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Department of Economics

Working Paper

Monica Billio, Mila Getmansky, Andrew W. Lo and Loriana Pelizzon

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ISSN: 1827-3580
No. 21/WP/2011

Working Papers
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Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors

Monica Billio
Department of Economics
University Ca' Foscari Venice

Mila Getmansky
Isenberg School of Management
University of Massachusetts

Andrew W. Lo
MIT Sloan School of Management

Loriana Pelizzon
Department of Economics
University Ca' Foscari Venice

November 2011

Abstract
We propose several econometric measures of connectedness based on principal-components analysis and Granger-causality networks, and apply them to the monthly returns of hedge funds, banks, broker/dealers, and insurance companies. We find that all four sectors have become highly interrelated over the past decade, likely increasing the level of systemic risk in the finance and insurance industries through a complex and time-varying network of relationships. These measures can also identify and quantify financial crisis periods, and seem to contain predictive power in out-of-sample tests. Our results show an asymmetry in the degree of connectedness among the four sectors, with banks playing a much more important role in transmitting shocks than other financial institutions.

Keywords
Systemic Risk; Financial Institutions; Liquidity; Financial Crises.
JEL Codes
G12, G29, C51

Address for correspondence:
Monica Billio
Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta S.Giobbe
30121 Venezia - Italy
Phone: (++39) 041 2349170
Fax: (++39) 041 2349176
e-mail: billio@unive.it

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wp.dse@unive.it

Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

1 Introduction

The Financial Crisis of 2007–2009 has created renewed interest in systemic risk, a concept originally associated with bank runs and currency crises, but which is now applied more broadly to shocks to other parts of the financial system, e.g., commercial paper, money market funds, repurchase agreements, consumer finance, and OTC derivatives markets. Although most regulators and policymakers believe that systemic events can be identified after the fact, a precise definition of systemic risk seems remarkably elusive, even after the demise of Bear Stearns and Lehman Brothers in 2008, the government takeover of AIG in that same year, the "Flash Crash" of May 6, 2010, and the European sovereign debt crisis of 2011–2012.

By definition, systemic risk involves the financial *system*, a collection of interconnected institutions that have mutually beneficial business relationships through which illiquidity, insolvency, and losses can quickly propagate during periods of financial distress. In this paper, we propose two econometric methods to capture this connectedness—principal components analysis and Granger-causality networks—and apply them to the monthly returns of four types of financial institutions: hedge funds, and publicly traded banks, broker/dealers, and insurance companies. We use principal components analysis to estimate the number and importance of common factors driving the returns of these financial institutions, and we use pairwise Granger-causality tests to identify the network of statistically significant Granger-causal relations among these institutions.

Our focus on hedge funds, banks, broker/dealers, and insurance companies is not coincidental, but is motivated by the extensive business ties between them, many of which have emerged only in the last decade. For example, insurance companies have had little to do with hedge funds until recently. However, as they moved more aggressively into non-core activities such as insuring financial products, credit-default swaps, derivatives trading, and investment management, insurers created new business units that competed directly with banks, hedge funds, and broker/dealers. These activities have potential implications for systemic risk when conducted on a large scale (see Geneva Association, 2010). Similarly, the banking industry has been transformed over the last 10 years, not only with the repeal of the Glass-Steagall Act in 1999, but also through financial innovations like securitization that have blurred the distinction between loans, bank deposits, securities, and trading strategies. The types of business relationships between these sectors have also changed, with banks and insurers providing credit to hedge funds but also competing against them through their own proprietary trading desks, and hedge funds using insurers to provide principal protection on their funds while simultaneously competing with them by offering capital-market-intermediated insurance such as catastrophe-linked bonds.

For banks, broker/dealers, and insurance companies, we confine our attention to publicly listed entities and use their monthly equity returns in our analysis. For hedge funds—which are private partnerships—we use their monthly reported net-of-fee fund returns. Our emphasis on market returns is motivated by the desire to incorporate the most current information in our measures; market returns reflect information more rapidly than non-market-based measures such as accounting variables. In our empirical analysis, we consider the individual returns of the 25 largest entities in each of the four sectors, as well as asset- and market-capitalization-weighted return indexes of these sectors. While smaller institutions can also contribute to systemic risk,[1] such risks should be most readily observed in the largest entities. We believe our study is the first to capture the network of causal relationships between the largest financial institutions across these four sectors.

Our empirical findings show that linkages within and across all four sectors are highly dynamic over the past decade, varying in quantifiable ways over time and as a function of market conditions. Over time, all four sectors have become highly interrelated, increasing the channels through which shocks can propagate throughout the finance and insurance sectors. These patterns are all the more striking in light of the fact that our analysis is based on monthly returns data. In a framework where all markets clear and past information is fully impounded into current prices, we should not be able to detect significant statistical relationships on a monthly timescale.

Our principal components estimates and Granger-causality tests also point to an important asymmetry in the connections: the returns of banks and insurers seem to have more significant impact on the returns of hedge funds and broker/dealers than vice versa. This asymmetry became highly significant prior to the Financial Crisis of 2007–2009, raising the possibility that these measures may be useful out-of-sample indicators of systemic risk. This pattern suggests that banks may be more central to systemic risk than the so-called shadow banking system. One obvious explanation for this asymmetry is the fact that banks lend

[1]For example, in a recent study commissioned by the G-20, the IMF (2009) determined that systemically important institutions are not limited to those that are the largest, but also include others that are highly interconnected and that can impair the normal functioning of financial markets when they fail.

capital to other financial institutions, hence the nature of their relationships with other counterparties is not symmetric. Also, by competing with other financial institutions in non-traditional businesses, banks and insurers may have taken on risks more appropriate for hedge funds, leading to the emergence of a "shadow hedge-fund system" in which systemic risk cannot be managed by traditional regulatory instruments. Yet another possible interpretation is that because they are more highly regulated, banks and insurers are more sensitive to value-at-risk changes through their capital requirements, hence their behavior may generate endogenous feedback loops with perverse externalities and spillover effects to other financial institutions.

In Section 2 we provide a brief review of the literature on systemic risk measurement, and describe our proposed measures in Section 3. The data used in our analysis is summarized in Section 4, and the empirical results are reported in Sections 5. The practical relevance of our measures as early warning signals is considered in Section 6, and we conclude in Section 7.

2 Literature review

Since there is currently no widely accepted definition of systemic risk, a comprehensive literature review of this rapidly evolving research area is difficult to provide. Like Justice Potter Stewart's description of pornography, systemic risk seems to be hard to define but we think we know it when we see it. Such an intuitive definition is hardly amenable to measurement and analysis, a pre-requisite for macroprudential regulation of systemic risk. A more formal definition is *any set of circumstances that threatens the stability of or public confidence in the financial system.*[2] Under this definition, the stock market crash of October 19, 1987 was not systemic, but the "Flash Crash" of May 6, 2010 was because the latter event called into question the credibility of the price discovery process, unlike the former. Similarly,

[2]For an alternate perspective, see De Bandt and Hartmann's (2000) review of the systemic risk literature, which led them to the following definition:

> A systemic crisis can be defined as a systemic event that affects a considerable number of financial institutions or markets in a strong sense, thereby severely impairing the general well-functioning of the financial system. While the "special" character of banks plays a major role, we stress that systemic risk goes beyond the traditional view of single banks' vulnerability to depositor runs. At the heart of the concept is the notion of "contagion", a particularly strong propagation of failures from one institution, market or system to another.

the 2006 collapse of the \$9 billion hedge fund Amaranth Advisors was not systemic, but the 1998 collapse of the \$5 billion hedge fund Long Term Capital Management (LTCM) was, because the latter event affected a much broader swath of financial markets and threatened the viability of several important financial institutions, unlike the former. And the failure of a few regional banks is not systemic, but the failure of a single highly interconnected money market fund can be.

While this definition does seem to cover most, if not all, of the historical examples of "systemic" events, it also implies that the risk of such events is multifactorial and unlikely to be captured by any single metric. After all, how many ways are there of measuring "stability" and "public confidence"? If we consider financial crises the realization of systemic risk, then Reinhart and Rogoff's (2009) volume encompassing eight centuries of crises is the new reference standard. If we focus, instead, on the four "L"s of financial crises—leverage, liquidity, losses, and linkages—several measures of the first three already exist.[3] However, the one common thread running through all truly systemic events is that they involve the financial *system*, i.e., the connections and interactions among financial stakeholders. Therefore, any measure of systemic risk must capture the degree of connectivity of market participants to some extent. Therefore, in this paper we choose to focus our attention on the fourth "L": linkages.

From a theoretical perspective, it is now well established that the likelihood of major financial dislocation is related to the degree of correlation among the holdings of financial institutions, how sensitive they are to changes in market prices and economic conditions (and the directionality, if any, of those sensitivities, i.e., causality), how concentrated the risks are among those financial institutions, and how closely linked they are with each other and the rest of the economy.[4] Three measures have been proposed recently to estimate these linkages:

[3]With respect to leverage, in the wake of the sweeping Dodd-Frank Financial Reform Bill of 2010, financial institutions are now obligated to provide considerably greater transparency to regulators, including the disclosure of positions and leverage. There are many measures of liquidity for publicly traded securities, e.g., Amihud and Mendelson (1986), Brennan, Chordia and Subrahmanyam (1998), Chordia, Roll and Subrahmanyam (2000, 2001, 2002), Glosten and Harris (1988), Lillo, Farmer, and Mantegna (2003), Lo, Mamaysky, and Wang (2001), Lo and Wang (2000), Pastor and Stambaugh (2003), and Sadka (2006). For private partnerships such as hedge funds, Lo (2001) and Getmansky, Lo, and Makarov (2004) propose serial correlation as a measure of their liquidity, i.e., more liquid funds have less serial correlation. Billio, Getmansky and Pelizzon (2011) use Large-Small and VIX factors as liquidity proxies in hedge fund analysis. And the systemic implications of losses are captured by CoVaR (Adrian and Brunnermeier, 2010) and SES (Acharya, Pedersen, Philippon, and Richardson, 2011).

[4]See, for example Acharya and Richardson (2009), Allen and Gale (1994, 1998, 2000), Battiston, Delli Gatti, Gallegati, Greenwald, and Stiglitz (2009), Brunnermeier (2009), Brunnermeier and Pedersen (2009),

Adrian and Brunnermeier's (2010) conditional value-at-risk (CoVaR), Acharya, Pedersen, Philippon, and Richardson's (2011) systemic expected shortfall (SES), and Huang, Zhou, and Zhu's (2011) distressed insurance premium (DIP). SES measures the expected loss to each financial institution conditional on the entire set of institutions' poor performance; CoVaR measures the value-at-risk (VaR) of financial institutions conditional on other institutions experiencing financial distress; and DIP measures the insurance premium required to cover distressed losses in the banking system.

The common theme among these three closely related measures is the magnitude of losses during periods when many institutions are simultaneously distressed. While this theme may seem to capture systemic exposures, it does so only to the degree that systemic losses are well represented in the historical data. But during periods of rapid financial innovation, newly connected parts of the financial system may not have experienced simultaneous losses, despite the fact that their connectedness implies an increase in systemic risk. For example, prior to the 2007–2009 crisis, extreme losses among monoline insurance companies did not coincide with comparable losses among hedge funds invested in mortgage-backed securities because the two sectors had only recently become connected through insurance contracts on collateralized debt obligations. Moreover, measures based on probabilities invariably depend on market volatility, and during periods of prosperity and growth, volatility is typically lower than in periods of distress. This implies lower estimates of systemic risk until after a volatility spike occurs, which reduces the usefulness of such a measure as an early warning indicator.

Of course, aggregate loss probabilities depend on correlations through the variance of the loss distribution (which is comprised of the variances and covariances of the individual institutions in the financial system). Over the last decade, correlations between distinct sectors of the financial system like hedge funds and the banking industry tend to become much higher *during and after* a systemic shock occurs, not before. Therefore, by conditioning on extreme losses, measures like CoVaR and SES are estimated on data that reflect unusually high correlations among financial institutions. This, in turn, implies that during non-crisis periods, correlation will play little role in indicating a build-up of systemic risk using such measures.

Our approach is to simply measure correlation directly and unconditionally—through

Gray (2009), Rajan (2006), Danielsson, Shin, and Zigrand (2011), and Reinhart and Rogoff (2009).

principal components analysis and by pairwise Granger-causality tests—and use these metrics to gauge the degree of connectedness of the financial system. During normal times, such connectivity may be lower than during periods of distress, but by focusing on unconditional measures of connectedness, we are able to detect new linkages between parts of the financial system that have nothing to do with simultaneous losses. In fact, while aggregate correlations may decline during bull markets—implying lower conditional loss probabilities—our measures show increased unconditional correlations among certain sectors and financial institutions, yielding finer-grain snapshots of linkages throughout the financial system.

Moreover, our Granger-causality-network measures have, by definition, a time dimension that is missing in conditional loss probability measures which are based on contemporaneous relations. In particular, Granger causality is defined as a predictive relation between past values of one variable and future values of another. Our out-of-sample analysis shows that these lead/lag relations are important, even after accounting for leverage measures, contemporaneous connections, and liquidity.

In summary, our two measures of connectedness complement the three conditional loss-probability-based measures, CoVaR, SES, and DIP in providing direct estimates of the statistical connectivity of a network of financial institutions' asset returns.

Our work is also related to Boyson, Stahel, and Stulz (2010) who investigate contagion from lagged bank- and broker-returns to hedge-fund returns. We consider these relations as well, but also consider the possibility of reverse contagion, i.e., causal effects from hedge funds to banks and broker/dealers. Moreover, we add a fourth sector—insurance companies—to the mix, which has become increasingly important, particularly during the most recent financial crisis.

Our paper is also related to Allen, Babus, and Carletti (2011) who show that the structure of the network—where linkages among institutions are based on the commonality of asset holdings—matters in the generation and propagation of systemic risk. In our work, we empirically estimate the network structure of financial institutions generated by stock-return interconnections.

3 Measures of connectedness

In this section we present two measures of connectedness that are designed to capture changes in correlation and causality among financial institutions. In Section 3.1, we construct a measure based on principal components analysis to identify increased correlation among the asset returns of financial institutions. To assign directionality to these correlations, in Sections 3.2 and 3.3 we use pairwise linear and nonlinear Granger-causality tests to estimate the network of statistically significant relations among financial institutions.

3.1 Principal components

Increased commonality among the asset returns of banks, broker/dealers, insurers, and hedge funds can be empirically detected by using principal components analysis (PCA), a technique in which the asset returns of a sample of financial institutions are decomposed into orthogonal factors of decreasing explanatory power (see Muirhead, 1982 for an exposition of PCA). More formally, let R^i be the stock return of institution i, $i = 1, \ldots, N$, let the system's aggregate return be represented by the sum $R^S = \sum_i R^i$, and let $\mathrm{E}[R^i] = \mu_i$ and $\mathrm{Var}[R^i] = \sigma_i^2$. Then we have:

$$\sigma_S^2 \quad = \quad \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\mathrm{E}[z_i z_j] \quad , \quad \text{where} \quad z_k \equiv (R^k - \mu_k)/\sigma_k \ , \ k = i, j \ , \tag{1}$$

where z_k is the standardized return of institution k and σ_S^2 is the variance of the system. We now introduce N zero-mean uncorrelated variables ζ_k for which

$$\mathrm{E}[\zeta_k\zeta_l] = \begin{cases} \lambda_k & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases} \tag{2}$$

and all the higher order co-moments are equal to those of the z's, where λ_k is the k-th eigenvalue. We express the z's as a linear combination of the ζ_k's

$$z_i = \sum_{k=1}^{N} L_{ik}\zeta_k, \tag{3}$$

where L_{ik} is a factor loading for ζ_k for an institution i. Thus we have

$$\mathrm{E}[z_i z_j] \quad = \quad \sum_{k=1}^{N}\sum_{l=1}^{N} L_{ik}L_{jl}\mathrm{E}[\zeta_k\zeta_l] \quad = \quad \sum_{k=1}^{N} L_{ik}L_{jk}\lambda_k \tag{4}$$

$$\sigma_S^2 \quad = \quad \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sigma_i\sigma_j L_{ik}L_{jk}\lambda_k \ . \tag{5}$$

PCA yields a decomposition of the variance-covariance matrix of returns of the N financial institutions into the orthonormal matrix of loadings L (eigenvectors of the correlation matrix of returns) and the diagonal matrix of eigenvalues Λ. Because the first few eigenvalues usually explain most of the variation of the system, we focus our attention on only a subset $n<N$ of them. This subset captures a larger portion of the total volatility when the majority of returns tend to move together, as is often associated with crisis periods. Therefore, periods when this subset of principal components explains more than some fraction H of the total volatility are indicative of increased interconnectedness between financial institutions.[5]

Defining the total risk of the system as $\Omega \equiv \sum_{k=1}^{N} \lambda_k$ and the risk associated with the first n principal components as $\omega_n \equiv \sum_{k=1}^{n} \lambda_k$, we compare the ratio of the two (i.e., the Cumulative Risk Fraction) to the pre-specified critical threshold level H to capture periods of increased interconnectedness:

$$\frac{\omega_n}{\Omega} \equiv h_n \geq H \ . \tag{6}$$

When the system is highly interconnected, a small number n of N principal components can explain most of the volatility in the system, hence h_n will exceed the threshold H. By examining the time variation in the magnitudes of h_n, we are able to detect increasing correlation among institutions, i.e., increased linkages and integration as well as similarities in risk exposures, which can contribute to systemic risk.

The contribution $\text{PCAS}_{i,n}$ of institution i to the risk of the system—conditional on a strong common component across the returns of all financial institutions ($h_n \geq H$)—is a univariate measure of connectedness for each company i, i.e.:

$$\text{PCAS}_{i,n} = \frac{1}{2} \frac{\sigma_i^2}{\sigma_S^2} \frac{\partial \sigma_S^2}{\partial \sigma_i^2} \bigg|_{h_n \geq H} \ . \tag{7}$$

It is easy to show that this measure also corresponds to the exposure of institution i to the total risk of the system, measured as the weighted average of the square of the factor loadings of the single institution i to the first n principal components, where the weights are

[5]In our framework, H is determined statistically as the threshold level that exhibits a statistically significant change in explaining the fraction of total volatility with respect to previous periods. The statistical significance is determined through simulation as described in Appendix A.

simply the eigenvalues. In fact:

$$\mathrm{PCAS}_{i,n} \quad = \quad \frac{1}{2}\frac{\sigma_i^2}{\sigma_S^2}\frac{\partial \sigma_S^2}{\partial \sigma_i^2}\bigg|_{h_n \geq H} \quad = \quad \sum_{k=1}^{n} \frac{\sigma_i^2}{\sigma_S^2} L_{ik}^2 \lambda_k \bigg|_{h_n \geq H} \quad . \tag{8}$$

Intuitively, since we are focusing on endogenous risk, this is both the contribution and the exposure of the i-th institution to the overall risk of the system given a strong common component across the returns of all institutions.

In Online Appendix O.1 we show how, in a Gaussian framework, this measure is related to the co-kurtosis of the multivariate distribution. When fourth co-moments are finite, PCAS captures the contribution of the i-th institution to the multivariate tail dynamics of the system.

3.2 Linear Granger causality

To investigate the dynamic propagation of shocks to the system, it is important to measure not only the degree of connectedness between financial institutions, but also the directionality of such relationships. To that end, we propose using Granger causality, a statistical notion of causality based on the relative forecast power of two time series. Time series j is said to "Granger-cause" time series i if past values of j contain information that helps predict i above and beyond the information contained in past values of i alone. The mathematical formulation of this test is based on linear regressions of R_{t+1}^i on R_t^i and R_t^j.

Specifically, let R_t^i and R_t^j be two stationary time series, and for simplicity assume they have zero mean. We can represent their linear inter-relationships with the following model:

$$\begin{aligned} R_{t+1}^i &= a^i R_t^i \;+\; b^{ij} R_t^j \;+\; e_{t+1}^i \;, \\ R_{t+1}^j &= a^j R_t^j \;+\; b^{ji} R_t^i \;+\; e_{t+1}^j \end{aligned} \tag{9}$$

where e_{t+1}^i and e_{t+1}^j are two uncorrelated white noise processes, and a^i, a^j, b^{ij}, b^{ji} are coefficients of the model. Then j Granger-causes i when b^{ij} is different from zero. Similarly, i Granger-causes j when b^{ji} is different from zero. When both of these statements are true, there is a feedback relationship between the time series.[6]

[6]We use the "Bayesian Information Criterion" (BIC; see Schwarz, 1978) as the model-selection criterion for determining the number of lags in our analysis. Moreover, we perform F-tests of the null hypotheses that the coefficients $\{b^{ij}\}$ or $\{b^{ji}\}$ (depending on the direction of Granger causality under consideration) are

In an informationally efficient financial market, short-term asset-price changes should not be related to other lagged variables,[7] hence a Granger-causality test should not detect any causality. However, in the presence of value-at-risk constraints or other market frictions such as transactions costs, borrowing constraints, costs of gathering and processing information, and institutional restrictions on shortsales, we may find Granger causality among price changes of financial assets. Moreover, this type of predictability may not easily be arbitraged away precisely because of the presence of such frictions. Therefore, the degree of Granger causality in asset returns can be viewed as a proxy for return-spillover effects among market participants as suggested by Danielsson, Shin, and Zigrand (2011), Battiston et al. (2009), and Buraschi et al. (2010). As this effect is amplified, the tighter are the connections and integration among financial institutions, heightening the severity of systemic events as shown by Castiglionesi, Periozzi, and Lorenzoni (2009) and Battiston et al. (2009).

Accordingly, we propose a Granger-causality measure of connectedness to capture the lagged propagation of return spillovers in the financial system, i.e., the network of Granger-causal relations among financial institutions.

We consider a GARCH(1,1) baseline model of returns:

$$\begin{aligned} R_t^i &= \mu_i + \sigma_{it}\epsilon_t^i \ , \ \ \epsilon_t^i \sim \mathrm{WN}(0,1) \\ \sigma_{it}^2 &= \omega_i + \alpha_i \left(R_{t-1}^i - \mu_i\right)^2 + \beta_i \sigma_{it-1}^2 \end{aligned} \tag{10}$$

conditional on the system information:

$$I_{t-1}^S = \mathfrak{S}\left(\left\{\left\{R_\tau^i\right\}_{\tau=-\infty}^{t-1}\right\}_{i=1}^N\right) \ , \tag{11}$$

where μ_i, ω_i, α_i, and β_i are coefficients of the model, and $\mathfrak{S}(\cdot)$ represents the sigma algebra. Since our interest is in obtaining a measure of connectedness, we focus on the dynamic propagation of shocks from one institution to others, controlling for return autocorrelation for that institution.

equal to zero.

[7] Of course, predictability may be the result of time-varying expected returns, which is perfectly consistent with dynamic rational expectations equilibria, but it is difficult to reconcile short-term predictability (at monthly and higher frequencies) with such explanations. See, for example, Getmansky, Lo, and Makarov (2004, Section 3) for a calibration exercise in which an equilibrium two-state Markov switching model is used to generate autocorrelation in asset returns, with little success.

A rejection of a linear Granger-causality test as defined in (9) on $\widetilde{R}_t^i = \frac{R_t^i}{\widehat{\sigma}_{it}}$, where $\widehat{\sigma}_{it}$ is estimated with a GARCH(1,1) model to control for heteroskedasticity, is the simplest way to statistically identify the network of Granger-causal relations among institutions, as it implies that returns of the i-th institution linearly depend on the past returns of the j-th institution:

$$\mathrm{E}\left[R_t^i \left| I_{t-1}^S\right.\right] = \mathrm{E}\left[R_t^i \left| \left\{\left(R_\tau^i - \mu_i\right)^2\right\}_{\tau=-\infty}^{t-2}, R_{t-1}^i, R_{t-1}^j, \left\{\left(R_\tau^j - \mu_j\right)^2\right\}_{\tau=-\infty}^{t-2}\right.\right] . \tag{12}$$

Now define the following indicator of causality:

$$(j \to i) = \begin{cases} 1 & \text{if } j \text{ Granger causes } i \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

and define $(j \to j) \equiv 0$. These indicator functions may be used to define the connections of the network of N financial institutions, from which we can then construct the following network-based measures of connectedness.

1. **Degree of Granger Causality**. Denote by the *degree of Granger causality* (DGC) the fraction of statistically significant Granger-causality relationships among all $N(N-1)$ pairs of N financial institutions:

$$\mathrm{DGC} \equiv \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} (j \to i) . \tag{14}$$

The risk of a systemic event is high when DGC exceeds a threshold K which is well above normal sampling variation as determined by our Monte Carlo simulation procedure (See Appendix B).

2. **Number of Connections**. To assess the systemic importance of single institutions, we define the following simple counting measures, where S represents the system:

$$\begin{array}{lrcl} \#\textbf{Out}: & (j \to S)|_{\mathrm{DGC} \geq K} & = & \frac{1}{N-1} \sum_{i \neq j} (j \to i)|_{\mathrm{DGC} \geq K} \\ \#\textbf{In}: & (S \to j)|_{\mathrm{DGC} \geq K} & = & \frac{1}{N-1} \sum_{i \neq j} (i \to j)|_{\mathrm{DGC} \geq K} \\ \#\textbf{In+Out}: & (j \longleftrightarrow S)|_{\mathrm{DGC} \geq K} & = & \frac{1}{2(N-1)} \sum_{i \neq j} (i \to j) + (j \to i)|_{\mathrm{DGC} \geq K} . \end{array} \tag{15}$$

#Out measures the number of financial institutions that are significantly Granger-caused by institution j, *#In* measures the number of financial institutions that signif-

icantly Granger-cause institution j, and *#In+Out* is the sum of these two measures.

3. **Sector-Conditional Connections**. Sector-conditional connections are similar to (15), but they condition on the type of financial institution. Given M types (four in our case: banks, broker/dealers, insurers, and hedge funds), indexed by $\alpha, \beta = 1, \ldots, M$, we have the following three measures:

   #Out-to-Other :

$$\left((j|\alpha) \to \sum_{\beta \neq \alpha} (S|\beta) \right)\Bigg|_{\mathrm{DGC} \geq K} \quad = \quad \frac{1}{(M-1)N/M} \sum_{\beta \neq \alpha} \sum_{i \neq j} \left((j|\alpha) \to (i|\beta) \right)\Big|_{\mathrm{DGC} \geq K} \tag{16}$$

   #In-from-Other :

$$\left(\sum_{\beta \neq \alpha} (S|\beta) \to (j|\alpha) \right)\Bigg|_{\mathrm{DGC} \geq K} \quad = \quad \frac{1}{(M-1)N/M} \sum_{\beta \neq \alpha} \sum_{i \neq j} \left((i|\beta) \to (j|\alpha) \right)\Big|_{\mathrm{DGC} \geq K} \tag{17}$$

   #In+Out-Other :

$$\left((j|\alpha) \longleftrightarrow \sum_{\beta \neq \alpha} (S|\beta) \right)\Bigg|_{\mathrm{DGC} \geq K} \quad = \quad \frac{\sum_{\beta \neq \alpha} \sum_{i \neq j} \left((i|\beta) \to (j|\alpha) \right) + \left((j|\alpha) \to (i|\beta) \right)\Big|_{\mathrm{DGC} \geq K}}{2(M-1)N/M} \tag{18}$$

   where *#Out-to-Other* is the number of other types of financial institutions that are significantly Granger-caused by institution j, *#In-from-Other* is the number of other types of financial institutions that significantly Granger-cause institution j, and *#In+Out-Other* is the sum of the two.

4. **Closeness**. *Closeness* measures the shortest path between a financial institution and all other institutions reachable from it, averaged across all other financial institutions. To construct this measure, we first define j as weakly causally C-connected to i if there exists a causality path of length C between i and j, i.e., there exists a sequence of nodes $k_1, \ldots, k_{C-1}$ such that:

$$(j \to k_1) \times (k_1 \to k_2) \cdots \times (k_{C-1} \to i) \quad \equiv \quad (j \stackrel{C}{\to} i) \quad = \quad 1 \ . \tag{19}$$

Denote by C_{ji} the length of the shortest C-connection between j to i:

$$C_{ji} \quad \equiv \quad \min_C \left\{ C \in [1, N-1] : (j \xrightarrow{C} i) = 1 \right\}, \tag{20}$$

where we set $C_{ji} = N-1$ if $(j \xrightarrow{C} i)=0$ for all $C \in [1, N-1]$. The closeness measure for institution j is then defined as:

$$C_{jS}|_{\mathrm{DGC} \geq K} = \frac{1}{N-1} \sum_{i \neq j} C_{ji} (j \xrightarrow{C} i) \Big|_{\mathrm{DGC} \geq K} \ . \tag{21}$$

5. **Eigenvector Centrality**. The *eigenvector centrality* measures the importance of a financial institution in a network by assigning relative scores to financial institutions based on how connected they are to the rest of the network. First define the adjacency matrix A as the matrix with elements:

$$[A]_{ji} \quad = \quad (j \to i) \ . \tag{22}$$

The eigenvector centrality measure is the eigenvector v of the adjacency matrix associated with eigenvalue 1, i.e., in matrix form:

$$Av = v \ . \tag{23}$$

Equivalently, the eigenvector centrality of j can be written as the sum of the eigenvector centralities of institutions caused by j:

$$v_j|_{\mathrm{DGC} \geq K} = \sum_{i=1}^{N} [A]_{ji} \, v_i|_{\mathrm{DGC} \geq K} \ . \tag{24}$$

If the adjacency matrix has non-negative entries, a unique solution is guaranteed to exist by the Perron-Frobenius theorem.

3.3 Nonlinear Granger causality

The standard definition of Granger causality is linear, hence it cannot capture nonlinear and higher-order causal relationships. This limitation is potentially relevant for our purposes since we are interested in whether an increase in riskiness (e.g., volatility) in one financial institution leads to an increase in the riskiness of another. To capture these higher-order effects, we consider a second causality measure in this section that we call *nonlinear Granger causality*, which is based on a Markov-switching model of asset returns.[8] This nonlinear extension of Granger causality can capture the effect of one financial institution's return on the future mean and variance of another financial institution's return, allowing us to detect the volatility-based interconnectedness hypothesized by Danielsson, Shin, and Zigrand (2011), for example.

More formally, consider the case of hedge funds and banks, and let $Z_{h,t}$ and $Z_{b,t}$ be Markov chains that characterize the expected returns (μ) and volatilities (σ) of the two financial institutions, respectively, i.e.:

$$R_{j,t} \quad = \quad \mu_j(Z_{j,t}) \ + \ \sigma_j(Z_{j,t})u_{j,t}, \tag{25}$$

where $R_{j,t}$ is the excess return of institution j in period t, $j = h, b$, $u_{j,t}$ is independently and identically distributed (IID) over time, and $Z_{j,t}$ is a two-state Markov chain with transition probability matrix $\mathbf{P}_{z,j}$ for institution j.

We can test the nonlinear causal interdependence between these two series by testing the two hypotheses of causality from $Z_{h,t}$ to $Z_{b,t}$ and vice versa (the general case of nonlinear Granger-causality estimation is considered in the Appendix C). In fact, the joint stochastic process $Y_t \equiv (Z_{h,t}, Z_{b,t})$ is itself a first-order Markov chain with transition probabilities:

$$P(Y_t \,|\, Y_{t-1}) \quad = \quad P(Z_{h,t}, Z_{b,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) \ , \tag{26}$$

where all the relevant information from the past history of the process at time t is represented by the previous state, i.e., regimes at time $t-1$. Under the additional assumption that the transition probabilities do not vary over time, the process can be defined as a Markov chain with stationary transition probabilities, summarized by the transition matrix $\boldsymbol{P}$. We can

[8]Markov-switching models have been used to investigate systemic risk by Chan, Getmansky, Haas and Lo (2006) and to measure value-at-risk by Billio and Pelizzon (2000).

then decompose the joint transition probabilities as:

$$
\begin{aligned}
P(Y_t|Y_{t-1}) &= P(Z_{h,t}, Z_{b,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) \\
&= P(Z_{b,t} \,|\, Z_{h,t}, Z_{h,t-1}, Z_{b,t-1}) \times P(Z_{h,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) \; .
\end{aligned} \tag{27}
$$

According to this decomposition and the results in Appendix C, we run the following two tests of nonlinear Granger causality:

1. Granger non-causality from $Z_{h,t}$ to $Z_{b,t}$ ($Z_{h,t} \not\Rightarrow Z_{b,t}$):

   Decompose the joint probability:

$$
\begin{aligned}
P(Z_{h,t}, Z_{b,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) &= P(Z_{h,t} \,|\, Z_{b,t}, Z_{h,t-1}, Z_{b,t-1}) \times \\
&\qquad P(Z_{b,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) \; .
\end{aligned} \tag{28}
$$

   If $Z_{h,t} \not\Rightarrow Z_{b,t}$, the last term becomes

$$
P(Z_{b,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) = P(Z_{b,t} \,|\, Z_{b,t-1}) \; . \tag{29}
$$

2. Granger non-causality from $Z_{b,t}$ to $Z_{h,t}$ ($Z_{b,t} \not\Rightarrow Z_{h,t}$):

   This requires that if $Z_{b,t} \not\Rightarrow Z_{h,t}$, then:

$$
P(Z_{h,t} \,|\, Z_{h,t-1}, Z_{b,t-1}) = P(Z_{h,t} \,|\, Z_{h,t-1}) \; . \tag{30}
$$

4 The data

For the main analysis, we use monthly returns data for hedge funds, broker/dealers, banks, and insurers, described in more detail in Sections 4.1 and 4.2. Summary statistics are provided in Section 4.3.

4.1 Hedge funds

We use individual hedge-fund data from the TASS Tremont database. We use the September 30, 2009 snapshot of the data, which includes 8,770 hedge funds in both Live and Defunct databases.

Our hedge-fund index data consists of aggregate hedge-fund index returns from the CS/Tremont database from January 1994 to December 2008, which are asset-weighted indexes of funds with a minimum of $10 million in assets under management, a minimum one-year track record, and current audited financial statements. The following strategies are included in the total aggregate index (hereafter, known as Hedge Funds): Dedicated Short Bias, Long/Short Equity, Emerging Markets, Distressed, Event Driven, Equity Market Neutral, Convertible Bond Arbitrage, Fixed Income Arbitrage, Multi-Strategy, and Managed Futures. The strategy indexes are computed and rebalanced monthly and the universe of funds is redefined on a quarterly basis. We use net-of-fee monthly excess returns. This database accounts for survivorship bias in hedge funds (Fung and Hsieh, 2000). Funds in the TASS Tremont database are similar to the ones used in the CS/Tremont indexes, however, TASS Tremont does not implement any restrictions on size, track record, or the presence of audited financial statements.

4.2 Banks, broker/dealers, and insurers

Data for individual banks, broker/dealers, and insurers are obtained from the University of Chicago's Center for Research in Security Prices Database, from which we select the monthly returns of all companies with SIC Codes from 6000 to 6199 (banks), 6200 to 6299 (broker/dealers), and 6300 to 6499 (insurers). We also construct value-weighted indexes of banks (hereafter, called Banks), broker/dealers (hereafter, called Brokers), and insurers (hereafter, called Insurers).

4.3 Summary statistics

Table 1 reports annualized mean, annualized standard deviation, minimum, maximum, median, skewness, kurtosis, and first-order autocorrelation coefficient ρ_1 for individual hedge funds, banks, broker/dealers, and insurers from January 2004 through December 2008. We choose the 25 largest financial institutions (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) in each of the four index categories. Brokers have the highest annual mean of 23% and the highest standard deviation of 39%. Hedge funds have the lowest mean, 12%, and the lowest standard deviation, 11%. Hedge funds have the highest first-order autocorrelation of 0.14, which is particularly striking when compared to the small negative autocorrelations of broker/dealers (−0.02), banks (−0.09), and insurers (−0.06). This finding is consistent with the hedge-fund industry's higher exposure to illiquid assets and return-smoothing (see Getmansky, Lo, and Makarov, 2004).

We calculate the same statistics for different time periods that will be considered in the empirical analysis: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. These periods encompass both tranquil, boom, and crisis periods in the sample. For each 36-month rolling-window time period the largest 25 hedge funds, broker/dealers, insurers, and banks are included. In the last period, 2006–2008 which is characterized by the recent Financial Crisis, we observe the lowest mean across all financial institutions: 1%, −5%, −24%, and −15% for hedge funds, broker/dealers, banks, and insurers, respectively. This period is also characterized by very large standard deviations, skewness, and kurtosis. Moreover, this period is unique, as all financial institutions exhibit positive first-order autocorrelations.

5 Empirical analysis

In this section, we implement the measures defined in Section 3 using historical data for individual company returns corresponding to the four sectors of the finance and insurance industries described in Section 4. Section 5.1 contains the results of the principal components analysis applied to returns of individual financial institutions, and Sections 5.2 and 5.3 report the outcomes of linear and nonlinear Granger-causality tests, respectively, including simple visualizations via network diagrams.

5.1 Principal components analysis

Since the heart of systemic risk is commonality among multiple institutions, we attempt to measure commonality through PCA applied to the individual financial and insurance companies described in Section 4 over the whole sample period, 1994–2008. The time-series results for the Cumulative Risk Fraction (i.e., eigenvalues) are presented in Figure 1a. The time-series graph of eigenvalues for all principal components (PC1, PC2–10, PC11–20, and PC21–36) shows that the first 20 principal components capture the majority of return variation during the whole sample, but the relative importance of these groupings varies considerably. The time periods when few principal components explain a larger percentage of total variation are associated with an increased interconnectedness between financial institutions

	Mean	SD	Min	Max	Median	Skew.	Kurt.	Autocorr.
Full Sample								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	12%	11%	-7%	8%	12%	-0.24	4.40	0.14
Brokers	23%	39%	-21%	32%	14%	0.23	3.85	-0.02
Banks	16%	26%	-17%	19%	17%	-0.05	3.71	-0.09
Insurers	15%	28%	-17%	21%	15%	0.04	3.84	-0.06
January 1994 to December 1996								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	14%	15%	-8%	12%	12%	0.25	3.63	0.08
Brokers	23%	29%	-15%	22%	21%	0.26	3.63	-0.09
Banks	29%	23%	-12%	16%	29%	-0.05	2.88	0.00
Insurers	20%	22%	-11%	17%	16%	0.20	3.18	-0.06
January 1996 to December 1998								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	13%	18%	-15%	11%	18%	-1.12	6.13	0.15
Brokers	31%	43%	-29%	37%	26%	0.06	5.33	-0.03
Banks	34%	30%	-23%	22%	35%	-0.53	5.17	-0.10
Insurers	24%	29%	-19%	21%	24%	-0.13	3.60	-0.03
January 1999 to December 2001								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	14%	11%	-6%	9%	11%	0.08	3.99	0.15
Brokers	28%	61%	-26%	55%	-2%	0.76	4.19	-0.03
Banks	13%	33%	-19%	24%	8%	0.21	3.26	-0.10
Insurers	10%	41%	-22%	34%	2%	0.62	4.21	-0.16
January 2002 to December 2004								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	9%	7%	-4%	5%	9%	-0.03	4.05	0.21
Brokers	10%	32%	-20%	21%	10%	-0.11	3.13	-0.01
Banks	14%	22%	-14%	15%	15%	-0.12	3.18	-0.12
Insurers	12%	24%	-17%	16%	14%	-0.19	3.81	0.02
January 2006 to December 2008								
	Mean	**SD**	**Min**	**Max**	**Median**	**Skew.**	**Kurt.**	**Autocorr.**
Hedge Funds	1%	13%	-12%	5%	10%	-1.00	5.09	0.26
Brokers	-5%	40%	-33%	27%	6%	-0.52	4.69	0.16
Banks	-24%	37%	-34%	22%	-8%	-0.57	5.18	0.05
Insurers	-15%	39%	-40%	28%	1%	-0.84	8.11	0.07

Table 1: Summary statistics for monthly returns of individual hedge funds, broker/dealers, banks, and insurers for the full sample: January 2004 to December 2008, and five time periods: 1994-1996, 1996-1998, 1999-2001, 2002-2004, and 2006-2008. The annualized mean, annualized standard deviation, minimum, maximum, median, skewness, kurtosis, and first-order autocorrelation are reported. We choose 25 largest financial institutions (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) in each of the four financial institution sectors.

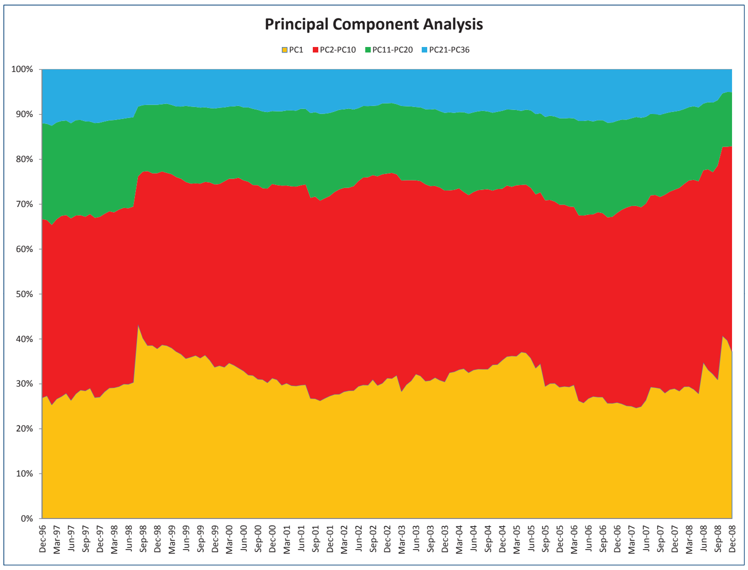
as described in Section 3.1. In particular, Figure 1a shows that the first principal component is very dynamic, capturing from 24% to 43% of return variation, increasing significantly during crisis periods. The PC1 eigenvalue was increasing from the beginning of the sample, peaking at 43% in August 1998 during the LTCM crisis, and subsequently decreased. The PC1 eigenvalue started to increase in 2002 and stayed high through 2005 (the period when the Federal Reserve intervened and raised interest rates), declining slightly in 2006–2007, and increasing again in 2008, peaking in October 2008. As a result, the first principal component explained 37% of return variation over the Financial Crisis of 2007–2009. In fact, the first 10 components explained 83% of the return variation over the recent financial crisis, which was the highest compared to all other sub-periods.

In addition, we tabulate eigenvalues and eigenvectors from the principal components analysis over five time periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. The results in Table 2 show that the first 10 principal components capture 67%, 77%, 72%, 73%, and 83% of the variability among financial institutions in these five time periods, respectively. The first principal component explains 33% of the return variation on average. The first 10 principal components explain 74% of the return variation on average, and the first 20 principal components explain 91% of the return variation on average, as shown by the Cumulative Risk Fractions in Figure 1a.

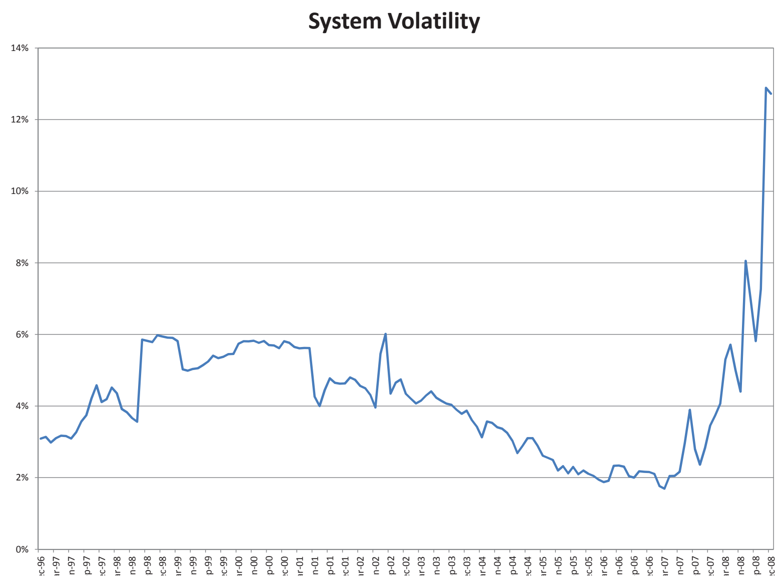
We also estimate the variance of the system, σ_S^2, from the GARCH(1,1) model.[9] Figure 1b depicts the system variance from January 2004 to December 2008. Both the system variance and the Cumulative Risk Fraction increase during the LTCM (August 2008) and the Financial Crisis of 2007–2009 (October 2008) periods. The correlation between these two aggregate indicators is 0.41. Not only is the first principal component able to explain a large proportion of the total variance during these crisis periods, but the system variance greatly increased as well.

Table 2 contains the mean, minimum, and maximum of our PCAS measures defined in (8) for the 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008 periods. These measures are quite persistent over time for all financial and insurance institutions, but we

[9] For every 36-month window, we calculate the average of returns for all 100 institutions and we estimate a GARCH(1,1) model on the resulting time series. For each window, we select the GARCH variance of the last observation. We prefer to report the variance of the system estimated with the GARCH(1,1) model rather than just the variance estimated for different windows because in this way we have a measure that is reacting earlier to the shocks. However, even if we use the variance for each window, we still observe similar dynamics, only smoothed.



(a)



(b)

Figure 1: Principal components analysis of the monthly standardized returns of individual hedge funds, broker/dealers, banks, and insurers over January 1994 to December 2008: (a) 36-month rolling-window estimates of the Cumulative Risk Fraction (i.e., eigenvalues) that correspond to the fraction of total variance explained by principal components 1–36 (PC 1, PC 2–10, PC 11–20, and PC 21–36); (b) system variance from the GARCH(1,1) model.

Sector		PCAS 1	PCAS 1-10	PCAS 1-20
		PCAS x 10^5		
	1994 to 1996			
Hedge Funds	Mean	**0.04**	**0.19**	**0.24**
	Min	0.00	0.00	0.00
	Max	0.18	1.08	1.36
Brokers	Mean	**0.22**	**0.50**	**0.65**
	Min	0.01	0.12	0.20
	Max	0.52	1.19	1.29
Banks	Mean	**0.14**	**0.31**	**0.42**
	Min	0.02	0.13	0.16
	Max	0.41	0.90	1.45
Insurers	Mean	**0.12**	**0.29**	**0.40**
	Min	0.01	0.12	0.15
	Max	0.42	1.32	2.23
	1996 to 1998			
Hedge Funds	Mean	**0.04**	**0.11**	**0.13**
	Min	0.00	0.00	0.00
	Max	0.15	0.44	0.55
Brokers	Mean	**0.22**	**0.62**	**0.71**
	Min	0.02	0.05	0.09
	Max	0.63	3.79	4.06
Banks	Mean	**0.13**	**0.23**	**0.28**
	Min	0.05	0.13	0.16
	Max	0.21	0.39	0.56
Insurers	Mean	**0.10**	**0.24**	**0.30**
	Min	0.02	0.06	0.11
	Max	0.33	1.57	2.08
	1999 to 2001			
Hedge Funds	Mean	**0.00**	**0.05**	**0.07**
	Min	0.00	0.00	0.00
	Max	0.03	0.24	0.28
Brokers	Mean	**0.12**	**1.30**	**1.71**
	Min	0.00	0.06	0.11
	Max	0.44	5.80	7.14
Banks	Mean	**0.20**	**0.33**	**0.42**
	Min	0.05	0.09	0.13
	Max	0.71	1.45	1.93
Insurers	Mean	**0.29**	**0.52**	**0.63**
	Min	0.03	0.18	0.25
	Max	0.76	2.30	3.00

Sector		PCAS 1	PCAS 1-10	PCAS 1-20
		PCAS x 10^5		
	2002 to 2004			
Hedge Funds	Mean	**0.01**	**0.04**	**0.04**
	Min	0.00	0.00	0.00
	Max	0.22	0.28	0.32
Brokers	Mean	**0.31**	**0.53**	**0.65**
	Min	0.02	0.14	0.21
	Max	1.05	1.55	1.91
Banks	Mean	**0.16**	**0.25**	**0.30**
	Min	0.02	0.08	0.10
	Max	0.51	0.60	0.76
Insurers	Mean	**0.12**	**0.28**	**0.37**
	Min	0.01	0.11	0.13
	Max	0.40	0.91	1.14
	2006 to 2008			
Hedge Funds	Mean	**0.02**	**0.10**	**0.11**
	Min	0.00	0.00	0.00
	Max	0.16	1.71	1.91
Brokers	Mean	**0.21**	**0.41**	**0.51**
	Min	0.05	0.12	0.16
	Max	0.53	1.88	3.00
Banks	Mean	**0.12**	**0.42**	**0.46**
	Min	0.00	0.13	0.14
	Max	0.34	1.43	1.54
Insurers	Mean	**0.25**	**0.47**	**0.51**
	Min	0.01	0.05	0.06
	Max	0.71	1.76	1.84

Sample Period	PC 1	PC 1-10	PC 1-20
Cumulative Risk Fraction			
Hedge Funds, Brokers, Banks, Insurers			
1994 to 1996	**27%**	**67%**	**88%**
1996 to 1998	**38%**	**77%**	**92%**
1999 to 2001	**27%**	**72%**	**90%**
2002 to 2004	**35%**	**73%**	**91%**
2006 to 2008	**37%**	**83%**	**95%**

Table 2: Mean, minimum, and maximum values for PCAS 1, PCAS 1–10, and PCAS 1–20. These measures are based on the monthly returns of individual hedge funds, broker/dealers, banks, and insurers for the five time periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. Cumulative Risk Fraction (i.e., eigenvalues) is calculated for PC 1, PC 1-10, and PC 1-20 for all five time periods.

find variation in the sensitivities of the financial sectors to the four principal components. PCAS 1–20 for broker/dealers, banks, and insurers are on average 0.85, 0.30, and 0.44, respectively for the first 20 principal components. This is compared to 0.12 for hedge funds, which represents the lowest average sensitivity out of the four sectors. However, we also find variation in our PCAS measure for individual hedge funds. For example, the maximum PCAS 1–20 for hedge funds in the 2006–2008 time period is 1.91.

As a result, hedge funds are not greatly exposed to the overall risk of the system of financial institutions. Broker/dealers, banks, and insurers have greater PCAS, thus, result in greater connectedness. However, we still observe large cross-sectional variability, even among hedge funds.[10]

We explore the out-of-sample performance of our PCAS measures (individually and jointly with our Granger-causality-network measures) during the crisis periods in Section 6.

5.2 Linear Granger-causality tests

To fully appreciate the impact of Granger-causal relationships among various financial institutions, we provide a visualization of the results of linear Granger-causality tests presented in Section 3.2, applied over 36-month rolling sub-periods to the 25 largest institutions (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) in each of the four index categories.[11]

The composition of this sample of 100 financial institutions changes over time as assets under management change, and as financial institutions are added or deleted from the sample. Granger-causality relationships are drawn as straight lines connecting two institutions, color-coded by the type of institution that is "causing" the relationship, i.e., the institution at date-t which Granger-causes the returns of another institution at date $t+1$. Green indicates a broker, red indicates a hedge fund, black indicates an insurer, and blue indicates a bank. Only those relationships significant at the 5% level are depicted. To conserve space, we

[10] We repeated the analysis by filtering out heteroskedasticity with a GARCH(1,1) model and adjusting for autocorrelation in hedge funds returns using the algorithm proposed by Getmansky, Lo, and Makarov (2004), and the results are qualitatively the same. These results are available upon request.

[11] Given that hedge-fund returns are only available monthly, we impose a minimum of 36 months to obtain reliable estimates of Granger-causal relationships. We also used a rolling window of 60 months to control the robustness of the results. Results are provided upon request.

tabulate results only for two of the 145 36-month rolling-window sub-periods in Figures 2 and 3: 1994–1996 and 2006–2008. These are representative time-periods encompassing both tranquil and crisis periods in the sample.[12] We see that the number of connections between different financial institutions dramatically increases from 1994–1996 to 2006–2008.

[12]To fully appreciate the dynamic nature of these connections, we have created a short animation using 36-month rolling-window network diagrams updated every month from January 1994 to December 2008, which can be viewed at `http://web.mit.edu/alo/www`.

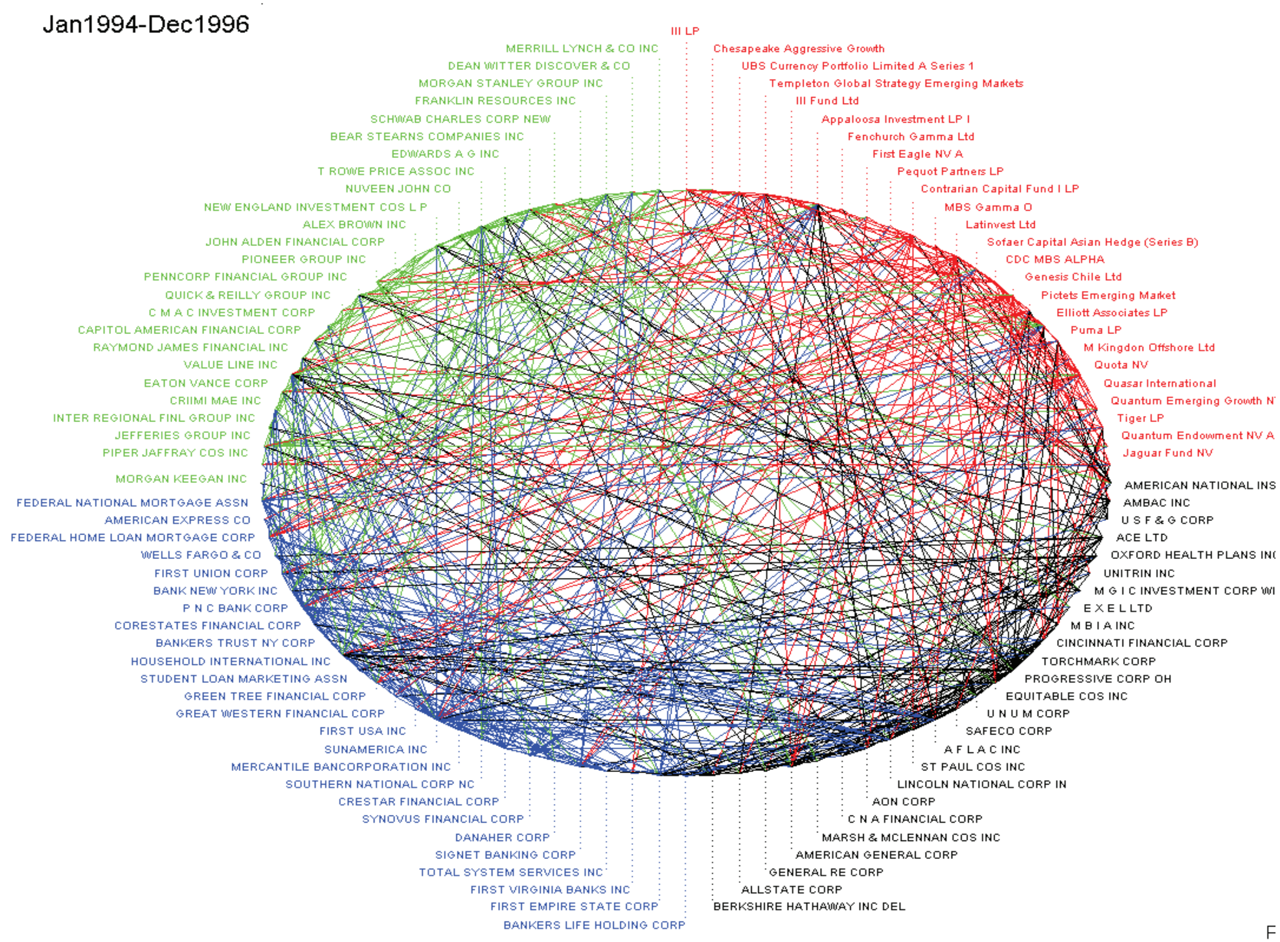


Figure 2: Network Diagram of Linear Granger-causality relationships that are statistically significant at the 5% level among the monthly returns of the 25 largest (in terms of average AUM) banks, broker/dealers, insurers, and hedge funds over January 1994 to December 1996. The type of institution causing the relationship is indicated by color: green for broker/dealers, red for hedge funds, black for insurers, and blue for banks. Granger-causality relationships are estimated including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

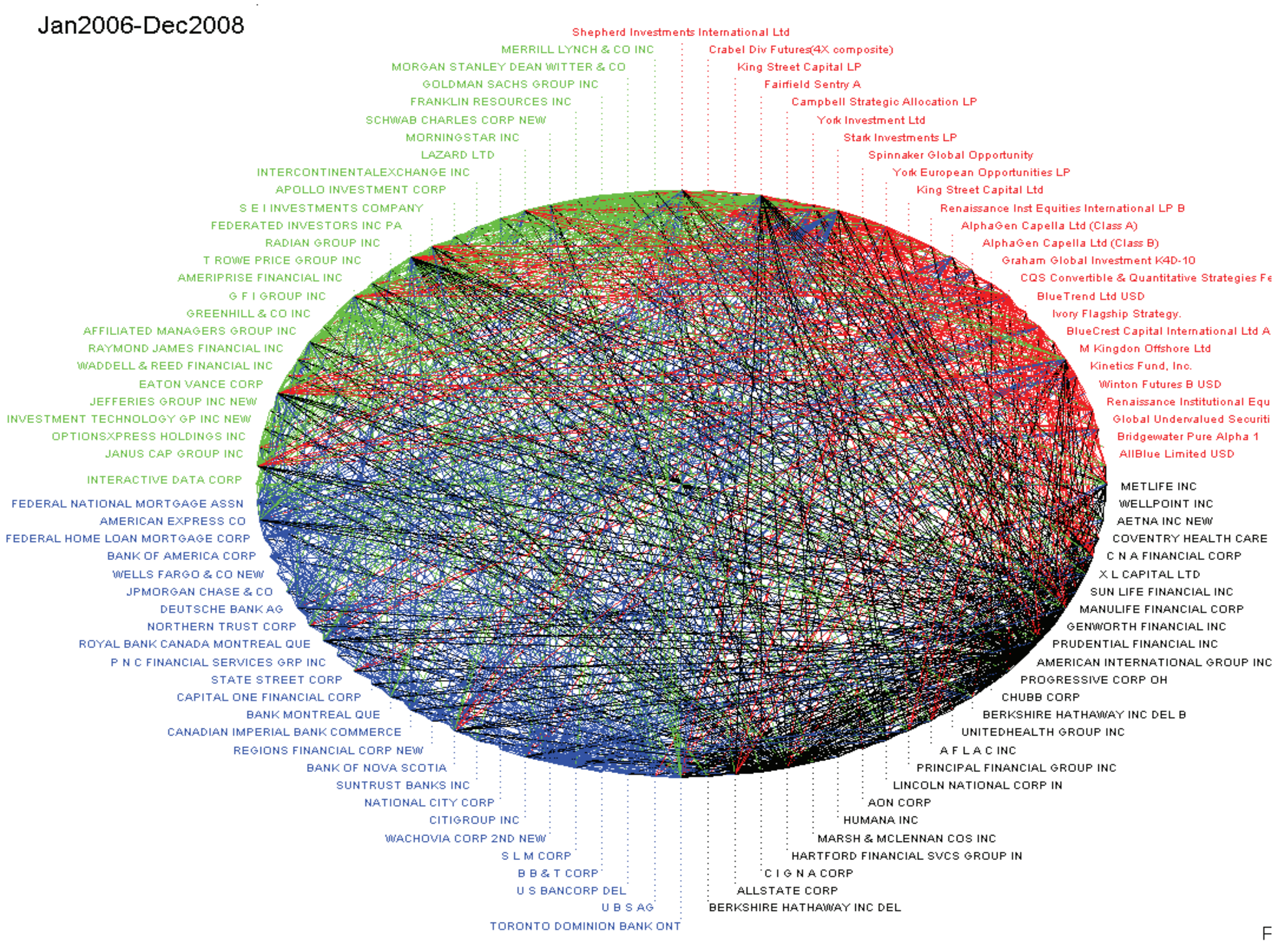


Figure 3: Network diagram of linear Granger-causality relationships that are statistically significant at the 5% level among the monthly returns of the 25 largest (in terms of average AUM) banks, broker/dealers, insurers, and hedge funds over January 2006 to December 2008. The type of institution causing the relationship is indicated by color: green for broker/dealers, red for hedge funds, black for insurers, and blue for banks. Granger-causality relationships are estimated including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

For our five time periods: (1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008), we also provide summary statistics for the monthly returns of the 100 largest (with respect to market value and AUM) financial institutions in Table 3, including the asset-weighted autocorrelation, the normalized number of connections,[13] and the total number of connections.

We find that Granger-causality relationships are highly dynamic among these financial institutions. Results are presented in Table 3 and Figures 2 and 3. For example, the total number of connections between financial institutions was 583 in the beginning of the sample (1994–1996), but it more than doubled to 1,244 at the end of the sample (2006–2008). We also find that during and before financial crises the financial system becomes much more interconnected in comparison to more tranquil periods. For example, the financial system was highly interconnected during the 1998 LTCM crisis and the most recent Financial Crisis of 2007–2009. In the relatively tranquil period of 1994–1996, the total number of connections as a percentage of all possible connections was 6% and the total number of connections among financial institutions was 583. Just before and during the LTCM 1998 crisis (1996–1998), the number of connections increased by 50% to 856, encompassing 9% of all possible connections. In 2002–2004, the total number of connections was just 611 (6% of total possible connections), and that more than doubled to 1244 connections (13% of total possible connections) in 2006–2008, which was right before and during the recent Financial Crisis of 2007–2009 according to Table 3. Both the 1998 LTCM crisis and the Financial Crisis of 2007–2009 were associated with liquidity and credit problems. The increase in interconnections between financial institutions is a significant systemic risk indicator, especially for the Financial Crisis of 2007–2009 which experienced the largest number of interconnections compared to other time-periods.[14]

The time series of the number of connections as a percent of all possible connections is depicted in black in Figure 4, against a threshold of 0.055, the 95th percentile of the simulated distribution obtained under the hypothesis of no causal relationships, depicted in red. Following the theoretical framework of Section 3.2, this figure displays the DGC measure which indicates greater connectedness when DGC exceeds the threshold. According to Figure 4, the number of connections are large and significant during the 1998 LTCM crisis,

[13] The normalized number of connections is the fraction of all statistically significant connections (at the 5% level) between the N financial institutions out of all $N(N-1)$ possible connections.

[14] The results are similar when we adjust for the S&P 500, and are available upon request.

	Sector	Asset Weighted AutoCorr	# of Connections as % of All Possible				# of Connections			
			TO				TO			
			Hedge Funds	Brokers	Banks	Insurers	Hedge Funds	Brokers	Banks	Insurers
January 1994 to December 1996										
FROM	All	-0.07	6%				583			
	Hedge Funds	0.03	7%	3%	6%	6%	41	21	36	37
	Brokers	-0.15	3%	5%	6%	4%	18	29	36	24
	Banks	-0.03	6%	7%	9%	7%	40	46	54	44
	Insurers	-0.10	5%	6%	6%	9%	33	38	35	51
January 1996 to December 1998										
FROM	All	-0.03	9%				856			
	Hedge Funds	0.08	14%	6%	5%	3%	82	38	30	20
	Brokers	-0.04	13%	9%	9%	9%	81	53	54	57
	Banks	-0.09	11%	8%	11%	10%	71	52	65	64
	Insurers	0.02	9%	9%	7%	6%	57	54	44	34
January 1999 to December 2001										
FROM	All	-0.09	5%				520			
	Hedge Funds	0.17	5%	5%	5%	9%	32	32	33	58
	Brokers	0.03	8%	9%	3%	5%	53	52	19	29
	Banks	-0.09	5%	3%	4%	7%	30	17	25	42
	Insurers	-0.20	5%	3%	2%	6%	32	16	14	36
January 2002 to December 2004										
FROM	All	-0.08	6%				611			
	Hedge Funds	0.20	10%	3%	9%	5%	61	20	56	29
	Brokers	-0.09	8%	4%	4%	6%	53	23	26	39
	Banks	-0.14	9%	3%	4%	5%	55	16	24	30
	Insurers	0.00	8%	6%	9%	6%	48	40	55	36
January 2006 to December 2008										
FROM	All	0.08	13%				1244			
	Hedge Funds	0.23	10%	13%	5%	13%	57	82	31	83
	Brokers	0.23	12%	17%	9%	12%	78	102	55	73
	Banks	0.02	23%	12%	10%	9%	142	74	58	54
	Insurers	0.12	13%	16%	12%	16%	84	102	73	96

Table 3: Summary statistics of asset-weighted autocorrelations and linear Granger-causality relationships (at the 5% level of statistical significance) among the monthly returns of the largest 25 banks, broker/dealers, insurers, and hedge funds (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) for five sample periods: 1994-1996, 1996-1998, 1999-2001, 2002-2004, and 2006-2008.The normalized number of connections, and the total number of connections for all financial institutions, hedge funds, broker/dealers, banks, and insurers are calculated for each sample including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

2002–2004 (a period of low interest rates and high leverage among financial institutions), and the recent Financial Crisis of 2007–2009.

If we compare Figure 4 and Figure 1, we observe that all the aggregate indicators—the fraction explained by the first PC (h_1), the number of connections as a percent of all possible connections, and the financial system variance from the GARCH model—move in tandem during the two financial crises of 1998 and 2008, respectively. However, there are several periods where they move in opposite directions, e.g., 2002–2005. We find that these measures exhibit statistically significant contemporaneous and lagged correlations. For example, contemporaneous correlation between h_1 and the number of connections is 0.50, and correlation between system variance and the number of connections is 0.43. However, these connectedness measures are not perfectly correlated. For example, during 2001–2006, the system variance was decreasing, but other measures were increasing. As a result, all these measures seem to be capturing different facets of connectedness, as suggested by the out-of-sample analysis in Section 6.[15]

By measuring Granger-causality-network connections among individual financial institutions, we find that during the 1998 LTCM crisis (1996–1998 period), hedge funds were greatly interconnected with other hedge funds, banks, broker/dealers, and insurers. Their impact on other financial institutions was substantial, though less than the total impact of other financial institutions on them. In the aftermath of the crisis (1999–2001 and 2002–2004 time periods), the number of financial connections decreased, especially links affecting hedge funds. The total number of connections clearly started to increase just before and at the beginning of the recent Financial Crisis of 2007–2009 (2006–2008 time period). In that time period, hedge funds had significant bi-lateral relationships with insurers and broker/dealers. Hedge funds were highly affected by banks (23% of total possible connections), though they did not reciprocate in affecting the banks (5% of total possible connections). The number of significant Granger-causal relations from banks to hedge funds, 142, was the highest between these two sectors across all five sample periods. In comparison, hedge funds Granger-caused only 31 banks. These results for the largest individual financial institutions suggest that banks may be of more concern than hedge funds from the perspective of connectedness, though hedge funds may be the "canary in the cage" that first experience losses

[15]More detailed analysis of the significance of Granger-causal relationships is provided in the robustness analysis of Appendix B.

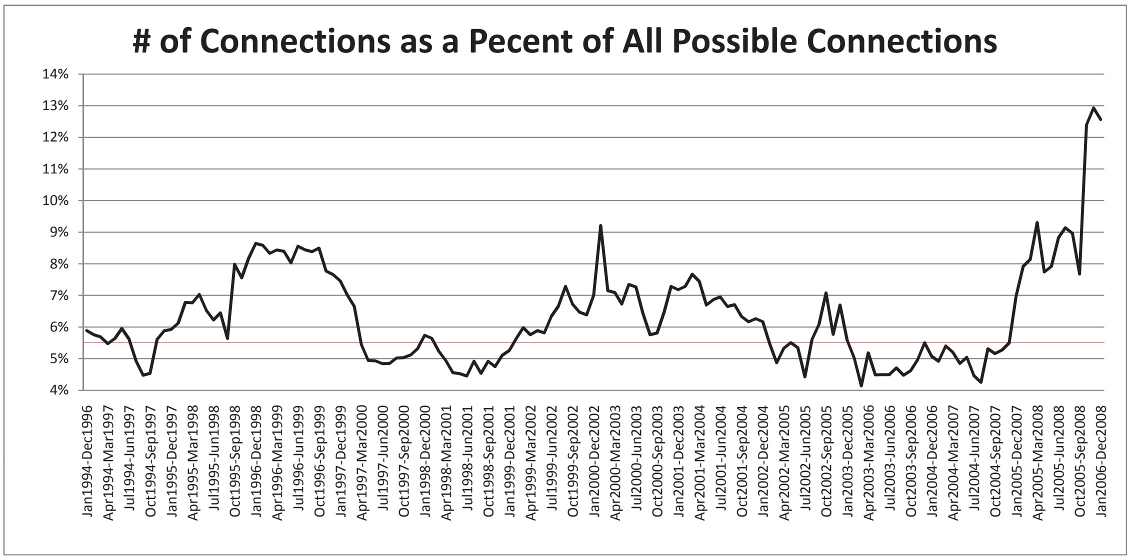


Figure 4: The time series of linear Granger-causality relationships (at the 5% level of statistical significance) among the monthly returns of the largest 25 banks, broker/dealers, insurers, and hedge funds (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) for 36-month rolling-window sample periods from January 1994 to December 2008. The number of connections as a percentage of all possible connections (our DGC measure) is depicted in black against 0.055, the 95% of the simulated distribution obtained under the hypothesis of no causal relationships depicted in red. The number of connections is estimated for each sample including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

when financial crises hit.[16]

Lo (2002) and Getmansky, Lo, and Makarov (2004) suggest using return autocorrelations to gauge the illiquidity risk exposure, hence we report asset-weighted autocorrelations in Table 3. We find that the asset-weighted autocorrelations for all financial institutions were negative for the first four time periods, however, in 2006–2008, the period that includes the recent financial crisis, the autocorrelation becomes positive. When we separate the asset-weighted autocorrelations by sector, we find that during all periods, hedge-fund asset-weighted autocorrelations were positive, but were mostly negative for all other financial institutions.[17] However, in the last period (2006–2008), the asset-weighted autocorrelations became positive for all financial institutions. These results suggest that the period of the Financial Crisis of 2007–2009 exhibited the most illiquidity and connectivity among financial institutions.

In summary, we find that, on average, all companies in the four sectors we studied have become highly interrelated and generally less liquid over the past decade, increasing the level of connectedness in the finance and insurance industries.

To separate contagion and common-factor exposure, we regress each company's monthly returns on the S&P 500 and re-run the linear Granger causality tests on the residuals. We find the same pattern of dynamic interconnectedness between financial institutions, and the resulting network diagrams are qualitatively similar to those with raw returns, hence we omit them to conserve space.[18] We also explore whether our connectedness measures are related to return predictability, which we capture via the following factors: inflation, industrial production growth, the Fama-French factors, and liquidity (detailed results are reported in Online Appendix O.3). We find that our main Granger-causality results still hold after adjusting for these sources of predictability. It appears that connections between financial institutions are not contemporaneously priced and cannot be exploited as a trading strategy. The analysis on predictability highlights the fact that our measures are not related to traditional macroeconomic or Fama-French firm-specific factors, and are capturing connections

[16] These results are also consistent if we consider indexes of hedge funds, broker/dealers, banks, and insurers. The results are available in Online Appendix O.2.

[17] Starting in the October 2002–September 2005 period, the overall system and individual financial-institution 36-month rolling-window autocorrelations became positive and remained positive through the end of the sample.

[18] Network diagrams for residual returns (from a market-model regression against the S&P 500) are available upon request.

beyond the usual common drivers of asset-return fluctuations.

For completeness, in Table 4 we present summary statistics for the other network measures proposed in Section 3.2, including the various counting measures of the number of connections, and measures of centrality. These metrics provide somewhat different but largely consistent perspectives on how the Granger-causality network of banks, broker/dealers, hedge funds, and insurers changed over the past 15 years.[19]

[19]To compare these measures with the classical measure of correlation, see Online Appendix O.4.

	In			Out			In+Out			In-from-Other			Out-to-Other			In+Out-Other			Closeness Centrality			Eigenvector Centrality		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Jan1994-Dec1996																								
Hedge Funds	1	5.28	16	0	5.40	15	3	10.68	22	0	3.64	16	0	3.76	13	1	7.40	16	1.85	5.91	99.00	0.00	0.07	0.25
Brokers	0	5.36	13	1	4.28	9	3	9.64	19	0	4.52	11	0	3.56	9	2	8.08	17	1.91	1.96	1.99	0.01	0.05	0.17
Banks	2	6.44	24	1	7.36	30	4	13.80	36	1	5.00	21	1	5.76	26	3	10.76	29	1.70	1.93	1.99	0.00	0.09	0.32
Insurers	1	6.24	16	1	6.28	29	5	12.52	31	0	4.76	13	0	4.96	22	2	9.72	24	1.71	1.94	1.99	0.01	0.08	0.38
Jan1996-Dec1998																								
Hedge Funds	0	11.64	49	2	6.80	22	4	18.44	63	0	8.36	34	0	3.52	14	1	11.88	42	1.78	1.93	1.98	0.02	0.05	0.21
Brokers	0	7.88	29	0	9.80	44	2	17.68	44	0	6.36	22	0	6.56	31	2	12.92	31	1.56	5.86	99.00	0.00	0.09	0.36
Banks	0	7.72	26	1	10.08	25	1	17.80	38	0	6.52	17	1	7.24	21	1	13.76	29	1.75	1.90	1.99	0.01	0.09	0.22
Insurers	0	7.00	22	1	7.56	22	2	14.56	43	0	6.20	18	1	5.28	20	1	11.48	33	1.78	1.92	1.99	0.00	0.07	0.25
Jan1999-Dec2001																								
Hedge Funds	0	5.88	27	1	6.20	22	4	12.08	29	0	4.60	27	0	4.92	18	2	9.52	29	1.78	1.94	1.99	0.00	0.09	0.20
Brokers	0	4.68	16	1	6.12	12	2	10.80	21	0	3.40	11	0	4.00	9	0	7.40	14	1.88	1.94	1.99	0.01	0.12	0.28
Banks	0	3.64	9	1	4.56	10	3	8.20	14	0	2.32	8	0	3.36	8	1	5.68	13	1.90	1.95	1.99	0.01	0.06	0.13
Insurers	0	6.60	21	0	3.92	9	2	10.52	29	0	4.28	18	0	2.64	7	1	6.92	23	1.91	5.92	99.00	0.00	0.06	0.16
Jan2002-Dec2004																								
Hedge Funds	1	8.68	26	0	6.64	16	2	15.32	39	0	6.24	19	0	4.20	15	0	10.44	30	1.84	5.89	99.00	0.00	0.08	0.21
Brokers	0	3.96	12	0	5.64	21	1	9.60	22	0	3.16	9	0	3.52	20	1	6.68	20	1.79	9.86	99.00	0.00	0.08	0.23
Banks	0	6.44	24	0	5.00	14	2	11.44	33	0	4.20	16	0	2.80	10	1	7.00	18	1.86	9.87	99.00	0.00	0.07	0.19
Insurers	0	5.36	19	0	7.16	29	0	12.52	33	0	4.20	13	0	5.24	25	0	9.44	27	1.71	5.89	99.00	0.00	0.09	0.36
Jan2006-Dec2008																								
Hedge Funds	1	14.44	49	0	10.12	36	1	24.56	60	1	12.16	43	0	7.84	31	1	20.00	47	1.64	9.82	99.00	0.00	0.05	0.21
Brokers	2	14.40	44	0	12.32	51	5	26.72	57	2	11.12	36	0	9.20	41	4	20.32	43	1.48	5.84	99.00	0.00	0.06	0.32
Banks	1	8.68	36	2	13.12	51	6	21.80	55	1	7.44	30	0	7.44	38	4	14.88	42	1.48	1.87	1.98	0.01	0.07	0.30
Insurers	2	12.24	29	2	14.20	71	4	26.44	73	1	8.92	21	1	10.84	56	2	19.76	58	1.28	1.86	1.98	0.00	0.07	0.49

Table 4: Summary statistics of network measures of linear Granger-causality networks (at the 5% level of statistical significance) among the monthly returns of the largest 25 banks, broker/dealers, insurers, and hedge funds (as determined by average AUM for hedge funds and average market capitalization for the other three types of institutions during the time period considered) for five sample periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. The normalized and total number of connections for all four types of financial institutions are calculated for each sample including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

In addition, we explore whether our Granger-causality-network measures are related to firm characteristics for two sub-periods: October 2002 to September 2005 and July 2004 to June 2007. We select these two sub-periods to provide examples of high and low levels of the number of connections, respectively, as Figure 4 shows. We conduct a rank regression of all Granger-causality-network measures on firm-specific characteristics such as size, leverage, liquidity, and industry dummies for these sub-periods.[20] The results are inconclusive. For the October 2002 to September 2005 period, we find that size is negatively related to the In-from-Other measure, suggesting that smaller firms are more likely to be affected by firms from other industries. Market beta positively affects the Out-to-Other measure. The first order autocorrelation is significantly associated with the In+Out measure. However, these results are not robust for the July 2004 to June 2007 period. As a result, we do not find any consistent relationships between the Granger-causality-network measures and firm characteristics.

We explore the out-of-sample performance of our Granger-causality measures (individually and jointly with our PCAS measures) including firm characteristics during the crisis periods in Section 6.

5.3 Nonlinear Granger-causality tests

Table 5 presents p-values of nonlinear Granger-causality likelihood ratio tests (see Section 3.3) for the monthly residual returns indexes of Banks, Brokers, Insurers, and Hedge Funds over the two samples: 1994–2000 and 2001–2008. Given the larger number of parameters in nonlinear Granger-causality tests as compared to linear Granger-causality tests, we use monthly indexes instead of the returns of individual financial institutions and two longer sample periods. Index returns are constructed by value-weighting the monthly returns of individual institutions as described in Section 4. Residual returns are obtained from regressions of index returns against the S&P 500 returns. Index results for linear Granger-causality tests are presented in Online Appendix O.2. This analysis shows that causal relationships are even stronger if we take into account both the level of the mean and the level of risk that these financial institutions may face, i.e., their volatilities. The presence of strong nonlinear Granger-causality relationships is detected in both samples. Moreover, in the 2001–2008

[20]We thank the referee for suggesting this line of inquiry. Rank regressions are available from the authors upon request.

sample, we find that almost all financial institutions were affected by the past level of risk of other financial institutions.[21]

	Sector	TO			
		Hedge Funds	Brokers	Banks	Insurers
1994 to 2000					
FROM	**Hedge Funds**		**0.00**	**0.00**	**0.00**
FROM	**Brokers**	**0.00**		0.24	0.75
FROM	**Banks**	**0.02**	**0.00**		0.78
FROM	**Insurers**	0.07	0.82	0.93	
2001 to 2008					
FROM	**Hedge Funds**		**0.00**	**0.01**	0.09
FROM	**Brokers**	**0.00**		**0.00**	0.94
FROM	**Banks**	0.21	**0.01**		**0.00**
FROM	**Insurers**	0.37	**0.00**	**0.00**	

Table 5: p-values of nonlinear Granger-causality likelihood ratio tests for the monthly residual returns indexes (from a market-model regression against S&P 500 returns) of Banks, Brokers, Insurers, and Hedge Funds for two sub-samples: January 1994 to December 2000, and January 2001 to December 2008. Statistics that are significant at the 5% level are shown in bold.

Note that linear Granger-causality tests provide causality relationships based only on the means, whereas nonlinear Granger-causality tests also take into account the linkages among the volatilities of financial institutions. With nonlinear Granger-causality tests we find more interconnectedness between financial institutions compared to linear Granger-causality results, which supports the endogenous volatility feedback relationship proposed by Danielsson, Shin, and Zigrand (2011). The nonlinear Granger-causality results are also consistent with the results of the linear Granger-causality tests in two respects: the connections are increasing over time, and even after controlling for the S&P 500, shocks to one financial institution are likely to spread to all other financial institutions.

6 Out-of-sample results

One important application of any systemic risk measure is to provide early warning signals to regulators and the public. To this end, we explore the out-of-sample performance of our PCAS and Granger-causality measures in Sections 6.1 and 6.2, respectively. In particular,

[21] We consider only pairwise Granger causality due to significant multicollinearity among the returns.

following the approach of Acharya et al. (2011), we consider two 36-month samples, October 2002–September 2005 and July 2004–June 2007, as estimation periods for our measures, and the period from July 2007–December 2008 as the "out-of-sample" period encompassing the Financial Crisis of 2007–2009. The two sample periods (October 2002–September 2005 and July 2004–June 2007) have been selected to provide two different examples characterized by high and low levels of connectedness in the sample. In the October 2002–September 2005 period, the Cumulative Risk Fraction measure h_n is statistically larger than the threshold H (as shown in Appendix A) and the number of Granger-causality connections are statistically different than zero as shown by Figure 4. In the July 2004–June 2007, h_n is statistically smaller than H and the number of Granger-causality connections is not statistically different from zero. In Section 6.3, we show that even with the time-varying levels of significance during this period, both measures still yield useful out-of-sample indications of the recent financial crisis.

6.1 PCAS results

When the Cumulative Risk Fraction, h_n, is large, this means that we observe a significant amount of connectedness among financial institutions. To identify when this percentage is large, i.e., to identify a threshold H, we use a simulation approach described in Appendix A. We find that h_1 (i.e., when $n=1$) should be larger than 33.74% (i.e., H is 33.74%) to exhibit a large degree of connectedness (where n is the fraction of eigenvalues considered, as defined in Section 3.1). When $n=10$ and 20, H is estimated to be 74.48% and 91.67%, respectively.

For each of the four financial and insurance categories we consider the top 25 financial institutions as determined by the average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered, yielding 100 entities in all. These financial institutions are ranked from 1 to 100 according to their PCAS.

To evaluate the out-of-sample performance of PCAS, we first compute the maximum percentage financial loss (Max%Loss) suffered by each of the 100 institutions during the crisis period from July 2007 to December 2008.[22] We then rank all financial institutions from

[22]The maximum percentage loss for a financial institution is defined to be the difference between the market capitalization of the institution (fund size in the case of hedge funds) at the end of June 2007 and the minimum market capitalization during the period from July 2007 to December 2008 divided by the market capitalization or fund size of the institution at the end of June 2007. An additional loss metric is the

1 to 100 according to Max%Loss. We then estimate univariate regressions for Max%Loss rankings on the institutions' PCAS rankings. We consider PCAS 1, PCAS 1–10, and PCAS 1–20 measures as reported in (8). The results are reported in Table 6 for two samples: October 2002–September 2005 and July 2004–June 2007. For each regression, we report the β coefficient, the t-statistic, p-value, and the Kendall (1938) τ rank-correlation coefficient.

We find that companies more exposed to the overall risk of the system, i.e., those with larger PCAS measures, were more likely to suffer significant losses during the recent crisis. In this respect, the PCAS measure is similar to the MES measure proposed by Acharya et al. (2011). Institutions that have the largest exposures to the 20 largest principal components (the most contemporaneously interconnected) are those that lose the most during the crisis.

As Table 6 shows for the October 2002 to September 2005 period, the rank correlation indicates a strict relationship between PCAS and losses during the recent Financial Crisis of 2007–2009. The beta coefficients are all significant at the 5% level, showing that PCAS correctly identifies firms that will be more affected during crises, i.e., will face larger losses.

However, the percentage of volatility explained by the principal components decreased in July 2004–June 2007, consistent with Figure 1a. In this case, there is not a strict relationship between the exposure of a single institution to principal components and the losses it may face during the crisis. We have repeated the analysis using the PCAS measure described in (8) where σ_i^2 and σ_S^2 are estimated using a GARCH(1,1) model, and the results are qualitatively unchanged.

6.2 Granger-causality results

We use the same estimation and out-of-sample periods to evaluate our Granger-causality-network measures as in Section 6.1, and for each financial institution, we compute 8 Granger-causality-network measures. As before, for each of the four categories of financial institutions, we consider the top 25 as determined by the average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered, yielding 100 entities in all. As with the PCAS measure, we rank financial institutions from 1 to 100 according to Granger-causality-network measures.[23]

maximum dollar loss, Max$Loss, however, this is highly driven by size. We do believe that size is a relevant factor for systemic risk, thus we chose to include the largest 100 financial institutions in our analysis. We concentrate on Max%Loss in order to capture additional drivers of connectedness.

[23]The institution with the highest value of a measure is ranked 1 and the one with the lowest is ranked 100. However, for the Closeness measure, the ranking is reversed: an institution with the lowest Closeness

Measure	Max % Loss			
	Coeff	t-stat	p-value	Kendall τ
October 2002 to September 2005				
PCAS 1	**0.35**	**3.46**	**0.00**	**0.25**
PCAS 1-10	**0.29**	**2.83**	**0.01**	**0.22**
PCAS 1-20	**0.29**	**2.83**	**0.01**	**0.22**
July 2004 to June 2007				
PCAS 1	0.11	1.10	0.28	0.09
PCAS 1-10	0.07	0.73	0.47	0.06
PCAS 1-20	0.09	0.91	0.37	0.07

Table 6: Regression coefficients, t-statistics, p-values, and Kendall (1938) τ rank-correlation coefficients for regressions of Max%Loss loss on PCAS 1, PCAS 1–10, and PCAS 1–20. The maximum percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007. These measures are calculated over two samples: October 2002–September 2005 and July 2004–June 2007. Statistics that are significant at the 5% level are displayed in bold.

To evaluate the predictive power of these rankings, we repeated the out-of-sample analysis done with PCAS measures by first ranking all financial institutions from 1 to 100 according to Max%Loss. We then estimate univariate regressions for Max%Loss rankings on the institutions' Granger-causality-network rankings. The results are reported in Table 7 for two samples: October 2002–September 2005 and July 2004–June 2007. For each regression, we report the β coefficient, the t-statistic, p-value, and the Kendall (1938) τ rank-correlation coefficient.

We find that Out, Out-to-Other, In+Out-Other, Closeness, and Eigenvector Centrality are significant determinants of the Max%Loss variable.

Based on the Closeness and Eigenvector Centrality measures, financial institutions that are highly connected are the ones that suffered the most during the Financial Crisis of 2007–2009. However, the institutions that declined the most during the Crisis were the ones that greatly affected other institutions—both their own and other types—and not the institutions that were affected by others. Both Out and Out-to-Other are significant, whereas In and In-from-Other are not.

measure is ranked 1, and the one with the highest is 100.

Measure	Max % Loss			
	Coeff	t-stat	p-value	Kendall τ
October 2002 to September 2005				
# of "In" Connections	0.03	0.25	0.80	0.02
# of "Out" Connections	**0.23**	**2.23**	**0.03**	**0.16**
# of "In+Out" Connections	0.16	1.51	0.13	0.11
# of "In-from-Other" Connections	0.12	1.15	0.25	0.09
# of "Out-to-Other" Connections	**0.32**	**3.11**	**0.00**	**0.22**
# of "In+Out Other" Connections	**0.23**	**2.23**	**0.03**	**0.15**
Closeness	**0.23**	**2.23**	**0.03**	**0.16**
Eigenvector Centrality	**0.24**	**2.31**	**0.02**	**0.16**
July 2004 to June 2007				
# of "In" Connections	-0.01	-0.07	0.94	-0.01
# of "Out" Connections	**0.25**	**2.53**	**0.01**	**0.20**
# of "In+Out" Connections	0.19	1.89	0.06	0.13
# of "In-from-Other" Connections	-0.02	-0.19	0.85	-0.02
# of "Out-to-Other" Connections	0.17	1.68	0.10	0.13
# of "In+Out Other" Connections	0.09	0.84	0.41	0.06
Closeness	**0.25**	**2.53**	**0.01**	**0.20**
Eigenvector Centrality	**0.24**	**2.44**	**0.02**	**0.17**

Table 7: Regression coefficients, t-statistics, p-values, and Kendall (1938) τ rank-correlation coefficients for regressions of Max%Loss loss on Granger-causality-network measures. The maximum percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007. These measures are calculated over two samples: October 2002–September 2005 and July 2004–June 2007. Statistics that are significant at the 5% level are displayed in bold.

6.3 Combined measures

To evaluate the predictive power of PCA and Granger-causality-network measures, we first compute the maximum percentage financial loss (Max%Loss) suffered by each of the 100 institutions during the crisis period from July 2007 to December 2008. We then rank all financial institutions from 1 to 100 according to Max%Loss. We then estimate univariate regressions for Max%Loss rankings on the institutions' connectedness rankings. The results are reported in Table 8 for two samples: October 2002–September 2005 and July 2004–June 2007. For each regression, we report the β coefficient, the t-statistic, p-value, and the Kendall (1938) τ rank-correlation coefficient.

We find that Out, Out-to-Other, In+Out-Other, Closeness, Eigenvector Centrality, and PCAS are significant determinants of the Max%Loss variable. Based on the Closeness and Eigenvector Centrality measures, financial institutions that are systemically important and are very interconnected are the ones that suffered the most during the Financial Crisis of 2007–2009. However, the institutions that declined the most during the Crisis were the ones that greatly affected other institutions—both their own and other types—and not the institutions that were affected by others. Both Out and Out-to-Other are significant, whereas In and In-from-Other are not. The top names in the Out and Out-to-Other categories include Wells Fargo, Bank of America, Citigroup, Federal National Mortgage Association, UBS, Lehman Brothers Holdings, Wachovia, Bank New York, American International Group, and Washington Mutual.[24]

In addition to causal relationships, contemporaneous correlations between financial institutions served as predictors of the crisis. Based on the significance of the PCAS 1 measure,[25] companies that were more correlated with other companies and were more exposed to the overall risk of the system, were more likely to suffer significant losses during the recent crisis.[26] As early as 2002–2005, important connections among these financial institutions were established that later contributed to the Financial Crisis and the subsequent decline of many of them.[27]

[24]The top 20 ranked financial institutions with respect to the Out-to-Other measure are listed in Table A.1 in Appendix D.

[25]PCAS 1–10 and PCAS 1–20 are also significant. The same applies if we calculate PCAS 1, PCAS 1-10, or PCAS 1-20 using GARCH(1,1) model. Results are available from the authors upon request.

[26]The significance of the PCAS measures decreased in July 2004–June 2007. This is consistent with the result in Figure 1 where, for the monthly return indexes, the first principal component captured less return variation during this time period than in the October 2002–September 2005 period.

[27]We also consider time periods just before and after October 2002–September 2005 that show a signifi-

Variable	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
October 2002 to September 2005																
Intercept	18.57	1.63	11.76	1.10	12.76	1.18	17.49	1.55	9.55	0.90	12.78	1.18	11.76	1.10	12.01	1.11
Leverage	0.19	1.78	**0.22**	2.13	**0.21**	1.99	0.18	1.71	**0.25**	2.40	**0.21**	1.97	**0.22**	2.13	**0.22**	2.11
PCAS 1	**0.40**	3.07	**0.33**	2.61	**0.37**	2.88	**0.39**	2.99	**0.27**	2.06	**0.34**	2.64	**0.33**	2.61	**0.34**	2.63
Size	0.07	0.53	0.04	0.31	0.07	0.58	0.08	0.61	0.05	0.37	0.09	0.67	0.04	0.31	0.04	0.30
ρ_1	-0.13	-1.24	-0.12	-1.22	-0.17	-1.61	-0.14	-1.28	-0.11	-1.04	-0.16	-1.55	-0.12	-1.22	-0.10	-1.02
# of "In" Connections	0.04	0.35														
# of "Out" Connections			**0.25**	2.50												
# of "In+Out" Connections					**0.22**	2.12										
# of "In-from-Other" Connections							0.07	0.62								
# of "Out-to-Other" Connections									**0.32**	2.98						
# of "In+Out Other" Connections											**0.23**	2.12				
Closeness													**0.25**	2.50		
Eigenvector Centrality															**0.22**	2.18
R-square	0.19		0.25		0.23		0.19		0.27		0.23		0.25		0.23	
July 2004 to June 2007																
Intercept	19.03	1.35	3.91	0.29	-1.20	-0.08	20.27	1.44	4.70	0.33	6.71	0.44	3.91	0.29	10.15	0.77
Leverage	0.18	1.71	**0.24**	2.28	**0.22**	2.11	0.18	1.71	**0.23**	2.18	**0.21**	1.98	**0.24**	2.28	**0.21**	1.99
PCAS 1	0.20	1.44	0.20	1.54	**0.28**	2.06	0.20	1.42	0.23	1.73	0.25	1.81	0.20	1.54	0.20	1.49
Size	-0.01	-0.08	-0.01	-0.04	0.03	0.20	-0.01	-0.07	0.00	0.03	0.00	0.00	-0.01	-0.04	-0.01	-0.06
ρ_1	0.17	1.51	0.18	1.75	**0.21**	1.97	0.16	1.47	0.18	1.70	0.20	1.82	0.18	1.75	0.13	1.22
# of "In" Connections	0.05	0.49														
# of "Out" Connections			**0.30**	3.03												
# of "In+Out" Connections					**0.29**	2.83										
# of "In-from-Other" Connections							0.03	0.30								
# of "Out-to-Other" Connections									**0.26**	2.47						
# of "In+Out Other" Connections											0.20	1.79				
Closeness													**0.30**	3.03		
Eigenvector Centrality															**0.26**	2.57
R-square	0.09		0.17		0.16		0.08		0.14		0.12		0.17		0.15	

Table 8: Parameter estimates of a multivariate rank regression of Max%Loss for each financial institution during July 2007–December 2008 on PCAS 1, leverage, size, first order autocorrelation, and Granger-causality-network measures. The maximum percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007. PCAS 1, leverage, size, first order autocorrelation, and Granger-causality-network measures are calculated over October 2002–September 2005 and July 2004–June 2007. Parameter estimates that are significant at the 5% level are shown in bold.

It is possible that some of our results can be explained by leverage effects.[28] Leverage has the effect of a magnifying glass, expanding small profit opportunities into larger ones, but also expanding small losses into larger losses. And when unexpected adverse market conditions reduce the value of the corresponding collateral, such events often trigger forced liquidations of large positions over short periods of time. Such efforts to reduce leverage can lead to systemic events as we have witnessed during the recent crisis. Since leverage information is not directly available, for publicly traded banks, broker/dealers, and insurers, we estimate their leverage as the ratio of Total Assets minus Equity Market Value to Equity Market Value. For hedge funds, we use reported average leverage for a given time period. Using these crude proxies, we find that estimated leverage is positively related to future losses (Max%Loss). We also adjusted for asset size (as determined by AUM for hedge funds and market capitalization for broker/dealers, insurers, and banks) and the results are not altered by including this additional regressor. In all regressions, asset size is not significant for Max%Loss. This may be due to the fact that our analysis is concentrated on large financial institutions (the top 25 for each sector).

Leverage is also problematic, largely because of illiquidity—in the event of a margin call on a leveraged portfolio, forced liquidations may cause even larger losses and additional margin calls, ultimately leading to a series of insolvencies and defaults as financial institutions withdraw credit. Lo (2002) and Getmansky, Lo, and Makarov (2004) suggest using return autocorrelation to gauge the illiquidity risk exposure of a given financial institution, hence the multivariate regression of Table 8 is estimated by including the first-order autocorrelation of monthly returns as an additional regressor.

These robustness checks lead us to conclude that, in both sample periods (October 2002–September 2005 and July 2004–June 2007), our results are robust—measures based on Granger causality and principal components analysis seem to be predictive of the Financial Crisis of 2007–2009.

We also investigate the relationship between our measures of connectedness and the realized tail risk measured by correlation between each firm and the financial system during the Financial Crisis of 2007–2009. The analysis shows that our measures of contemporaneous connections between financial institutions, PCAS 1, 1–10, and 1–20 are all highly related to

cant number of interconnections, and the results are still significant for Out, Out-to-Other, In+Out-Other, Closeness, Eigenvector Centrality, and PCAS measures.

[28]We thank Lasse Pedersen and Mark Carey for suggesting this line of inquiry.

this realized tail risk (rank regression p-values equal to zero). However, Granger-causality-network measures are not consistently correlated with tail risk, indicating that they are capturing the spillover of losses that are not contemporaneous, but contribute to losses of financial institutions in the wake of systemic shocks.[29]

Finally, we consider spillover effects by measuring the performance of firms highly connected to the best-performing and worst-performing firms. Specifically, during the crisis period we rank 100 firms by performance and construct quintiles from this ranking. Using the "Out" measure of connectedness, we find that firms that have the highest number of significant connections to the worst-performing firms (1st quintile) do worse than firms that are less connected to these poor performers. More specifically, firms in the 2nd quintile exhibit 119 connections with the 1st quintile, and those that have the smallest number of connections (69) with the 1st quintile perform the best, i.e., they are in the 5th quintile. This pattern suggests that there are, indeed, spillover effects in performance that are being captured by Granger-causality networks.

7 Conclusion

The financial system has become considerably more complex over the past two decades as the separation between hedge funds, mutual funds, insurance companies, banks, and broker/dealers have blurred thanks to financial innovation and deregulation. While such complexity is an inevitable consequence of competition and economic growth, they are accompanied by certain consequences, including much greater interdependence.

In this paper, we propose to quantify that interdependence using principal components analysis and Granger-causality networks. Principal components analysis provides a broad view of connections among all four groups of financial institutions, and Granger-causality networks capture the intricate web of pairwise statistical relations among individual firms in the finance and insurance industries. Using monthly returns data for hedge-fund indexes and portfolios of publicly traded banks, insurers, and broker/dealers, we show that such indirect measures are indeed capable of picking up periods of market dislocation and distress, and have promising out-of-sample characteristics. These measures seem to capture unique and different facets of the finance and insurance sectors. For example, over the recent sample

[29]Results are available from the authors upon request.

period, our empirical results suggest that the banking and insurance sectors may be even more important sources of connectedness than other parts, which is consistent with the anecdotal evidence from the recent financial crisis. The illiquidity of bank and insurance assets, coupled with fact that banks and insurers are not designed to withstand rapid and large losses (unlike hedge funds), make these sectors a natural repository for systemic risk.

The same feedback effects and dynamics apply to bank and insurance capital requirements and risk management practices based on VaR, which are intended to ensure the soundness of individual financial institutions, but may amplify aggregate fluctuations if they are widely adopted. For example, if the riskiness of assets held by one bank increases due to heightened market volatility, to meet its VaR requirements the bank will have to sell some of these risky assets. This liquidation may restore the bank's financial soundness, but if all banks engage in such liquidations at the same time, a devastating positive feedback loop may be generated unintentionally. These endogenous feedback effects can have significant implications for the returns of financial institutions, including autocorrelation, increased correlation, changes in volatility, Granger causality, and, ultimately, increased systemic risk, as our empirical results seem to imply.

As long as human behavior is coupled with free enterprise, it is unrealistic to expect that market crashes, manias, panics, collapses, and fraud will ever be completely eliminated from our capital markets. The best hope for avoiding some of the most disruptive consequences of such crises is to develop methods for measuring, monitoring, and anticipating them. By using a broad array of tools for gauging the topology of the financial network, we stand a better chance of identifying "black swans" when they are still cygnets.

Appendix

In this Appendix we provide robustness checks and more detailed formulations and derivations for our connectedness measures. We conduct PCA significance tests in Section A. Tests for statistical significance of Granger-causality-network measures are in Section B. Section C provides some technical details for nonlinear Granger-causality tests. Finally, Section D provides a list of systemically important institutions based on our measures.

A PCA significance tests

For the PCA analysis we employ a 36-month rolling estimate of the principal components over the 1994–2008 sample period. According to Figure 1 we observe significant changes around August 1998, September 2005, and November 2008 for the first principal component. Below we devise a test for structural changes in the estimates within the PCA framework across all sample periods to test the significance of these changes.

Defining the Total Risk of the system as $\Omega = \sum_{k=1}^{N} \lambda_k$ and Cumulative Risk at n eigenvalue as $\omega_n = \sum_{k=1}^{n} \lambda_k$, the Cumulative Risk Fraction is:

$$\frac{\omega_n}{\Omega} \equiv h_n \tag{A.1}$$

where N is the total number of eigenvalues, λ_k is the k-th eigenvalue, and h_n is the fraction of total risk explained by the first n eigenvalues.

In our analysis we consider 100 institutions and 145 overlapping 36-month time periods. Therefore, h_1 is the fraction of total risk corresponding to the first principal component for each period. For each of the 145 periods, we calculate h_1, and select the time periods corresponding to the lowest quintile of the h_1 measure (the 20% of the 145 periods having the lowest h_1). We refer to the mean of the lowest quintile h_1 measures as the threshold H.

Excluding periods with h_1 values above the lowest quintile, we averaged elements of covariance matrix over the remaining periods obtaining an average covariance matrix, which we used in simulating 100 multivariate normal series for $1,000$ times.

For each simulation we compute h_n for each integer n and compute the mean, 95%, 99%, and 99.5% confidence intervals of the simulated distributions. We then test whether h_n, the fraction of total risk explained by the first n eigenvalues, for each rolling-window time periods considered in the analysis is statistically different by checking if it is outside the significance bounds of the simulated distribution.

Figure A.1 presents results for the following 36-month rolling periods: September 1995–August 1998, October 2002–September 2005, December 2005–November 2008, February 1994–January 1997, and April 1995–March 1998. For September 1995–August 1998, October 2002–September 2005, and December 2005–November 2008 we observe statistically significant changes; however, not for the April 1995–March 1998 and February 1994–January 1997 periods. This is consistent with our analysis in Figure 1 where we observe significant changes around August 1998, September 2005, and November 2008 for the first principal component.

B Significance of Granger-causality network measures

In Figure 4 we graph the total number of connections as a percentage of all possible connections we observe in the real data at the 5% significance level (in black) against 0.055, the 95th percentile of the simulated distribution obtained under the hypothesis of no causal relationships (in red). We find that for the 1998–1999, 2002–2004, and 2007-2008 periods, the number of causal relationships observed far exceeds the number obtained purely by chance. Therefore, for these time-periods we can affirm that the observed causal relationships are statistically significant.[30]

To test whether Granger-causal relationships between individual financial and insurance institutions are due to chance, we conduct a Monte Carlo simulation analysis. Specifically, assuming independence among financial institutions, we randomly simulate 100 time series representing the 100 financial institutions' returns in our sample, and test for Granger causality at the 5% level among all possible causal relationships (as in the empirical analysis in Section 5.2, there are a total of 9,900 possible causal relationships), and record the number of significant connections. We repeat this exercise 500 times, and the resulting distribution is given in Figure A.2a. This distribution is centered at 0.052, which represents the fraction of significant connections among all possible connections under the null hypothesis of no statistical relation among any of the financial institutions. The area between 0.049 and 0.055 captures 90% of the simulations. Therefore, if we observe more than 5.5% of significant relationships in the real data, our results are unlikely to be the result of type I error.

[30]The results are similar for the 1%-level of significance.

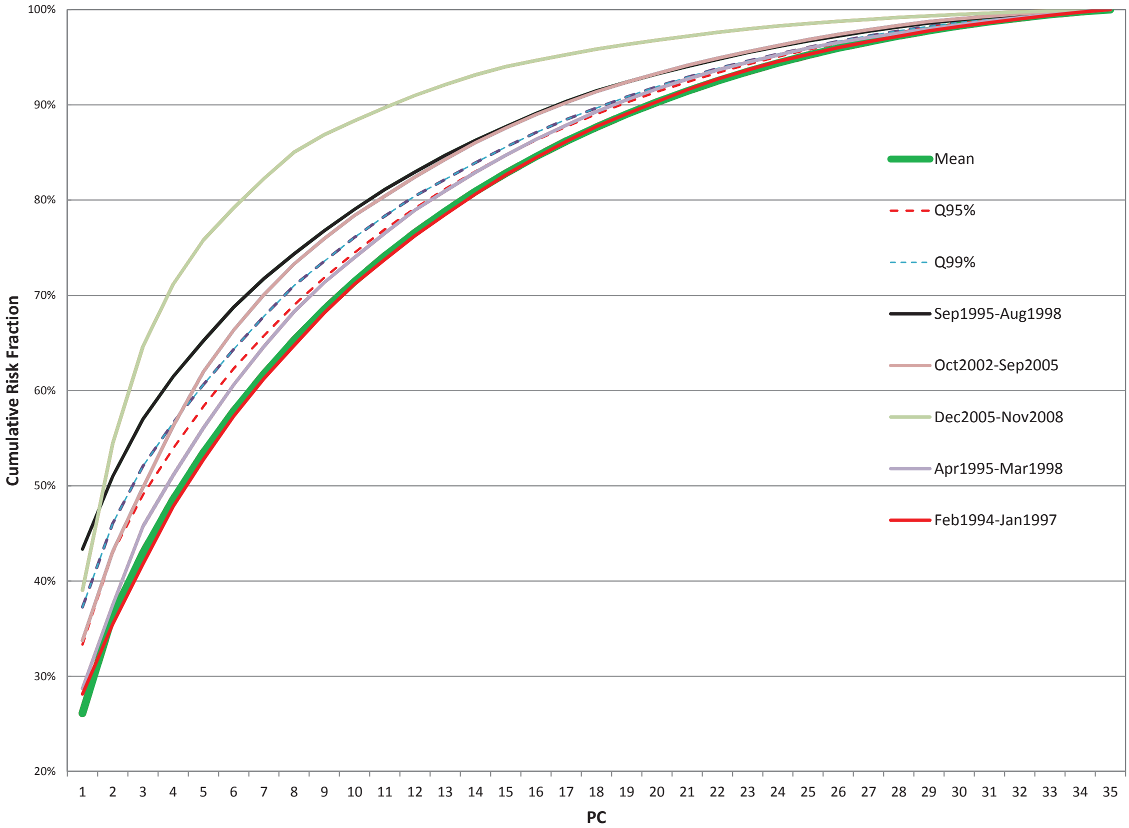
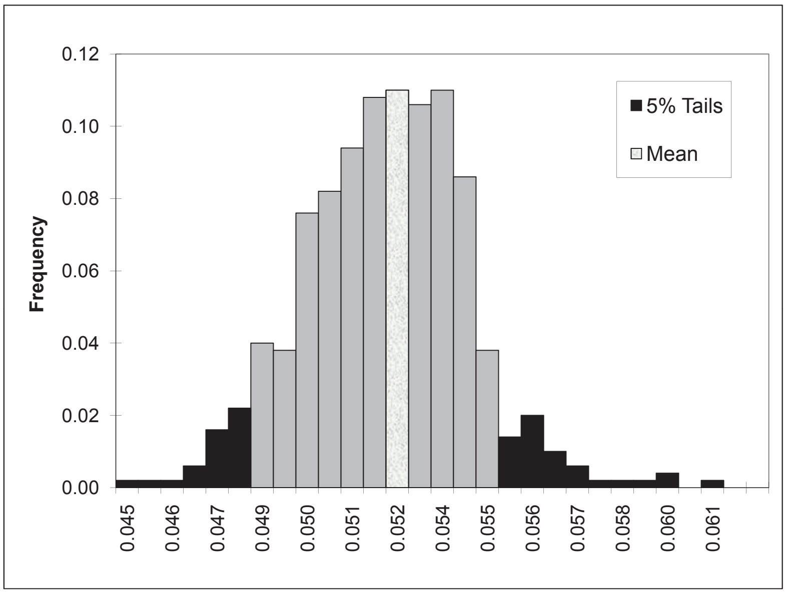
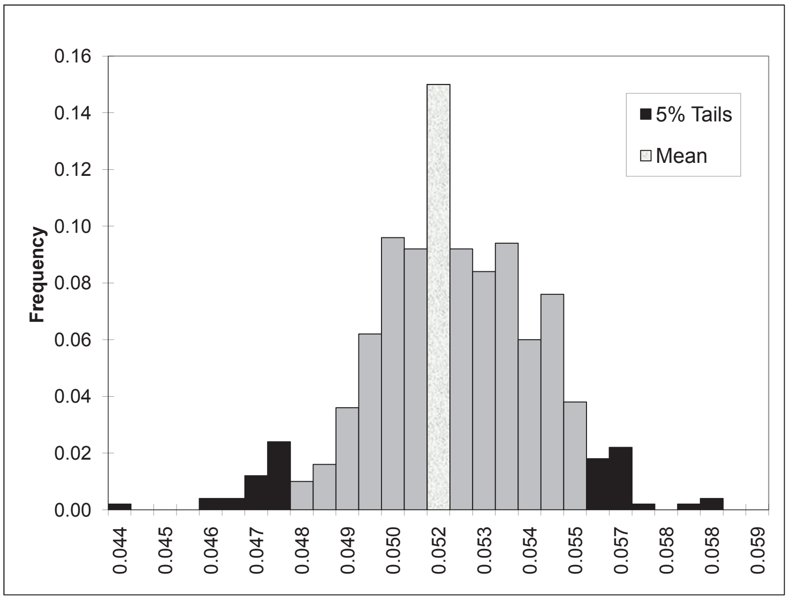


Figure A.1: The fraction of total risk explained by the first 36 principal components. The mean of the lowest quintile and the 95% and the 99% confidence intervals of the simulated distributions are graphed. For the simulated distributions we exclude periods with h_1 values above the lowest quintile. Tests of significance of the differences in the Cumulative Risk Fraction are presented for the following 36-month rolling periods: September 1995–August 1998, October 2002–September 2005, December 2005–November 2008, February 1994–January 1997, and April 1995–March 1998.



(a)



(b)

Figure A.2: Histograms of simulated Granger-causal relationships between financial institutions. 100 time series representing 100 financial institutions's returns are simulated and tested for Granger casuality at the 5% level. The number of significant connections out of all possible connections is calculated for 500 simulations. In histogram (a), independence among financial institutions is assumed. In histogram (b), contemporaneous correlation among financial institutions, captured through the dependence on the S&P 500 is allowed.

We also conduct a similar simulation exercise under the null hypothesis of contemporaneously correlated returns with the S&P 500, but no causal relations among financial institutions. The results are essentially the same, as seen in the histogram in Figure A.2b: the histogram is centered around 0.052, and the area between 0.048 and 0.055 captures 90% of the simulations.

The following provides a step-by-step procedure for identifying Granger-causal linkages:

Because we wish to retain the contemporaneous dependence structure among the individual time series, our working hypothesis is that the dependence arises from a common factor, i.e., the S&P 500. Specifically, to simulate 100 time series (one for each financial institution), we start with the time-series data for these institutions and filter out heteroskedastic effects with a GARCH(1,1) process, as in the linear Granger-causality analysis of Section 5.2. We then regress the standardized residuals on the returns of the S&P 500 index: $R_t^{\text{S\&P500}}$:

$$\widetilde{R}_t^i = \alpha_i \;+\; \beta_i R_t^{\text{S\&P500}} \;+\; \sigma_i \epsilon_t^i \quad , \quad i = 1, \ldots, 100 \quad , \quad \epsilon_t^i \text{ IID } \mathcal{N}(0,1) \tag{A.2}$$

and store the parameter estimates $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\sigma}_i$, to calibrate our simulation's data-generating process, where "IID" denotes independently and identically distributed random variables.

Next, we simulate 36 monthly returns (corresponding to the 3-year period in our sample) of the common factor and the residual returns of the 100 hypothetical financial institutions. Returns of the common factor come from a normal random variable with mean and standard deviation set equal to that of the S&P 500 return, $Y_{jt}^{\text{S\&P500}}$. The residuals η_{jt}^i are IID standard normal random variables. We repeat this simulation 500 times and obtain the resulting population of our simulated series Y_{jt}^i:

$$Y_{jt}^i = \hat{\alpha}_i \;+\; \hat{\beta}_i Y_{jt}^{\text{S\&P500}} \;+\; \hat{\sigma}_i \eta_{jt}^i \quad , \quad i = 1, \ldots, 100 \quad , \quad j = 1, \ldots, 500, t = 1....36 \tag{A.3}$$

For each simulation j, we perform our Granger-causality analysis and calculate the number of significant connections, and compute the empirical distribution of the various test statistics which can then be used to assess the statistical significance of our empirical findings.

In summary, using several methods we show that our Granger-causality results are not due to chance.

C Nonlinear Granger causality

In this section we provide a framework for conducting nonlinear Granger-causality tests.

Let us assume that $Y_t = (S_t, Z_t)$ is a first-order Markov process (or Markov chain) with transition probabilities:

$$P(Y_t|Y_{t-1}, ..., Y_0) = P(Y_t|Y_{t-1}) = P(S_t, Z_t|S_{t-1}, Z_{t-1}). \tag{A.4}$$

Then, all the information from the past history of the process, which is relevant for the transition probabilities in time t, is represented by the previous state of the process, i.e., the state in time $t-1$. Under the additional assumption that transition probabilities do not vary over time, the process is defined as a Markov chain with stationary transition probabilities, summarized in the transition matrix Π.

We can further decompose the joint transition probabilities as follows:

$$\Pi = P(Y_t|Y_{t-1}) = P(S_t, Z_t|S_{t-1}, Z_{t-1}) = P(S_t|Z_t, S_{t-1}, Z_{t-1}) \times P(Z_t|S_{t-1}, Z_{t-1}). \tag{A.5}$$

and thus define the Granger non-causality for a Markov chain as:

Definition 1 *Strong one-step ahead non-causality for a Markov chain with stationary transition probabilities, i.e., Z_{t-1} does not strongly cause S_t given S_{t-1} if:*

$$\forall t \quad P(S_t|S_{t-1}, Z_{t-1}) = P(S_t|S_{t-1}) \ . \tag{A.6}$$

Similarly, S_{t-1} does not strongly cause Z_t given Z_{t-1} if:

$$\forall t \quad P(Z_t|Z_{t-1}, S_{t-1}) = P(Z_t|Z_{t-1}) \ . \tag{A.7}$$

The Granger non-causality tests in this framework are based on the transition matrix Π that can be represented using an alternative parametrization. The transition matrix Π can, in fact, be represented through a logistic function. More specifically, when we consider two-state

Markov chains, the joint probability of S_t and Z_t can be represented as follows:

$$\begin{aligned} P(S_t, Z_t|S_{t-1}, Z_{t-1}) &= P(S_t|Z_t, S_{t-1}, Z_{t-1}) \times P(Z_t|S_{t-1}, Z_{t-1}) \\ &= \frac{\exp(\alpha' V_t)}{1+\exp(\alpha' V_t)} \times \frac{\exp(\beta' U_t)}{1+\exp(\beta' U_t)}, \end{aligned} \tag{A.8}$$

where

$$\begin{aligned} V_t &= (1, Z_t)' \otimes (1, S_{t-1})' \otimes (1, Z_{t-1})' \\ &= (1, Z_{t-1}, S_{t-1}, S_{t-1}Z_{t-1}, Z_t, Z_tZ_{t-1}, Z_tS_{t-1}, Z_tZ_{t-1}S_{t-1})', \end{aligned} \tag{A.9}$$

the vectors α and β have dimensions (8×1) and (4×1), respectively,

$$U_t = (1, S_{t-1}, Z_{t-1}, Z_{t-1}S_{t-1})' = (1, Z_{t-1})' \otimes (1, S_{t-1})', \tag{A.10}$$

where $\otimes$ denotes the Kronecker product. U_t is an invertible linear transformation of:

$$U_t^{\star} = \left[(1-S_{t-1})\,(1-Z_{t-1})\,, S_{t-1}\,(1-Z_{t-1})\,, (1-S_{t-1})\,Z_{t-1}, S_{t-1}Z_{t-1}\right]', \tag{A.11}$$

that represents the four mutually exclusive dummies representing the four states of the process at time $t-1$, i.e., $[00,\ 10,\ 01,\ 11]'$. Given this parametrization, the conditions for strong one-step ahead non-causality are easily determined as restrictions on the parameter space.

To impose Granger non-causality (as in Definition 1), it is necessary that the dependence on S_{t-1} disappears in the second term of the decomposition. Thus, it is simply required that the parameters of the terms of U_t depending on S_{t-1} are equal to zero:

$$H_{S \nRightarrow Z} \quad (S \nRightarrow Z): \quad \beta_2 = \beta_4 = 0\ . \tag{A.12}$$

Under $H_{S \nRightarrow Z}$, S_{t-1} does not strongly cause one-step ahead Z_t given Z_{t-1}. The terms S_{t-1} and $S_{t-1}Z_{t-1}$ are excluded from U_t, hence $P(Z_t|S_{t-1}, Z_{t-1}) = P(Z_t|Z_{t-1})$.

Both hypotheses can be tested in a bivariate regime-switching model using a Wald test or a Likelihood ratio test. In the empirical analysis, bivariate regime-switching models have been estimated by maximum likelihood using the Hamilton's filter (Hamilton (1994)) and in all our estimations we compute the robust covariance matrix estimators (often known as the sandwich estimator) to calculate the standard errors (see Huber (1981) and White (1982)).

D Systemically important financial institutions

Another robustness check of our connectedness measures is to explore their implications for individual financial institutions. In this section we provide a simple comparison between the rankings of individual institutions according to our measures with the rankings based on subsequent financial losses. Consider first the Out-to-Other Granger-causality-network measure, estimated over the October 2002–September 2005 sample period. We rank all financial institutions based on this measure, and the 20 highest-scoring institutions are presented in Table A.1, along with the 20 highest-scoring institutions based on the maximum percentage loss (Max%Loss) during the crisis period from July 2007 to December 2008.[31] We find an overlap of 7 financial institutions between these two rankings.

In Table 8 we showed that in addition to Out-to-Other, Leverage and PCAS were also significant in predicting Max%Loss. Therefore, it is possible to sharpen our prediction by ranking financial institutions according to a simple aggregation of all three measures. To that end, we multiply each institution's ranking according to Out-to-Other, Leverage, and PCAS 1–20 by their corresponding beta coefficients from Table 8, sum these products, and then re-rank all financial institutions based on this aggregate sum. The 20 highest-scoring institutions according to this aggregate measure, estimated using date from October 2002–September 2005, are presented in Table A.1. In this case we find an overlap of 12 financial institutions (among the top 20) and most of the rest (among the top 30) with financial institutions ranked on Max%Loss. This improvement in correspondence and reduction in "false positives" suggest that our aggregate ranking may be useful in identifying systemically important entities.

[31] The first 11 financial institutions in Max%Loss ranking were bankrupt, therefore, representing the same Max%Loss equalled to 100%.

Out-to-Other	Aggregate Measure	Max Percentage Loss
WELLS FARGO & CO NEW	DEUTSCHE BANK AG	Perry Partners LP
PROGRESSIVE CORP OH	U B S AG	EDWARDS A G INC
BANK OF AMERICA CORP	FEDERAL NATIONAL MORTGAGE ASSN	Canyon Value Realization (Cayman) Ltd (A)
STEWART W P & CO LTD	Tomasetti Investment LP	C I T GROUP INC NEW
UNITEDHEALTH GROUP INC	LEHMAN BROTHERS HOLDINGS INC	Tomasetti Investment LP
INVESTMENT TECHNOLOGY GP INC NEW	C I G N A CORP	BEAR STEARNS COMPANIES INC
CITIGROUP INC	JEFFERIES GROUP INC NEW	ACE LTD
U B S AG	CITIGROUP INC	LEHMAN BROTHERS HOLDINGS INC
FEDERAL NATIONAL MORTGAGE ASSN	INVESTMENT TECHNOLOGY GP INC NEW	WASHINGTON MUTUAL INC
AMERICAN EXPRESS CO	LINCOLN NATIONAL CORP IN	Kingate Global Ltd USD Shares
AMBAC FINANCIAL GROUP INC	AMERICAN INTERNATIONAL GROUP INC	FEDERAL HOME LOAN MORTGAGE CORP
Kingate Global Ltd USD Shares	BEAR STEARNS COMPANIES INC	FEDERAL NATIONAL MORTGAGE ASSN
T ROWE PRICE GROUP INC	ACE LTD	RADIAN GROUP INC
JEFFERIES GROUP INC NEW	C I T GROUP INC NEW	AMERICAN INTERNATIONAL GROUP INC
X L CAPITAL LTD	WASHINGTON MUTUAL INC	AMBAC FINANCIAL GROUP INC
M B N A CORP	RAYMOND JAMES FINANCIAL INC	STEWART W P & CO LTD
M B I A INC	BANK OF AMERICA CORP	M G I C INVESTMENT CORP WIS
Graham Global Investment K4D-10	STEWART W P & CO LTD	WACHOVIA CORP 2ND NEW
AMERICAN INTERNATIONAL GROUP INC	PROGRESSIVE CORP OH	HARTFORD FINANCIAL SVCS GROUP IN
ACE LTD	HARTFORD FINANCIAL SVCS GROUP IN	X L CAPITAL LTD

Table A.1: Granger-causality-network-based measures for a sample of 100 financial institutions consisting of the 25 largest banks, broker/dealers, insurers, and hedge funds (as determined by average AUM for hedge funds and average market capitalization for broker/dealers, insurers, and banks during the time period considered) for the sample period from October 2002 to September 2005. Only the 20 highest-scoring institutions based on Out-to-Other and aggregate measures are displayed. The aggregate measure is an aggregation of the Out-to-Other, Leverage and PCAS 1-20 measures. The maximum percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007. All connections are based on Granger-causality-network statistics at the 5% level of statistical significance.

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