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Implied volatilities of American options with cash dividends: an application to Italian Derivatives Market (IDEM)

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Abstract. In this contribution, we study options on assets which pay discrete dividends. We focus on American options, as when dealing with equities, most traded options are of American-type. In particular, we analyze implied volatilities in the model proposed by Haug *et al.* [12] and in the binomial model, with an application to the Italian Derivatives Market.

Keywords: Options on stocks, discrete dividends, implied volatilities.

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1 Introduction

Evaluation of options on stocks which pay dividends is an important problem from a practical viewpoint, which has received a lot of attention in the financial literature, but has not been settled in a satisfactory way. Different methods have been proposed for the pricing of both European and American options (for a survey we may refer to Haug [10]). Haug and Haug [11], Beneder and Vorst [1], Bos *et al.* [3], and Bos and Vandermark [4]) propose volatility adjustments which take into account the timing of the dividend; de Matos *et al.* [6] derive arbitrarily accurate lower and upper bounds for the value of European options on a stock paying a discrete dividend. Haug *et al.* [12] provide an integral representation formula that can be considered the exact solution to problems of evaluating both European and American call options and European put options.

The effect of a discrete dividend payment on American option prices is different than for European options. While for European-style options the pricing problem basically arises from mis-specifying the variance of the underlying process, for American options the impact on the optimal exercise strategy is more important. As well known, it is never optimal to exercise an American call option on non-dividend paying stocks before maturity. As a result, the American call has the same value as its European counterpart. In the presence of dividends, early exercise is optimal when it leads to an alternative income stream, i.e. dividends from the stock for a call and interest rates on cash for a put option. In the case of a discrete dividend, the call option may be optimally exercised right before the ex-dividend date, while for a put it may be optimal to exercise at any time till maturity. Simple adjustments like subtracting the present value of the dividend from the asset spot price make little sense for American options.

Approximations to the value of an American call on a dividend paying stock have been suggested by Black [2] (this is basically the *escrowed* dividend method), and by Roll [15], Geske [8] and [9], and Whaley [18] (hence we will refer to the RGW model). Nevertheless, the RGW model does not yield good results in many cases of practical interest. Moreover, in some instances it may allow for arbitrage opportunities (as also pointed out in Haug *et al.* [12]).

Lattice methods (Cox *et al.* [5]) are commonly used for the pricing of both European and American options. The evaluation of options using binomial methods is particularly easy to implement and efficient at standard conditions, but when assuming discrete dividends it becomes computationally intensive. In the absence of dividends, or when dividends are assumed proportional to the stock price, the binomial tree reconnects. As a result, the number of nodes at each step grows linearly. The hypothesis of a proportional dividend yield can be accepted as an approximation of dividends paid in the long term, but it is not acceptable in a short period of time during which the stock pays a dividend in cash and its amount is often known in advance or estimated with appropriate accuracy. If during the life of the option a dividend of amount D is paid, at each node after the ex-dividend date the tree is no-longer recombing and a new binomial tree has to be generated and evaluated. As a consequence, the total number of nodes increases considerably. Schroder [16] describes how to implement discrete dividends in a recombining tree. The approach is based on the escrowed dividend process idea, but the method leads to significant pricing errors.

2 Valuing equity options in the presence of a single cash dividend

We assume that dividends are a pure cash amount D to be paid at a specified ex-dividend date t_D. Empirically, one observes that at the ex-dividend date the stock price drops. In order to exclude arbitrage opportunities, the jump in the stock price should be equal to the size of the net dividend. Dividend payments during the life of the option imply lower call and higher put premia.

Dividends affect option prices through their effect on the underlying stock price. Since in the case of cash dividends we cannot use the proportionality argument, the price dynamics depends on the timing of the dividend payment. In a continuous time setting, the underlying price dynamics is assumed to satisfy the following stochastic differential equation

$$\begin{aligned} dS_t &= rS_t dt + \sigma S_t dW_t \qquad t \neq t_D \\ S_{t_D}^+ &= S_{t_D}^- - D_{t_D} \, , \end{aligned} \tag{1}$$

where $S_{t_D}^-$ and $S_{t_D}^+$ denote the stock price an instant before and after the jump at time t_D, respectively. Due to this discontinuity, the solution to equation (1) is no longer lognormal but in the form

$$S_t = S_0 e^{(r-\sigma^2/2)t+\sigma W_t} - D_{t_D} e^{(r-\sigma^2/2)(t-t_D)+\sigma W_{t-t_D}} I_{\{t \geq t_D\}} \, , \tag{2}$$

where I_A denotes the indicator function of A.

Haug *et al.* [12] (henceforth HHL) derived an exact expression for the fair price of a European call option on a cash dividend paying stock. The basic idea is that after the dividend payment, option pricing reduces to simple Black-Scholes formula for a non-dividend paying stock. Before t_D one considers the discounted expected value of the BS formula adjusted for the dividend payment. In the geometric Brownian motion setup, the HHL formula is

$$c_{HHL}(S_0, T; D, t_D) = e^{-rt_D} \int_d^\infty c_{BS}(S_x - D, T - t_D) \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \, , \tag{3}$$

where $d = \frac{\log(D/S_0)-(r-\sigma^2/2)t_D}{\sigma\sqrt{t_D}}$, $S_x = S_0 e^{(r-\sigma^2/2)t_D+\sigma\sqrt{t_D}x}$, and $c_{BS}(S_x - D, T - t_D)$ is the BS formula with time to maturity $T - t_D$. The price of a European put option with a discrete dividend can be obtained by exploiting put-call parity results.

For the American call option, since early exercise is only optimal instantaneously prior to the ex-dividend date, one can merely replace relation (3) with

$$C_{HHL}(S_0, T; D, t_D,) = e^{-rt_D} \int_d^\infty \max\left\{S_x - X, c_{BS}(S_x - D, T - t_D)\right\} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \, . \tag{4}$$

For American put options, early exercise may be optimal even in the absence of dividends. Since no analytical solutions for both the option price and the exercise strategy are available, one is generally forced to numerical solutions, such as lattice approaches.

A method which performs very efficiently and can be applied to both European and American call and put options is a binomial method[1] which maintains the recombining feature and is based on an interpolation idea proposed by Vellekoop and Nieuwenhuis [17] (see also Nardon and Pianca [14]).

For an American option, the method can be described as follows: a standard binomial tree is constructed without considering the payment of the dividend (with $S_{ij} = S_0 u^j d^{i-j}$, $u = e^{\sigma\sqrt{T/n}}$, and $d = 1/u$), then it is evaluated by backward induction from maturity until the dividend payment; at the node corresponding to an ex-dividend date (at step n_D), we approximate the continuation value V_{n_D} using the following linear interpolation

$$V(S_{n_D,j}) = \frac{V(S_{n_D,k+1}) - V(S_{n_D,k})}{S_{n_D,k+1} - S_{n_D,k}} \left(S_{n_D,j} - S_{n_D,k}\right) + V(S_{n_D,k})\,, \tag{5}$$

for $j = 0, 1, \ldots, n_D$ and $S_{n_D,k} \leq S_{n_D,j} \leq S_{n_D,k+1}$; then continue backward along the tree.

Such a method can be easily extended to the valuation of option with multiple dividends.

Let us observe that numerical problems may arise when the dividend is paid very close to the evaluation date, due to the fact that at the early stages of the tree the number of nodes may be not sufficient to compute interpolation (5). Ad hoc solutions have to be used. Such solutions have to take into account that the stopping and continuation regions and the early exercise strategies (and the early exercise boundaries) are different for American put and call options.

Another problem is related to the fact that in some cases, in particular when dividends are too high, negative prices may arise. As a solution, we have imposed an absorbing barrier at zero: when the dividend is higher than the underlying price, the ex-dividend underlying price is set at zero (and the dividend is not fully paid due to limited liability).

3 Implied volatilities when the underlying asset pays discrete dividends

Usually, derivative pricing theory assumes that stocks pay known dividends, both in size and timing. Moreover, new dividends are often supposed to be equal to the former ones. Even if these assumptions might be too strong, in this work we assume that we know both the amount of dividends and times in which they are paid.

Formulas (3) and (4) can be used to derive implied volatilities and implied dividends from market data. In particular, formula (4) can be numerically inverted in order to compute the implied volatilities from the prices of American equity options of the Italian Derivatives Market (IDEM). We apply such a procedure to obtain implied volatilities of the stock prices in FTSEMIB index.

Let us observe also that dividend policies are not uniform for all the assets in FTSEMIB index. With reference to the year 2008 (hence when considering the dividend which will be paid in 2009), there are some firms that pay no dividends at all (this choice has been

[1] The interpolation procedure here described can be applied also to other numerical schemes, such a finite difference schemes for the pricing of European and American options.

justified by the difficulties implied by the recent financial crisis); for example FIAT does not pay dividends in 2009. Some firms pay a dividend (or the remainder of the dividends already paid in the end of 2008): on 21 September ENI pays a dividend of 0.50 euros, and Parmalat pays a dividend of 0.041 euros, ENEL pays a dividend of 0.10 euro on 23 November; such payments are an anticipation of the dividends for the year 2009. Dividends can be paid in cash: normally in euro, but sometimes dividends are also given in dollars (such as STM), hence one has to evaluate currency risk. Alternatively dividends are paid issuing new shares of stock (in a number which is proportional to the shares already held), or could be a mixture of stocks and cash. For example, on 18 May Unicredit distributed 29 new shares of stock for every 159 shares already owned. Generali pays a cash dividend of $D = 0.15$ and moreover one share of stock is distributed every 25 shares already owned.

Taking into account in the model all such different dividend policies is a tough task. In particular, if we want to extend the model to multiple dividends (this is the case of dividends paid twice a year), this can be done but at a higher computational cost. For example STM pays dividends quarterly.

In the next section, some examples are discussed.

4 Empirical experiments

In this section we analyze implied volatilities of American options of the Italian derivatives market, written on stocks which pay one or two dividends during during the life of the option.

It is worth noting that the computation and numerical inversion of both formulas (3) and (4) entail some drawbacks concerning the approximation of the integral in order to obtain accurate results. In particular, difficulties arise when considering dividends paid very near in the future or very close to the option's maturity. Truncation of the integral domain has also to be chosen carefully.

In the numerical experiments we have used both HHL method and the interpolated binomial approach in order to obtain prices and volatilities of American call options written on single dividend paying stock. Whereas in the case of American put options and multiple dividends we used only the interpolated binomial method.

4.1 Single dividend

As a first example, we have considered American call and put options written on ENI stock, with maturity 18 December 2009. At the evaluation date t, where t is the 1 July, the underlying price is $S_t = 17.2$. A dividend $D = 0.50$ will be paid on 21 September 2009. The ex-dividend date is $t_D = 0.2247$ and time to maturity $\tau = T - t = 0.4658$. The risk-free interest rate is assumed to be $r = 0.02$.

Implied volatilities are obtained by numerically inverting the interpolated binomial method with 10 000 steps. In the computations we have considered option prices calculated as an average between the bid and ask prices. The results for different strike prices are reported in tables 1 and 2.

Let us consider the American call and put options written on ENEL stock, which pays part of the annual dividend in November. The options expire on the 18 December 2009. We have considered two trading dates: the 23 October and 6 November 2009. Option prices for different strike prices are reported in tables 3 and 4 (we considered bid and ask prices and also the average price).

At the evaluation date 23 October, the underlying price is $S_t = 4.193$. A dividend $D = 0.1$ is paid on the 23 November ($t_D - t = 0.0849$). The time to maturity is $T - t = 0.1534$. The risk-free interest rate is assumed to be $r = 0.005$. Implied volatilities are obtained in the interpolated binomial method with 1 000 steps. Figures 1 and 2 show the results.

Let us observe that, in some cases it was not possible to determine the implied volatilities due to mispricing of options. For example, the first five bid prices in table 3 for the call options are lower than the immediate exercise value $S_t - X$. Whereas the last three bid prices for the put options violates the condition[2]

$$P_t \geq \max\left(De^{-r(t_D-t)} + Xe^{-r(T-t)} - S_t,\, 0\right) . \tag{6}$$

At the evaluation date 6 November, the underlying price is $S_t = 4.082$. The time to maturity is $T - t = 0.1151$ and $t_D - t = 0.0466$. The risk-free interest rate is assumed to be $r = 0.005$. Implied volatilities are obtained in the interpolated binomial method with 2 000 steps. Figures 3 and 4 show the results.

Consider now the American call and put options written on STM stock, which pays a dividend $D = \$\,0.03$ on the 23 November. Let the euro/dollar exchange rate be approximately 1.5, then the dividend in euros is $D = 0.02$. The options expire on the 18 December 2009. As in previous case, we have considered two trading dates: the 2 and 6 November 2009. Option prices for different strike prices are reported in tables 5 and 6 (we considered bid, ask and average prices). The risk-free interest rate is assumed to be $r = 0.005$. Implied volatilities are obtained in the interpolated binomial method with 1 000 steps. The results are shown in figures 5- 8.

4.2 Multiple discrete dividends

The interpolated binomial method can be easily implemented also in the case of multiple dividends (see Nardon and Pianca [14]).

As an example, we have considered American call and put options written on STM stock, with maturity 18 December 2009. During the trading day t, where t is the 15 July, the underlying price is $S_t = 5.495$. The time to maturity is $\tau = T - t = 0.4274$.

STM pays dividends quarterly; dividends are in dollars. Annual dividend is 0.12 dollars; a dividend $D = \$\,0.03$ will be paid in August and in November 2009. The ex-dividend dates are: 24 August ($t_1 = 0.1096$) and 23 November $t_2 = 0.3589$. Let the euro/dollar exchange rate be approximately 1.4 and assuming that it remains constant over the life of the option[3], then the dividends are $D_1 = 0.0214286$ and $D_2 = 0.0214286$. The risk-free interest rate is assumed to be $r = 0.01$.

[2] Note that the condition holds in a frictionless market.

[3] This assumption seems to be not realistic, anyway in this example we consider a constant exchange rate. In the interpolated binomial method dividends need not to be of the same amount.

Implied volatilities are obtained by numerically inverting the interpolated binomial method with 10 000 steps. In the computations we have considered option prices calculated as an average between the bid and ask prices. The results for different strike prices are reported in tables 7 and 8.

5 Concluding remarks and further research

In this contribution, we studied American options on stocks which pay discrete dividends. In particular, we obtained implied volatilities considering the prices of options which trade on the Italian Derivatives Market.

Due to the computational efforts required by the method, and the fact the dividend policies are differentiate, one may wonder if it is possible to obtain implied volatilities which are not model-based, but derived using only market price of traded options, as for *variance swaps* (see Demeterfi *et al.* [7], and Jiang and Tian [13]). Along this line, a procedure which computes a volatility index is used by CBOE for the calculation of VIX.

As further research, pricing models in the presence of discrete dividends can also be extended in order to consider stochastic volatility, jumps and stochastic interest rates, non standard payoffs. Exotic options trade in OTC equity markets and are also embedded in warrants and other derivatives. The European and American options with cash dividends could also be used to value real options (e.g. real investment opportunities), when the underlying offers known discrete payouts.

References

[1] Beneder R., Vorst T. (2001), Options on dividend paying stocks. In: *Proceedings of the International Conference on Mathematical Finance*, Shanghai 2001

[2] Black F. (1975), Fact and fantasy in the use of options. *Financial Analysts Journal*, July-August, 36–72

[3] Bos R., Gairat A., Shepeleva A. (2003), Dealing with discrete dividends *Risk*, **16**, 109–112

[4] Bos R., Vandermark S. (2002), Finessing fixed dividends. *Risk*, **15**, 157–158

[5] Cox J.C., Ross S.A., Rubinstein M. (1979), Option pricing: a simplified approach. *Journal of Financial Economics*, **7**, 229–263

[6] de Matos J.A., Dilao R., Ferreira B. (2006), *The exact value for European options on a stock paying a discrete dividend.* Munich Personal RePEc Archive

[7] Demeterfi K.E., Derman E., Kamal M., Zou J. (1999), *More than You ever wanted to know about volatility swaps*, Quantitative Strategies Reseach Notes, NEGE, University of Minho, Goldman Sachs

[8] Geske R. (1979), The valuation of compound options. *Journal of Financial Economics*, **7**, 63–81

[9] Geske R. (1981), Comments on Whaley's note. *Journal of Financial Economics*, **9**, 213–215

[10] Haug, E.S. (2007), *The Complete Guide to Option Pricing Formulas.* McGraw-Hill, New York

[11] Haug E.S., Haug J. (1998), *A new look at pricing options with time varying volatility.* Working paper

[12] Haug E.S., Haug J., Lewis A. (2003), Back to basics: a new approach to discrete dividend problem. *Willmot Magazine*, **9**

[13] Jiang G.J., Tian Y.S. (2007), Extracting model-free volatility from option prices: an examination of the VIX index, *Journal of Derivatives*, **14**, 35–60

[14] Nardon M., Pianca P. (2009), Binomial algorithms for the evaluation of options on stocks with fixed per share dividends. In: Corazza M., Pizzi C. [eds], *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Springer-Italy

[15] Roll R. (1977), An analytical formula for unprotected American call options on stocks with known dividends. *Journal of Financial Economics*, **5**, 251–258

[16] Schroder M. (1988), Adapting the binomial model to value options on assets with fixed-cash payouts. *Financial Analysts Journal*, **44**, 54–62

[17] Vellekoop, M.H., Nieuwenhuis, J.W. (2006), Efficient pricing of derivatives on assets with discrete dividends. *Applied Mathematical Finance*, **13**, 265–284

[18] Whaley R.E. (1981), On the evaluation of American call options on stocks with known dividends. *Journal of Financial Economics*, **9**, 207–211

Table 1: Implied volatilities of American call options written on ENI with maturity 18 December 2009 ($S_t = 17.2$, $D = 0.50$, t_D = 21 September 2009)

X	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	21
$\hat{\sigma}$	0.3367	0.3222	0.3281	0.3027	0.2911	0.2656	0.2792	0.3126	0.2678	0.2642	0.2608	0.2553

Table 2: Implied volatilities of American put options written on ENI with maturity 18 December 2009 ($S_t = 17.2$, $D = 0.50$, t_D = 21 September 2009)

X	15	15.5	16	16.5	17	17.5	18	18.5	19
$\hat{\sigma}$	0.3115	0.3033	0.3026	0.2872	0.2789	0.2715	0.2592	0.2584	0.2553

Table 3: American call and put options prices on ENEL with maturity 18 December 2009, with $S_t = 4.193$, $D = 0.10$, t = 23 October ($T - t = 0.1534$) t_D = 23 November ($t_D = 0.0849$)

Am. call strike	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.0
Bid	0.7780	0.6890	0.5655	0.4710	0.3810	0.2985	0.2260	0.1025	0.0340	0.0065	0.0005	0.0005
Ask	0.9080	0.8090	0.6280	0.5285	0.4335	0.3370	0.2445	0.1130	0.0435	0.0175	0.0450	0.0200
Average	0.8430	0.7490	0.5968	0.4998	0.4073	0.3178	0.2353	0.1078	0.0388	0.0120	0.0228	0.0103
Am. put strike	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.0
Bid	0.0090	0.0155	0.0220	0.0315	0.0450	0.0655	0.0950	0.1880	0.3250	0.4865	0.6835	0.8790
Ask	0.0200	0.0215	0.0290	0.0370	0.0515	0.0720	0.1020	0.1975	0.3450	0.5365	0.7465	0.9460
Average	0.0145	0.0185	0.0255	0.0343	0.0483	0.0688	0.0985	0.1928	0.3350	0.5115	0.7150	0.9125

Table 4: American call and put options prices on ENEL with maturity 18 December 2009, with $S_t = 4.082$, $D = 0.10$, $t = 6$ November $(T - t = 0.1151)$ $t_D = 23$ November $(t_D = 0.0466)$

Am. call strike	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.8
Bid		0.6445	0.5455	0.4475	0.3585	0.2680	0.1900	0.1270	0.0360	0.0060	0.0005	0.0005
Ask		0.7210	0.6205	0.5205	0.4255	0.3235	0.2230	0.1390	0.0435	0.0155	0.0450	0.0450
Average		0.6828	0.5830	0.4840	0.3920	0.2958	0.2065	0.1330	0.0398	0.0108	0.0228	0.0228
Am. Put strike	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.8
Bid	0.0075	0.0085	0.0130	0.0195	0.0310	0.0500	0.0780	0.1200	0.2425	0.3905	0.5775	0.7745
Ask	0.0200	0.0200	0.0240	0.0310	0.0425	0.0615	0.0855	0.1260	0.2540	0.4535	0.6520	0.8515
Average	0.0138	0.0143	0.0185	0.0253	0.0368	0.0558	0.0818	0.1230	0.2483	0.4220	0.6148	0.8130

Table 5: American call and put options prices on STM with maturity 18 December 2009, with $S_t = 5.49$, $D = 0.02$, $t = 2$ November $(T - t = 0.1260)$ $t_D = 23$ November $(t_D = 0.0575)$

Am. call strike	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4
Bid						0.3305	0.2345	0.1600	0.1050	0.0660	0.0390
Ask						0.3640	0.2670	0.1890	0.1350	0.0970	0.0695
Average						0.3473	0.2508	0.1745	0.1200	0.0815	0.0543
Am. put strike	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4
Bid	0.0220	0.0405	0.0700	0.1130	0.1750	0.2565	0.3625	0.4845	0.6265	0.7845	0.9555
Ask	0.0505	0.0710	0.1010	0.1450	0.2065	0.2890	0.3945	0.5190	0.6665	0.8295	1.0040
Average	0.0363	0.0558	0.0855	0.1290	0.1908	0.2728	0.3785	0.5018	0.6465	0.8070	0.9798

Table 6: American call and put options prices on STM with maturity 18 December 2009, with $S_t = 5.5.63$, $D = 0.02$, $t = 6$ November $(T - t = 0.0.1151)$ $t_D = 23$ November $(t_D = 0.0466)$

Am. call strike	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6
Bid				0.5275	0.3925	0.2790	0.1860	0.1195	0.0740	0.0405	0.0200
Ask				0.5505	0.4110	0.2930	0.2020	0.1335	0.0905	0.0715	0.0505
Average				0.5390	0.4018	0.2860	0.1940	0.1265	0.0823	0.0560	0.0353
Am. put strike	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6
Bid	0.0175	0.0360	0.0655	0.1145	0.1740	0.2600	0.3680	0.4980	0.6495	0.8100	0.9870
Ask	0.0475	0.0655	0.0955	0.1315	0.1915	0.2795	0.3935	0.5320	0.6870	0.8590	1.0395
Average	0.0325	0.0508	0.0805	0.1230	0.1828	0.2698	0.3808	0.5150	0.6683	0.8345	1.0133

Table 7: Implied volatilities of American call options written on STM with maturity 18 December 2009 ($S_t = 5.495$, $D_i = \$\,0.03$, $i = 1, 2$, $t_i = 24$ August and 23 November 2009)

X	4	4.2	4.4	4.6	4.8	5	5.2	5.4	5.8	6	6.2	6.4	6.6
$\hat{\sigma}$	0.6568	0.7344	0.6054	0.5904	0.5854	0.5770	0.5717	0.5641	0.5486	0.5420	0.5352	0.5301	0.5264

Table 8: Implied volatilities of American put options written on STM with maturity 18 December 2009 ($S_t = 5.495$, $D_i = \$\,0.03$, $i = 1, 2$, $t_i = 24$ August and 23 November 2009)

X	3.9	4	4.2	4.4	4.6	4.8	5	5.2	5.4	5.8	6	6.2	6.4
$\hat{\sigma}$	0.6658	0.6561	0.6428	0.6296	0.6201	0.6063	0.5672	0.5782	0.5710	0.5692	0.5631	0.5555	0.5523

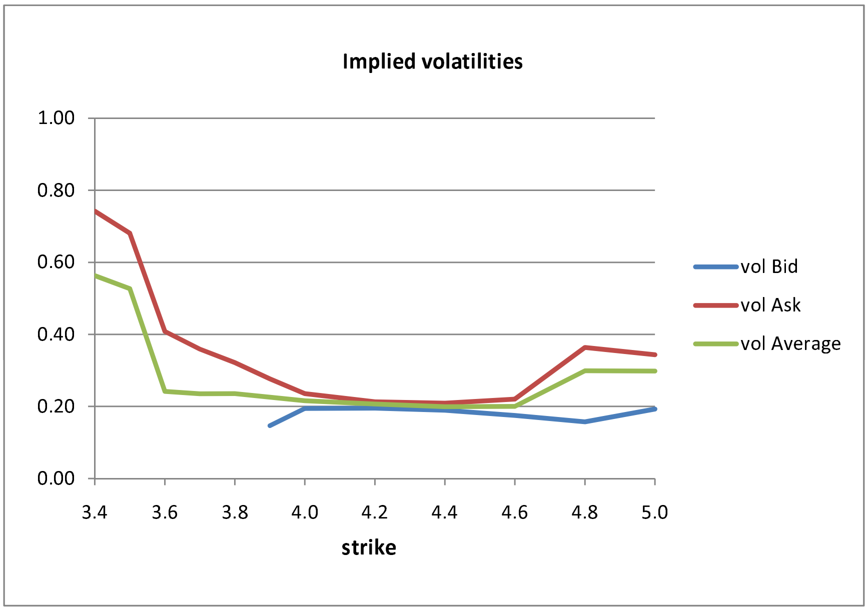

Figure 1: Implied volatilities of American call options prices on ENEL with maturity 18 December 2009, with $S_t = 4.193$, $D = 0.10$, $t = 23$ October, $t_D = 23$ November

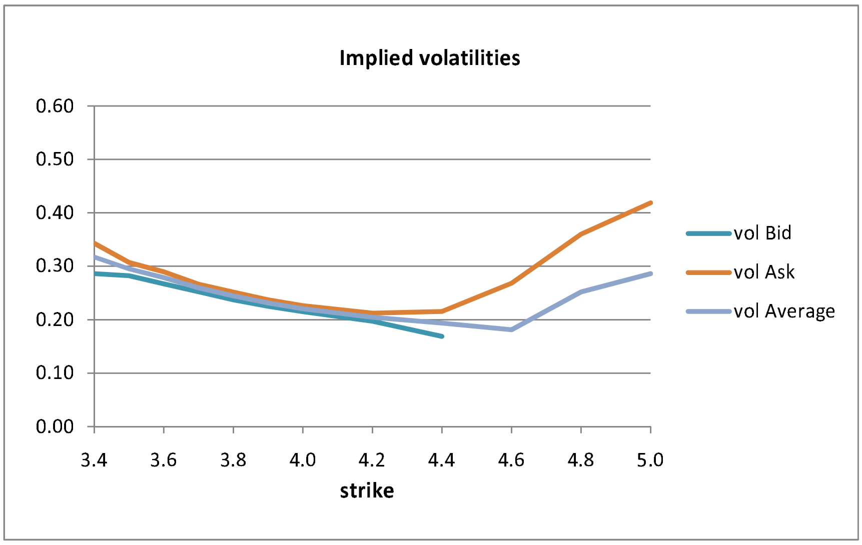

Figure 2: Implied volatilities of American put options prices on ENEL with maturity 18 December 2009, with $S_t = 4.193$, $D = 0.10$, $t\ = 23$ October, $t_D\ = 23$ November

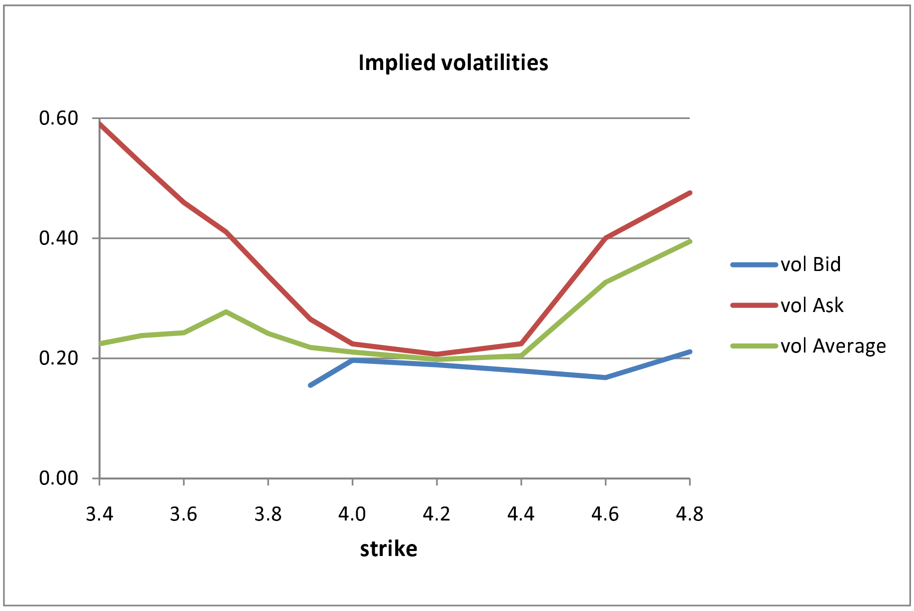

Figure 3: Implied volatilities of American call options prices on ENEL with maturity 18 December 2009, with $S_t = 4.082$, $D = 0.10$, $t\ = 6$ November, $t_D\ = 23$ November

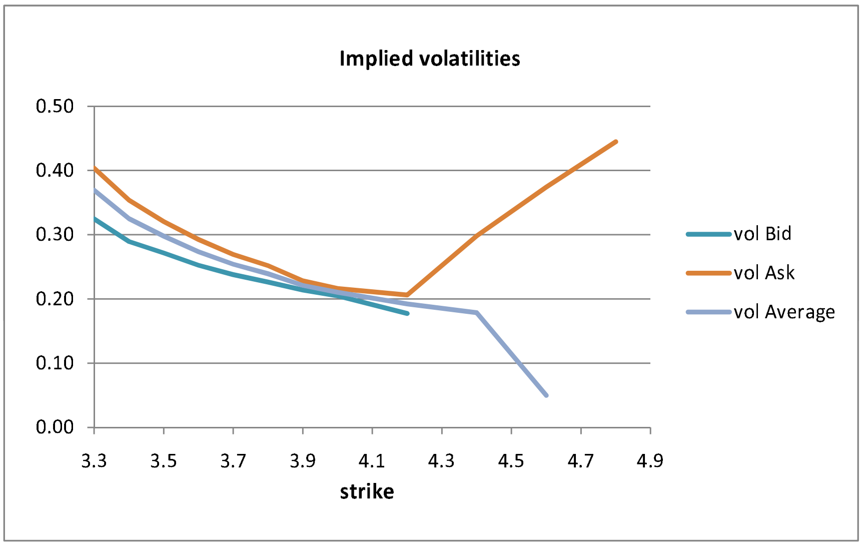

Figure 4: Implied volatilities of American put options prices on ENEL with maturity 18 December 2009, with $S_t = 4.082$, $D = 0.10$, $t = 6$ November, $t_D = 23$ November

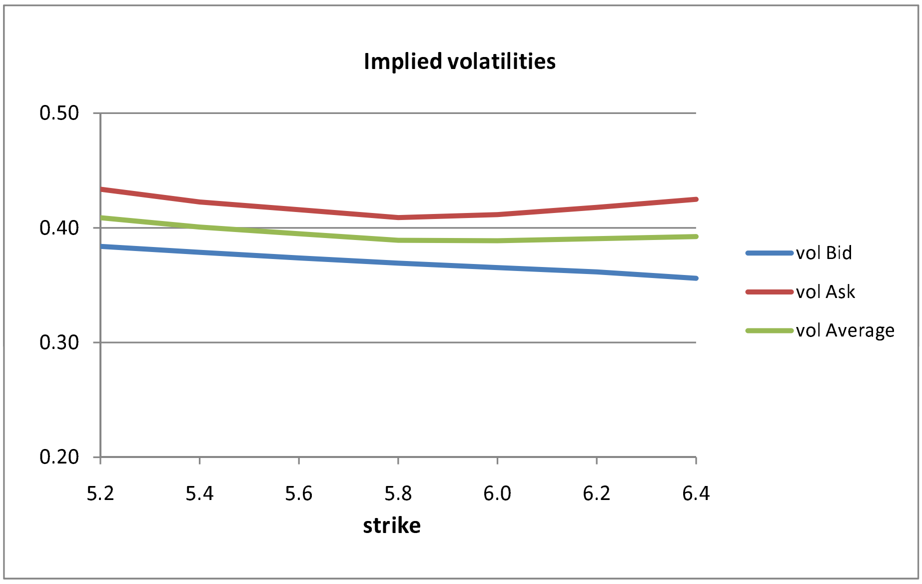

Figure 5: Implied volatilities of American call options prices on STM with maturity 18 December 2009, with $S_t = 5.49$, $D = 0.02$, $t\ = 2$ November, $t_D\ = 23$ November

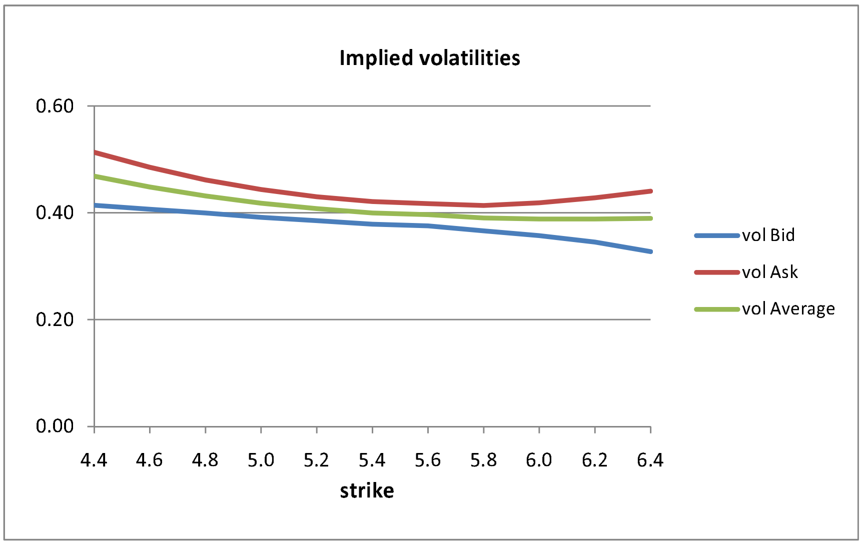

Figure 6: Implied volatilities of American put options prices on STM with maturity 18 December 2009, with $S_t = 5.49$, $D = 0.02$, $t = 2$ November, $t_D = 23$ November

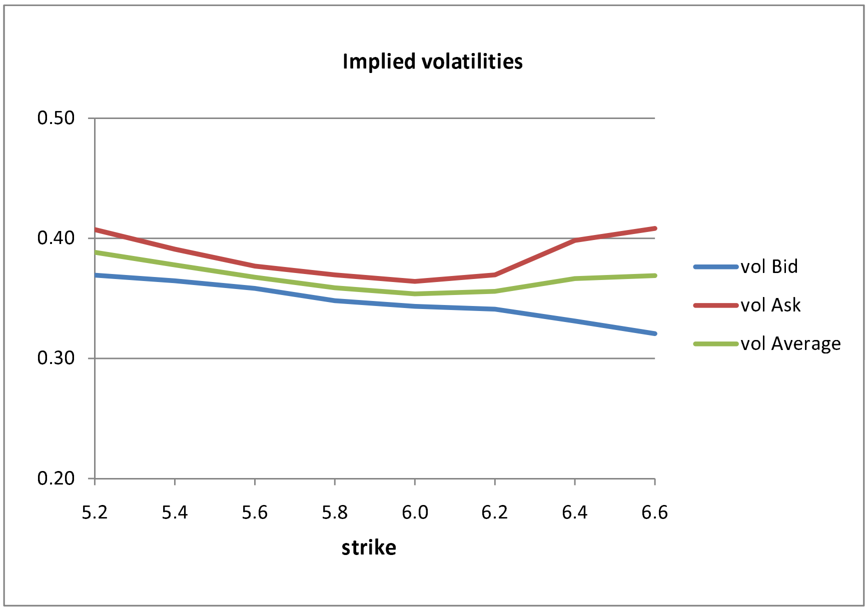

Figure 7: Implied volatilities of American call options prices on STM with maturity 18 December 2009, with $S_t = 5.63$, $D = 0.02$, $t = 6$ November, $t_D = 23$ November

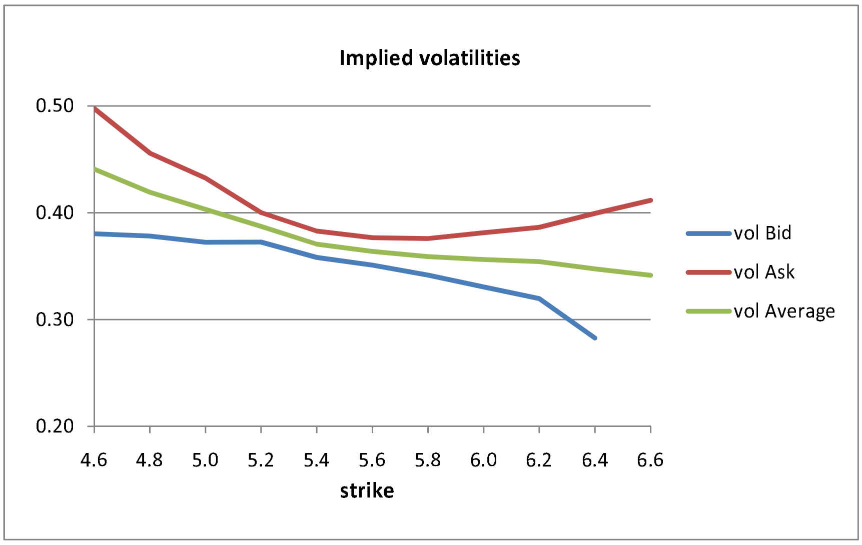

Figure 8: Implied volatilities of American put options prices on STM with maturity 18 December 2009, with $S_t = 5.63$, $D = 0.02$, $t\ = 6$ November, $t_D\ = 23$ November