Sigmund, 1998]- or other forms of *shadow of the future*[Axelrod and Dion, 1988]. Research on cooperation and groups formation also dedicated significant attention to ”other regarding preferences”, suggesting the idea that individuals may feel more altruistically towards similar others. This idea has been studied in sociological and economic theory under the label of *homophily*, namely the tendency of social actors to form ties with other actors similar to themselves in terms of several dimensions - race, culture, religion, occupation, attitudes, etc. [McPherson et al., 2001]. This concept has been variously used and developed in game theory, economic and organizational research: from tag-based cooperation

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models [Riolo et al., 2001], to spacial and other forms of clustering [Axelrod, 1984], to peer pressure in partners selection to form alliances [Kandel and Lazear, 1992]. In this paper we aim at contributing to this strand of literature analyzing the emergence of cooperation among unrelated individuals based on the preference for similarity, proposing a model with two peculiar features: the context informality and the presence of agents characterized by multi-dimensional types. This model shows features resembling standard social dilemmas in which cooperation is classically studied. Individuals choose to join a group anticipating they will experience a participation premium that is available to members only. This participation premium has an immaterial component, which has features of non rivalry among group members, although naturally it is excludable to non-members. Similarity across individuals is measured along a vector of individual characteristics (salient and general values), pertaining to their individual preferences. Values enter into the computation of individual utilities alongside the material endowment (building on the idea of homophily). As in [Alesina and La Ferrara, 2000], we assume that "if preferences are correlated with these characteristics, [this] is equivalent to saying that individuals prefer to join groups composed of individuals with preferences similar to their own". The utility function developed in this model is inspired from a concept found initially within the risk management literature, that of *Salient Value Similarity*, which introduces salient values as the relevant dimension for the perception of similarity. In our view, these values represent core standpoints of agents that cannot be subject to any adaptive processes. The other dimension of similarity is that of *general values*, which may be seen as more volatile positions than salient values, representing the perception of agents regarding the environment they are interacting in. As an example, the reader can think about potential cooperators of different races and diverse cultures, where the similarity among agents depends on an immutable trait (race) and on another cultural attribute that potentially can be adjusted and blended through repeated interaction and "contamination" among group members. Agents are heterogeneous in their "endowments" and at each time decide to join one of two groups or alternatively to stay on their own. Deciding to join, they automatically commit all their endowment as a contribution to the group, receiving as a payoff an equal share of the total contributions to the group plus the value coming from similarity (the immaterial component of the utility function). Agents are free to join and leave groups at any time, with no costs of entry or exit. If they decide to exit (that is to say, stay on their own), they will keep their initial endowment only. Interesting dynamics and the potential limits of similarity as a driver for cooperation will be observed increasing the number of "races" and variety of "cultures", when more information is available, and with the introduction of a memory parameter for agents' previous choices, which is interpreted as a precursor of trust-building processes developed among group members.

The paper is organized as follows. After an introductory section aimed at contextualizing this work within existing literature, Section 3 introduces the main concepts and the features of the model. Results are presented in Section 4, where we discuss both representative examples and aggregate data obtained running a large number of simulations. Section 5 concludes discussing specific examples of potential application of the model and suggesting directions for further extensions.

2 Background Theory

Collective action is a very important driver in economics and has rightfully attracted a lot of interest from both economic theory and empirical analysis. The emergence of stable groups of like-minded agents is at the basis of the creation of institutions, of the provision of specific goods and services and in general of the progress of human society. If a strand of literature has focused more on understanding how to incentivize players to cooperate and form stable cooperative groups, there is still uncertainty regarding the evolutionary dynamics of cooperation among similar agents, and in particular which other elements (wealth, information, memory and trust-building processes) can influence cooperation based on similarity. This paper is naturally framed in the context of cooperation and group formation research and represents an attempt to move forward in the investigation regarding cooperative action in *informal contexts*, which are characterized by:

- interactions sufficiently random with a very low probability of meeting the same person again (or where there is no possibility to precisely store information about previous encounters);
- absence of biased interactions [Riolo et al., 2001], such as agents embedded in two-dimensional spaces [Axelrod, 1984, Lomborg, 1996] or other context-preserving networks [Cohen et al., 2001];
- presence of negligible direct or indirect costs (or their complete absence) for cooperating or participating in a group.

These settings have been somehow less explored by agent-based literature, thus making them a challenging territory for both a theoretical and an empirical investigation. The present work is characterized by an informal setting, and by a specific choice concerning the formalization of the *utility function* (further details are available in 3): it features both a material and a non-material component, where the former is constituted by an equal share of total individual contributions to the group, and the latter is based on similarity, and summarizes the idea of homophily as a driver of utility for the agents. A lot of empirical evidence on the role of homophily has been provided by sociological and economic literature, showing how people prefer to connect, work, build relationships and play with similar individuals [McPherson et al., 2001, Lincoln and Miller, 1979, Huston and Levinger, 1978, Verbrugge, 1977]. The idea that similarity may in some ways foster cooperation is not new, either in experimental economics or in agent-based literature. In the former, it stems from an evolution of the experimentally founded fact that group identity or other forms of shared identity do support cooperative behavior among members (see [Akerlof and Kranton, 2000] for a seminal introduction to the role of identity in decision making) and increase uncooperative behavior among non-members (referred to as the in-group-out-group bias in [Chen and Li, 2009] and [Sosis and Ruffle, 2006]). In this work we refer to a specific formalization of similarity, that of *salient value similarity*. This concept has been developed in the risk management literature, where it is used in a slightly different way, but its main message is carried over to the present work: salient value similarity has been consistently found as a precursor of *social trust* - trust regarding the institutions we live in. [Poortinga and Pidgeon, 2006] describe salient value similarity as based on the idea that people use heuristics based on perceived similarities while making choices in complex environments, basing their judgments on the feeling that other persons or organizations have the same understanding of a specific situation. According to [Siegrist et al., 2000] ”Salient Values consist of the individual’s sense of what the important goals (ends) and or processes (means) are that should be followed in a particular situation” and are ”an aspect of the individuals understanding of the meaning of a specific situation”. The idea of salient values will be introduced in this work as the carrier of individual characteristics on which cooperation can be built, alongside another parameter, called *general values* representing less stringent individual features that also affect, although to a minor degree, the perceived similarity across subjects. Through these two parameters we are able to endow members with multidimensional types. This feature, to our knowledge, has not yet been proposed in the agent-based literature and has interesting implications for modeling cooperative and evolutionary processes somehow closer to reality. As previously mentioned, the possibility of using similarity as a driver for cooperation is part of a significant strand of literature devoted to agent-based models [Edmonds, 2006, Kim, 2010]. The evolutionary appeal of similarity has been established in the work of [Riolo et al., 2001] and subsequent works by the same authors, which have shown in an evolutionary model with inheritable tags that similarity can indeed breed cooperation. It is important to note that these results derive from setups in which homophily is based only on one dimension - there is one tag representing, for instance, only race or culture - and our setup improves from this state-of-the art proposing two-dimensional types, characterized by salient and general values Given this general setup, the groups that emerge in our model can be described as resulting from voluntary interaction, deliberately formed without a formal structure and based on mutual recognition of membership -given by the similarity perception. Thus, our results are twofold: on one side, we observe the emergence of groups of similar individuals able to overcome the risk of committing their resources to a group; on the other, we find that wealthier agents less willing to cooperate despite homophily preferences, due to the higher risk of being exploited by less wealthy individuals participating to the group. Evidence about the detrimental effect of wealth differences on participation to cooperative groups is already present in the literature. For example, [Lidenberg, 1982], in his investigation of *sharing groups*, shows how ”with increasing welfare per individual in a section of population, sharing groups will become smaller”. Another similar conclusion is reached in the work by [Hegselmann, 1994], which discusses and presents the Humboldt’s argument about the welfare state destroying networks of self-help through a modified version of the Prisoner’s Dilemma Game. Results show how the choice of cooperating in solidarity networks can become significantly less attractive if agents’ wealth is beyond a certain threshold. In the work by [Molinas, 1998] it is discussed how empirical evidences about the effect of wealth differences on cooperation are still controversial, mainly due to the specific context in which studies are developed. But still, in his review, it emerges how the majority of studies agree on the harmful effect of wealth inequalities for the emergence of cooperative structures. In the present work, we will analyze how preferences based on homophily considerations -which positively sustain cooperation- interact with contribution inequality -which, instead, has detrimental effect on participation-, and how the two are affected by changes in some parameters. In particular, we want to focus on participation levels resulting from the increment of the number of ”races” and variety of ”cultures” or the rate of information acquisition, and the introduction of a memory parameter for agents' previous choices -in the extended version of our model presented in section 3.1.

3 The model

In a nutshell, the model can be described as follows. A fixed number of heterogeneous agents are characterized by salient and general values. Agents consider the former as essential principles that are not subject to modifications or adaptation. General values, instead, are considered as less relevant issues. Groups are formed by agents that share their endowment and give members a utility that increases with the size (the sum of individual contributions) and the overall similarity of the group. In the presentation of the model, capital letters are assumed to denote quantities that stay constant, whereas small letters are assumed to denote variables that change with time. Assume K agents have N salient values $S_{ij}, i = 1,\dots,K, j = 1,\dots,N$ and are given a non-perishable endowment $E_i, i = 1,\dots,K$, that represent agent's potential contribution in joining a group. The stable, on-off nature of the salient values is stressed by supposing that they are drawn from the binary set $\{0,1\}$ and denote with $\overline{S}_i = (S_{i1}, S_{i2}, \dots, S_{iN})$ the vector of salient values of the ith subject. Agents are also equipped with general values that are represented by a real variable $0 \le V_i \le \varepsilon, i = 1,\dots,K$ and ε is a scale parameter. At any stage, agents can decide to stay alone or join one of the two groups: in the former case, they will keep their initial endowment, otherwise they will commit it as their individual contribution. Each agent at time t can be a member of the first or second group or be on his own. Let $\mathscr{G}_t^1, \mathscr{G}_t^2, \mathscr{G}_t^0$ be a partition of $\{1,\dots,K\}$ that keeps track of the choice of the agents at any given time t. In other words, $i \in \mathscr{G}_t^w$ if and only if the ith agent is in the wth group at time t (being the ”zero-group” the set of people that decided to stay out of either group). The participation to one group yields members utility through two components. The first one comes from the equal redistribution of the total contributions of the members of the group; the second is a non-material component that depends on the synergic interaction of the members that, in turn, is a function of the overall similarity of the characteristics of the agents (it can be thought as the benefit coming from homophily preferences). Define a similarity function between agents i_1 and i_2 as

$$sim(i_1,i_2) = \sum_{j=1}^{N} \mathbf{1}(S_{i_1,j} = S_{i_2,j}) - \frac{N}{2} - (V_{i_1} - V_{i_2})^2.$$

The first term in the similarity counts the number of equal salient values; the second term subtracts $N/2$, so that the sum of the first two terms is nonnegative when at least 50% of the salient values are concordant; finally, the third term is the squared difference of the general values of the agents. It is worth noting that the two parameters N and ε are related to each other: for a fixed N, a larger ε increases the importance of general values with respect to the salient ones. This formulation of similarity allows to model the idea that people have homophilic preferences and like being in a group with like-minded individuals, where this like-mindedness is measured along the two given dimensions of values -general and salient. In our formulation, similarity increases with common salient values but (exclusively) decreases with more different general values. Hence, the higher ε with respect to N, the less our agents will be willing to collaborate with other individuals, even in the presence of some consensus on salient matters. As we will see later on, for our purpose, we set the parameters of our benchmark environment in such a way that even the complete disagreement on general values between two agents is more than compensated by the agreement on all salient values. This choice has been made to stress the relevance of salient values in the computation of similarity and, consequently, in terms of utility. The utility of agent $i \in \mathscr{G}_t^w, w = 1,2$ is then:

$$p_{it}^w = \frac{1}{|\mathscr{G}_t^w|} \sum_{i \in \mathscr{G}_t^w} E_i + \sum_{k \in \mathscr{G}_t^w, k \neq i} sim(k,i).$$

The two terms of the payoff incorporate on the one side, the fact that in a group "the more, the merrier"; on the other hand, it is of concern not only how many members there are, but who they are. The first term, $\frac{1}{|\mathscr{G}_t^w|} \sum_{i \in \mathscr{G}_t^w} E_i$, redistributes equal shares of the total amount of resources that all agents bring to a group: the decision to take part in a project implies an effort on the part of individuals and the risk of sharing one's own endowment to build the common pie that will be equally divided among all the participants. The second term, $\sum_{k \in \mathscr{G}_t^w, k \neq i} sim(k,i)$, adds to each agent's utility the total sum of the pairwise similarities. For each agent, this total sum can be considered as a measure of the overall coherence of the group, that results in a higher return in terms of synergies for all the members. If $i \in \mathscr{G}_t^0$, the agent prefers to stay alone and his payoff for the current period is simply his own endowment E_i, i.e., $p_{it}^0 = E_i$. The option to stay out, to join or leave one of the two groups is available, at no cost, at any time t. This setting represents the informality needed to model groups defined without a formal structure. Agents' decisions will be based on partial information that is gathered at each time by randomly matching some members of groups (including agents "out" of any group). Hence, groups are dynamic structures that evolve and are shaped by in-group similarity and by the actions driven by the randomness of the matching process. Being aware that utilities are stochastic and dependent on the fluctuating composition of the groups, at each time, every agent randomly and independently meets P other agents, exchanging information about the size of groups, the contribution and the similarity of the matches. This data are used to compute a myopic estimate of the utility of being in a given group. Agents are myopic in the sense that they assume that the P agents they met are representative, in terms of values and contribution, of their whole group (i.e., they believe the sample has the same average value of similarity and the same average endowment of their group). In particular, fix i and assume that $\mathscr{A}_t$ is the set of P agents that meet i. Let

$$\mathscr{A}_t^1 = \mathscr{A}_t \cap \mathscr{G}_t^1,$$
$$\mathscr{A}_t^2 = \mathscr{A}_t \cap \mathscr{G}_t^2,$$
$$\mathscr{A}_t^0 = \mathscr{A}_t \cap \mathscr{G}_t^0,$$

be the subsets of matched agents that are in three $\mathscr{G}_t$, where we drop the reference to i to simplify notation. The agent works out the average endowment of the members of each set and the average similarity with them. The aforementioned quantities $\hat{e}_t^w$ and $\hat{m}_t^w$ for $w = 1,2$ are given by

$$\begin{aligned}\hat{e}_t^w &= \frac{1}{|\mathscr{A}_t^w|} \sum_{j \in \mathscr{A}_t^w} E_j;\\ \hat{m}_t^w &= \frac{1}{|\mathscr{A}_t^w|} \sum_{j \in \mathscr{A}_t^w} sim(i,j).\end{aligned}$$

Using this information, the ith agent can myopically estimate the utility that would result if he switches to one group, assuming the sample averages are representative of the whole group. Hence, estimated utilities in the three possible situations are

$$\pi_t^w = \hat{e}_t^w + |\mathscr{G}_t^w| \hat{m}_t^w, w = 1,2$$

and

$$\pi_t^0 = E_i.$$

The utility of choosing to stay out is set equal to E_i, thus the always available exit option from an informal group corresponds to the sure alternative of keeping one's initial endowment. The reason why individual endowment is not affected positively or negatively by the participation (or lack thereof) in a group lies in the informality of the environment. At the end of period t, agent i chooses to move to another group or to abandon altogether any group based on the highest estimated utility. In more detail, agent ith will move to group w at $t+1$ if

$$\pi_t^w = \max\{\pi_t^1, \pi_t^2, \pi_t^0\}.$$

This simple setup, called *basic* in what follows, can be used to computationally study how groups emerge and evolve on the basis of the similarity in values, and how wealth heterogeneity affects levels of participation.

3.1 Extensions

As more refined forms of reciprocal influence among agents can be conjectured, an extended model can take into account *memory effects*. The remaining part of this section outlines this enhancement. Agents are likely to realize that better utility estimates can be obtained by blending past measures with the novel information derived from sampling. Hence, they update a running measure of the benefits arising from participating to each group and the ith agents takes the decision to switch at time $t+1$ based on the highest among

$$\begin{aligned}\widehat{\pi}_t^1 &= \alpha \pi_{t-1}^1 + (1-\alpha)\pi_t^1\\ \widehat{\pi}_t^2 &= \alpha \pi_{t-1}^2 + (1-\alpha)\pi_t^2\\ \widehat{\pi}_t^0 &= E_i,\end{aligned}$$

where $0 \leq \alpha \leq 1$ is a memory-related coefficient. The basic setup can immediately be recovered by setting $\alpha = 0$ and the parameter α represents agents' memory, or stickiness: when $\alpha \cong 1$, agents will compute their estimates mainly using their previous results whereas for values of α close to zero, agents will rely more on their novel information. It can be argued that the introduction of a memory parameter could be traced back to some forms of indirect reciprocity: in our opinion this is not the case, since its formalization do not allow to store precise information about other group's members and the estimation of future payoff is still myopic in this respect. In the next sections, we will refer to the memory-extension as the *extended version* of the model. Another promising avenue of investigation is offered by introducing the possibility of a contamination processes among cooperating agents, concerning their cultural traits. In fact, agents may reasonably be willing to adapt their general values which, by definition, are more volatile and possibly can be modified to better fit the general values of other members of the group. Thus, the multi-dimensionality of of this setup offers the chance to have semi-moving types in which salient values represent the unchangeable traits of agents. Preliminary results on the effects of the contamination processes are shown in [Cruciani et al., 2013], where general values evolve towards the group members' average value depending on the time spent cooperating in the same group.

4 Results

This Section presents the simulation results for the basic and extended versions of the model, as described in Section 3. In this paper, results are presented in a specific instance in order to give the flavor of the main dynamics. The results of multiple simulations are then summarized in table form to provide a more comprehensive look at the average properties typically present in a large sample of groups that are generated for a given constellation of parameters' values. Table 1 shows the reference, or benchmark, values for the parameters that define a reasonable starting point for our investigation. These values were determined by trial-and-error and then modified, one at a time, to assess the incremental effects of single parametric variations. A number

Name	Value	Description
K	50	Number of agents
N	2	Number of salient values
ε	1.0	Amplitude of general value
P	2	Number of agents sampled (in computing expected utility)
α	0.0	Memory (in computing expected utility)
T	200	Periods
$\overline{E}$	10	Average initial endowment

Table 1 Parameters of the benchmark environment.

of 50 agents is considered, with 2 salient values and a real variable uniformly sampled in $[0,1]$ summarizing their general values. Each run of the model lasts 200 periods and, unless stated otherwise, endowments of agents are uniformly sampled in the interval $[0.5\overline{E}, 1.5\overline{E}]$, where $\overline{E} = 10$.

4.1 The Basic Model

This Section reports results of simulations with the Basic formulation of the model, where no memory is used by agents (i.e., $\alpha = 0$). What we are mostly interested in is how group composition, in terms of both numerosity and internal structure, is affected by the process of group formation driven by the random process of information gathering. In fact, assessing the value of cooperation requires not only understanding the impact of similarity perception on the emergence of cooperation (are groups formed?), but also on its stability and potential implications (does cooperation lead to homogeneous groups?)
The left panel of Figure 1 depicts the time series of the number of participants belonging to each group (labeled with different colors, with green indicating individuals staying out of either group). The right panel shows the average utilities of the members at each given time.

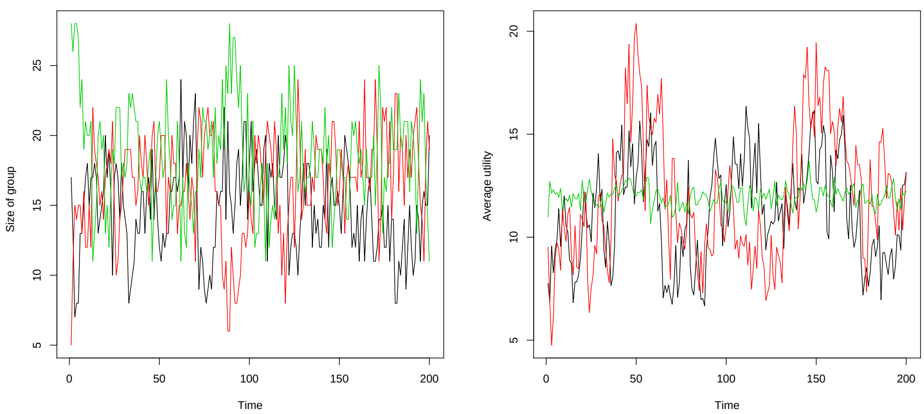

Fig. 1 Time series of the number of members (left) and average utility (right) for each group. Black, red and green lines denote the first, second group and the number of those who stay out, respectively.

The sizes of groups fluctuate widely: out of the 50 agents populating the model, the number of members of one group frequently goes from over 20 to well below 10. The reason of such marked fluctuation of groups' dimension is rooted in the volatile process of gathering information and in the resulting decision to join or abandon the groups they were in. The explorative nature of the group formation process is such that, interestingly, around period 90 most agents desert groups to stay on their own, as the green line clearly shows. The average utility of group members is not strictly related to the size of the groups, as the right panel of Figure 1 shows but, again, varies widely. While staying outside of any group yields roughly 12 on average, joining the second group around periods 50 or 150, say, produces a hefty utility close or even bigger than 20. The left graph of Figure 2 depicts the average similarity of

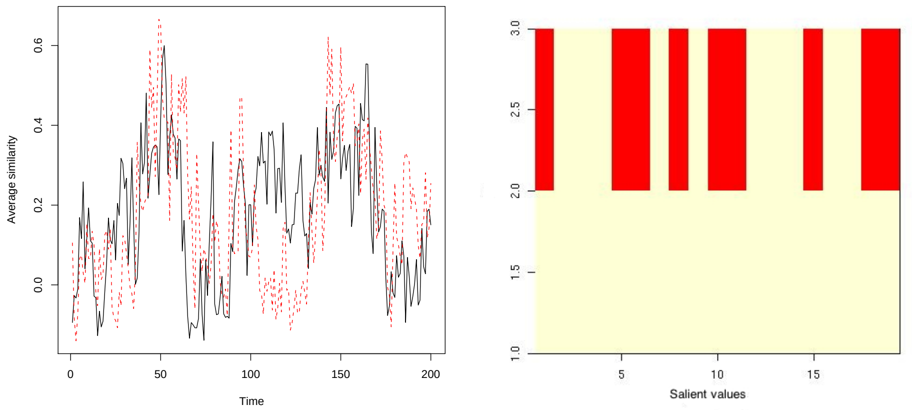

Fig. 2 Average similarity of the first (black) and second (red) groups (left). Salient values for group 1 members, yellow and red denoting “1” and “0”, respectively (right)

the members of the two groups. This quantity will be referred as *coherence* of the group in the remainder of the paper. Although there are significant variations in the average similarity over time, there are periods, like $t = 50$ or $t = 150$, where agents are grouped into fairly homogeneous groups. The right panel of the figure represents the salient values of the members of the first group at time 154, when its coherence peaks around 0.55. The bits are color-coded, with yellow and red denoting ”1” and ”0”, respectively. The picture shows that every member, at that time, shares at least one salient value (out of two) with every other peer, thus explaining the large average similarity. A plot of the similarity matrix is a useful tool to shed further light on the dynamics of the groups, in terms of size and internal coherence. Figure 3 shows two color-coded similarity matrices, relative to periods 50 (left) and 87 (right). In the matrices, members of the first, second and stay-out group are sequentially appended, and the (i, j) entry of the matrix represents the similarity of agents i and j, with yellow (red) denoting large (small) values. The first group is then shown on the bottom-left corner of the matrix whereas the second group is usually visible in the central part of the matrix, along the main diagonal. The upper-right corner represents the agents that do not belong to any group. The left panel shows the situation in period 50, where a homogeneous second group can clearly be seen in the bright block of entries $\{(i, j) : 13 \leq i, j \leq 29\}$. The first group appears to be made of less uniform agents in the bottom-left corner, where $1 \leq i, j \leq 12$. The previous figures show that sizes at time 50 are 12 and 17, with average similarities of 0.42 and 0.65 and average utilities of 12.92 and 20.37. The right panel of Figure 3 displays the similarity structure at

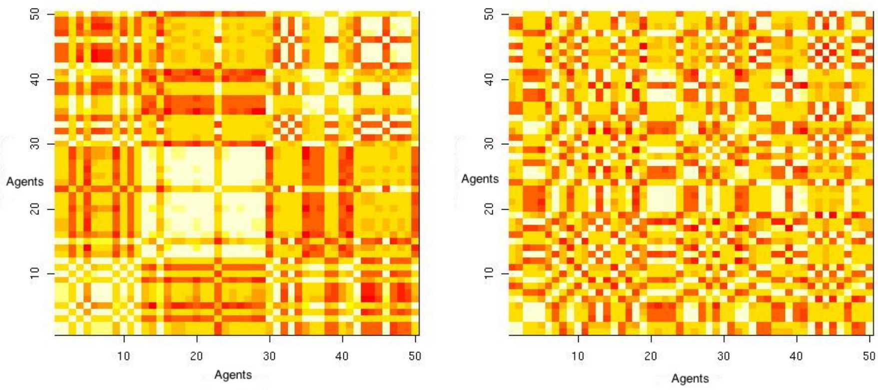

Fig. 3 Similarity matrices at times 50 (left) and 87 (right). The hue of entry (i, j) smoothly blends from bright yellow to dark red as the similarity decreases.

time 87 when, basically, groups are dismantled and agents are still in the way to form uniform groups. The first and second groups are barely visible despite their 14 and 11 members, the average similarities are -0.08 within both groups and, hence, the utilities are (only) 7.03 and 7.29, respectively. The difference in the two plots of Figure 3 visually confirms the general outcome that there is a remarkable time-variability in the groups that emerge in a single simulation. Figure 4 shows the time series of average contributions of the members of the groups. Typically, the endowments of agents that join in groups are smaller than the ones belonging to agents that opt to stay out. The result that individuals cooperate less when wealthier, depicted in a specific instance in Figure 4, is a very robust feature of the model (also with different configurations of parameters) and nicely matches already discussed results from previous studies [Lidenberg, 1982, Hegselmann, 1994, Molinas, 1998].
The basic formulation of the model introduces positive utility for cooperation with similar agents through joining the same group. Despite the completely random mechanism through which similarity is assessed, which leads to wide fluctuations, the model is able to reproduce interesting results coming from the reviewed literature and qualifies as a suitable tool to address the impact on ingroup similarity of the process of cooperation. The following section will address in more detail which parameters constellations are responsible for the features of the emerging groups.

4.1.1 Multiple Simulations

This section is dedicated to the description of more general features of the groups generated by the model as we change the level of some key parameters.The parameter changes we evaluated regard P - the number of individuals each agent randomly and independently meets when computing the expected utility of joining a different group, ε - the upper bound of the real variable representing general values of the population, and N, the number of salient values of agents. The parameter intervals

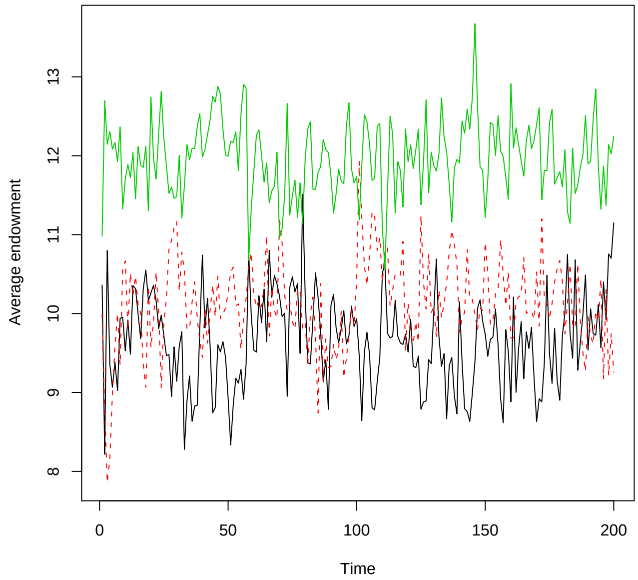

Fig. 4 Average contribution of groups members: black, red and green lines refer to the first and second group and to those who stay out.

used below have been defined after massive simulations, in order to present the most meaningful intervals in terms of observed dynamics both across and inside groups. We run 100 independent simulations and measure the average size of both groups, labeled generically ”small” and ”large”, together with the average size of the set of agents that decided to stay out, in order to discuss participation levels as some parameters change. Moreover, we compute the average *coherence* of the groups, the fraction of times in which the largest group changes (Sw), the average contribution of members (E) and their average utility (π). The last two values are normalized with the average endowment of the population $\overline{E}$. When computing any time-average, we discard the first 50 periods that are possibly affected by transient initial effects. The first analysis concerns P, the number of individuals each agent randomly and independently meets when computing the expected utility of joining a different group, that proxies the level of information that can be acquired within the population. The first panel of Table 2 shows, for example, that when agents sample $P = 1$ peer in each period, the smallest (largest) group has an average of 14.65 (16.74) members. The group of agents that stay out is normally larger (18.60 members) and the largest group changes on average every 4 periods (26%). Moreover, members of both groups are relatively poor, as shown by their endowments which is 93 or 94% of the average endowment of the population. The payoffs of agents belonging to either group is, however, substantially larger as they get a utility that is 100 and 114% of the average endowment of the population. Subjects that do not participate to groups are richer on

P		Size	Cohe	Sw	E	π
	Small	14.65	0.12	0.26	0.93	1.00
1	Large	16.74	0.18	-	0.94	1.14
	Out	18.60	-	-	1.11	1.11
	Small	15.58	0.29	0.34	0.94	1.29
2	Large	18.79	0.38	-	0.95	1.56
	Out	15.63	-	-	1.12	1.12
	Small	16.90	0.43	0.22	0.94	1.57
4	Large	21.65	0.43	-	0.96	1.79
	Out	11.45	-	-	1.14	1.14

Table 2 Time-averaged quantities for different values of P.

average (1.11) and, by definition, get exactly the very same payoff. The other panels show that the sizes of the groups are increasing in P. This result is likely to be related to better decisions taken by agents when a larger sample size is allowed for. This interpretation is corroborated by the higher utility for members of both the small and the large group that is due in turn to the increased coherence of both groups. The second parameter studied is ε (Table 3), the upper bound of the real variable representing general values of the population. Notice that the second panel, relative to the benchmark case where $\varepsilon = 1.0$, is exactly the same as in Table 2. The Table shows

ε		Size	Cohe	Sw	E	π
	Small	16.37	0.23	0.33	0.93	1.20
0.5	Large	18.51	0.27	-	0.94	1.35
	Out	15.13	-	-	1.14	1.14
	Small	15.58	0.29	0.34	0.94	1.29
1.0	Large	18.79	0.38	-	0.95	1.56
	Out	15.63	-	-	1.12	1.12
	Small	10.02	-0.03	0.26	0.88	0.72
2.0	Large	12.78	-0.01	-	0.92	0.81
	Out	27.20	-	-	1.08	1.08

Table 3 Time-averaged quantities for different values of ε.

that there are values of ε for which the coherence and size of both groups drops dramatically. When $\varepsilon = 2.0$, the disruptive diversity in the general values is such that joining a group is actually harmful in terms of utilities (as the beneficial similarity in salient values is too weak and few reasons are left to call them "salient" in such a situation). Once again, we find that richer individuals tend to remain out of the groups, looking at the average endowment of the stay-out group. Not surprisingly, the number of people choosing not to join either group increases with ε, for the reasons we have just discussed. The last parameter studied is N, the number of salient values of agents (Table 4). As N grows, it is more difficult for agents to join the "right" group, given that in the current version they can choose between two groups only. As an

N		Size	Cohe	Sw	E	π
	Small	19.06	0.36	0.16	0.95	1.57
1	Large	25.32	0.36	-	0.99	1.84
	Out	5.61	-	-	1.17	1.17
	Small	15.58	0.29	0.34	0.94	1.29
2	Large	18.79	0.38	-	0.95	1.56
	Out	15.63	-	-	1.12	1.12
	Small	16.49	0.10	0.29	0.96	0.96
3	Large	18.34	0.12	-	0.97	1.04
	Out	15.18	-	-	1.06	1.06

Table 4 Time-averaged quantities for different values of N.

example, the combinations of salient values can be interpreted as four different ethnic groups such as White, Blacks, Asian and Latinos. The problem of cooperation arises from having the possibility to join only one of the two available organizations, which cannot perfectly resemble racial divisions. We feel this is a realistic feature of the model that would otherwise yield trivial results if the number of groups could accommodate all the different types with negligible discordance. As a result, Table 4 shows how the average coherence and utility decrease as the number of salient values increases. From the joint inspection of Tables 2, 3 and 4, it appears that there is not a clear relationship between the switching measure Sw and P, N and ε. Such a link will instead be stronger in the model presented later. It is interesting to further

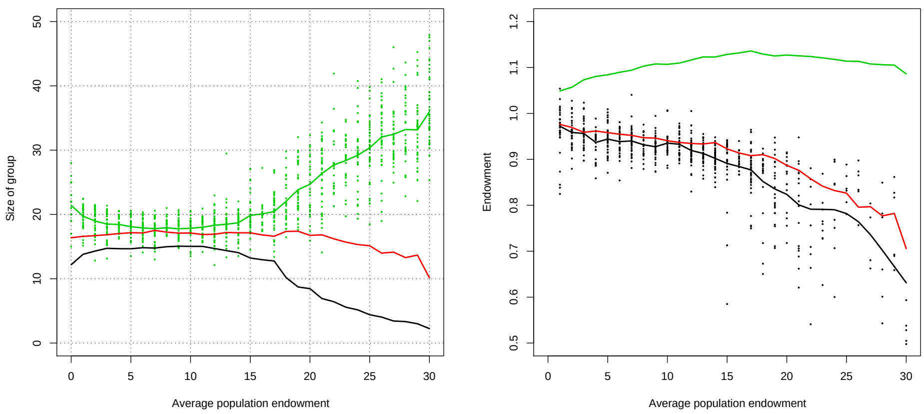

Fig. 5 Size of groups (left) and relative average endowment of members (right) as a function of the average endowment of the population $\overline{E}$. Black, red and green lines refer to the small and large groups and to those who stay out, respectively. The figure is based on 1000 simulations with a randomly sampled $\overline{E} \in \{0, 1, \ldots, 30\}$. Variations are shown only for group "out" (small) in the left (right) panel, for clarity of exposition.

explore the joint effect of the two components of the utility. Recall that one part is merely the equal share of the sum of the members' contributions, whereas the second (social) component is related to similarity. Figure 5 shows how the average size and the contribution of groups depend on the average endowment $\overline{E}$ of the population. Keeping fixed the other parameters, a larger (smaller) $\overline{E}$ makes joining a group less (more) convenient on a relative basis, as the profit from interaction is a little (substantial) part of agents' wealth. It is interesting to note that for the case $\overline{E} = 0$, the agents' utility is determined purely by the non-material part, thus the choice of joining or not is driven exclusively by their evaluation of similarities. The left panel of the Figure 5 shows that, as expected, an increase in the endowment pushes more agents to choose to stay out. The size of the two groups declines and, at the same time, the average endowment of the members of the groups shrinks, as can be seen on the right panel. In other words, a larger average endowment in the population reduces the size of the groups, which end up in attracting fewer and poorer agents. Synergies here defined can be thought both in terms of benefits coming from homophily preferences (liking to be in a group with like-minded individuals) and, borrowing from a recent survey by [Mesterton-Gibbons et al., 2011], in terms of the ability of a group to expand the pie of payoffs accessible to agents. The previous results show that N, ε, as well as $\overline{E}$, all have an impact on the synergy of the groups generated by the model. This outcome appears to be sensible as the number of salient values is likely to shape the willingness of agents to join together with the (possibly adverse) effect of significant general values. At the same time, wealthy populations with large $\overline{E}$ reap relatively little benefits from grouping and ultimately stay out, whereas smaller average endowments push agents to join in order to increase their utilities[1]

4.2 The Extended Model

This Section describes the case in which agents have some memory, characterized by a coefficient $\alpha > 0$, and estimate utility using a weighted average of past utilities and inferred information based on P samples. The introduction of the α parameter is another instance of increase in information for the myopic agents that populate the model. Differently from the case in which the number of sampled individuals is increased, improving the ability to keep track of the experience of joining different groups represent a first approximation of a learning process. As for the previous model, we first present a specific run and then aggregate many simulations to provide large-sample evidence of typical behavior. Let the parameters be given as in Table 1, with the exception that $\alpha = 0.4$. Figure 6 shows the size and average utilities of the three groups. The presence of memory produces a large and stable group (red line) that is always dominant in size and quite often yields the highest average utility. The smallest group (black line) includes roughly 10 members, leaving on average 15

[1] Cruciani et al. (2013) showed how within this model the convergence of values is able to reduce the effect of potential within-group segregation that kicks in when, for instance, better information is available and agents more quickly learn about potential partnership (through experience). This links the present work and its possible extensions to the literature on social identity, which points out that if subjects have similar values but different endowments, the willingness to support the lower-endowed with the same identity (high similarity) is increased. [Klor and Shayo, 2010].

agents on their own (green line). The right panel shows, if the initial transient effect is discarded, approximately steady utility for all groups. In particular, the performance

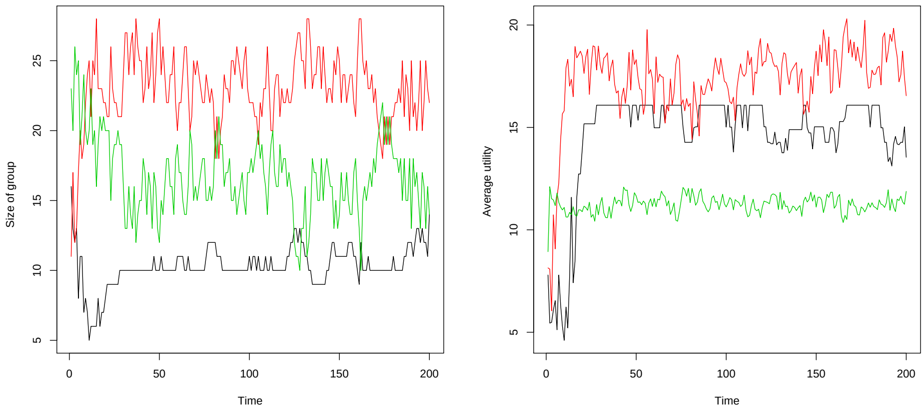

Fig. 6 Time series of the number of members (left) and average utility (right) for the three groups when the memory coefficient is $\alpha = 0.4$. Black, red and green lines denote the Small, Large and Stay-out group, respectively.

of the small group in terms of utility is relatively good, taking into account the difference in size with the dominant one. This is due to the internal large coherence of the smallest group that counterbalances its small size. Consistently with this result, we report that the average coherence of the two groups are 0.40 and 0.78 in this specific simulation. The left panel of Figure 7 shows the similarity matrix of agents in period 160. There is a small but extremely coherent first group on the bottom-left corner and a larger second group characterized by less similar agents, as shown by several darker hues. The relative stability of the groups that are formed with such a level of memory translates into a higher degree of similarity that lasts over a number of periods. The right panel of Figure 7 displays several statistics for a specific agent whose endowment, equal to 12.53, is shown as a dashed line. In particular, the upper (lower) black line shows the estimated utilities of joining the second (first) groups. The red line, often superimposed on one of the previous estimates, depicts the utility actually enjoyed by the agent. This individual mostly joins the second group, occasionally staying alone for brief periods. Clearly, the estimated utility to be in the first group (lower black line) never exceeds his endowment or the perceived benefit to join the second group (upper black line). Hence, the agent frequently stays in the second group, inflating his utilities that would have been much lower if alone or in the other group. As this example shows, introducing some memory allows agents to act correctly even if their decisions are based on a myopic estimate. In fact, the estimated utility for joining the second group is a reasonable guess of the actual outcome, given the fact that only $P = 2$ agents are sampled in each period. In other words, a positive

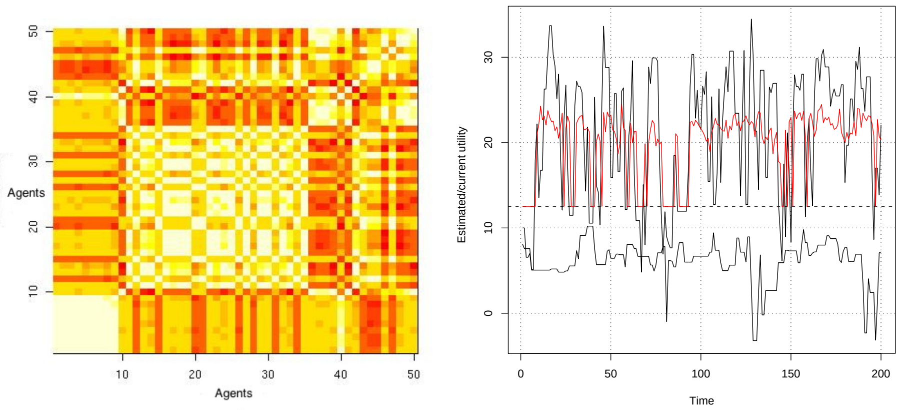

Fig. 7 Similarity matrix in period 160 when $\alpha = 0.4$ (left panel). Time series of the estimated utilities q_t^1, q_t^2 for a given agent (right panel). Actual utility and endowment of the agent are shown with red and dashed lines respectively.

α improves the quality of the estimates, reducing their variance, even in a myopic setting.

4.2.1 Multiple simulations

This section describes the more general features of the groups generated by the model once a memory parameter has been introduced. The structure of the presentation of the results mirrors the one of the previous Section, in which no memory was present, and, when computing any time-average, we discard the first 50 periods to avoid transient initial effects. Table 5 shows the changes in key variables for three levels of the

α		Size	Cohe	Sw	E	π
	Small	15.00	0.31	0.27	0.92	1.28
0.1	Large	18.63	0.37	-	0.95	1.54
	Out	16.37	-	-	1.12	1.12
	Small	16.27	0.47	0.19	0.92	1.56
0.4	Large	20.81	0.45	-	0.96	1.79
	Out	12.92	-	-	1.15	1.15
	Small	13.73	0.57	0.05	0.85	1.45
0.8	Large	24.56	0.43	-	0.96	1.88
	Out	11.70	-	-	1.18	1.18

Table 5 Average dynamics for different values of α.

memory parameter α. As α grows to 0.4, the size, coherence and average utilities increase significantly for both groups. When α grows to 0.8, it is in particular the large group that benefits from this change, attracting a much larger number of individuals. In both cases the number of agents deciding to stay out, instead, decreases markedly, but they remain the wealthiest group in the population. Coherently with the results of the basic model, the members of the large group always achieve a larger utility on average. Moreover, being in a group is always better than remaining out even in this extended model. Some memory appears to have long-lasting effects in that more stable groups are formed. This is confirmed by a dramatic drop in the switching rate pointing out that a dominant group quickly builds and persists for most periods.

5 Discussion and conclusion

We presented an agent-based model of groups in informal settings, in which cooperation is constructed through the flexible concept of perceived similarity. In our model, agents decide whether to join or abandon one of two possible groups, without any cost - due to the informality of the setting. At the end of each period, utilities are computed on the bases of the size of the group ("the more, the merrier") and the overall similarity of the group ("the more coherent, the better"). An innovative aspect of our model is the characterization of agents with some personal features, called *salient* and *general* values, which combine into what we called agents' multi-dimensional types: the former represent the standpoints agents take on matters related to a goal important for the group, whereas the latter describe agents' position about more negotiable issues. Together with agents' (heterogeneously distributed) endowment, similarity in values drives successful or unsuccessful cooperation. Individuals will cooperate, joining forces and sharing resources, if they perceive the group can increase their utility, which has two components: the average contribution of the group, and the sum of all pairwise similarities. The latter component represents the immaterial utility of being in a group with people one likes -reflecting homophily preferences- as they share a combination of common values. The model reproduces some known stylized facts, like the higher likelihood of poorer agents to join [Molinas, 1998, Lidenberg, 1982], and can be used to describe and interpret empirical examples of stable cooperative groups without direct or indirect reciprocity among members, or shadow of the future considerations. The basic formulation of our model aims at contributing to the strand of literature dealing with the evolution of cooperation based on peers' similarity. The evolution of cooperation based on agents' common features has recently received some attention in agent-based research, mainly because it seems to better represent real situations. Specifically, much attention has been devoted to the research on homophily, which explores how perception of similarities between individuals can foster cooperation sustaining trust-building processes (without the introduction of incentive schemes or reciprocity concepts). The work of [Riolo et al., 2001], for example, has shown in an evolutionary model with inheritable tags that similarity can indeed breed cooperation. Our model is consistent with their formalization of similarity, which is enriched by multi-dimensional types that would allow for a "contamination process". Indeed, the distinction between general and salient values, where salient are unchangeable binary values whose importance can never be overcome by the parameter summarizing general values, leads to a sophistication of the concept of similarity towards a better representation of ”real” economic agents. The basic formulation of the model shows that an imperfect perception of similarity is not enough to unequivocally create more cooperation, leading to significant fluctuations in the size of the two possible groups and on the number of individuals deciding not to join. When observed alongside the variation of other parameters, similarity-based cooperation creates interesting dynamics that have suggestive policy implications. For instance, increasing the level of information individuals have on potential group members (the parameter P) definitely supports cooperation over time, building more stable groups. This effect suggests that one way in which cooperation among informal partners can be supported is through the diffusion of general information, best practices and general standards that apply to the the potential partners.

The introduction of a memory parameter, in the extended version of the model, shows that the fewer agents deciding to stay out are still characterized by higher endowments than the rest of the population. More interestingly, some memory leads to the formation of more stable groups, with very low rates of switching and the presence of a dominant and persistent group for most of the periods. This effect suggests that informal groups can benefit from investments in tradition building by building reputations that reduce the need for direct information and support cooperation even in informal networks. We had already observed another instance of information improvement, when the number of sampled individuals were increased in section 4.1.1. Differently from what happens when P is increased, introducing a positive α not only improves the rate of participation to the groups - as happens with an increase in P - but the switching rate decreases drastically. This is due to the fact that introducing a positive memory parameter kickstarts a process of path dependence, whereby individuals keep track of a running average of their experiences in joining a group (or not).

There are a number of limitations in our work that point to potential avenues for future developments. Focusing on what we perceive are the most interesting issues, we plan to work on adaptation of general values and endogenization of the number of possible groups and of the memory coefficient α.

Assuming a fixed number of groups and a predetermined memory coefficient has clear shortcomings and may be inappropriate in certain circumstances. Some of the results suggest that the endogenization of α could be obtained letting agents choose which is the optimal level of memory they should have (with respect to their own characteristics) in order to maximize expected utility. Moreover, standard clustering algorithms could be used to establish benchmark groups of agents that can be compared with the groups produced by our model of social interactions. Preliminary results (not shown here) point to subtle but persistent differences in the clusters/groups obtained with the two methods and suggest that this fact may be due to potential synergies among agents that are only captured when the similarity perception is used by agents in a dynamic way. This could have interesting potential applications in interpreting empirical facts, or even suggesting new solutions in a wide range of environments, such as business organizations or socio-economic institutions.

From the literature on social norms, it emerges that the full contribution assumption is not as frequent as other forms of conditional response to the observed contributions of others. In particular, we observe that the modal contribution in different economics games (e.g trust game, public goods game, ultimatum games, etc.) ranges around half of the available endowment. [Guth et al., 1982, Ledyard, 1995, Berg et al., 1995] . Another interesting future development of this work regards the dynamics emerging from the implementation of other contribution levels, such as the half contribution hypothesis. In fact, preliminary evaluations of the impact of changing the full contribution assumption to half contribution showed that this further enhances the role of perceived similarity.

One last point deserves mention. Although our setting does not currently allow for the emergence of trust in its most standard way, the introduction of salient values as a medium to facilitate cooperation certainly goes in the direction of investigating what ultimately motivates trust-building processes. In the model described in this paper, there is no possibility for trust to emerge, as agents do not recall specific characteristics of other agents, but simply sample and make inferences on the average similarity of the group. Nevertheless, the perception of similarity even with respect to a group of indistinguishable individuals is enough to foster more cooperative behavior, facilitating the emergence of profitable groups. This points to the need of further understanding what is the exact relationship between similarity and trust building, which could become a potential avenue for further development of the current model.

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