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# INNOVATION BY LEADERS*

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A new rationale for the persistence of monopolies is based on a precommitment of the incumbent monopolist to invest in R&D. In a patent race, as long as entry is free, the Arrow effect disappears: the incumbent has more incentives to invest than any outsider. Paradoxically, a market with some persistence of monopoly is competitive, while one with continuous leapfrogging must hide some barriers to entry. When the size of innovations is endogenous, leaders invest in more radical innovations. If there is a sequence of innovations, cycling investment emerges. Finally, I apply the idea to a general equilibrium model of Schumpeterian growth with persistence of monopoly.

Who does research? Overwhelming evidence tells us that incumbent monopolists do a lot of research and their leadership persists through a number of innovations. This persistence of the monopolistic position drives the incentives to invest in Research & Development and indirectly enhances aggregate growth. Nevertheless the industrial organisation theory of innovation since the pathbreaking contribution of Arrow (1962) and the macroeconomic theory of Schumpeterian growth started by Aghion and Howitt (1992) do not provide clear arguments as to why leaders should innovate. Under free competition, the theory implies that leaders do not invest at all and a process of continuous leapfrogging between firms should characterise markets with technological progress. This paper provides a new rationale for the persistence of monopolistic positions and evaluates some of its microeconomic and macroeconomic consequences.

The literature on patent races has studied equilibrium outcomes in the market for innovations starting with Loury (1979) and Dasgupta and Stiglitz (1980).[1] The standard hypotheses of this literature assume decreasing returns to scale, fixed costs of innovations and a flow of investment in R&D (where either of the two may determine the arrival rate of innovations) and Nash-Cournot competition between firms. The participants of the patent race are the current monopolist of the market, who has a patent on the leading-edge quality of the product and a number of entrant firms trying to replace the patentholder. The main result is that the patentholder does less research than any other entrant and zero research under free entry because its incentives to invest in R&D are lower due to the Arrow (1962) effect: the expected gain of the patentholder is just the difference between expected profits obtained with the next technology and the current one, while the

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[1] See also Lee and Wilde (1980), Reinganum (1983, 1985*a*) and Beath *et al.* (1989) for important developments.

expected gain for each outsider is given by all the expected profits obtained with the next technology. Moreover, the investment of each firm, the equilibrium number of firms under the free-entry assumption and the aggregate investment in R&D are inefficient. Despite a present business-stealing effect *à la* Mankiw and Whinston (1986) creating a tendency toward too many firms, the discrepancy between private and social value of innovations creates an opposite tendency (since the latter is typically greater than the former), leaving the overall effect ambiguous.

The fact that patentholders do not invest in R&D implies a continuous leap-frogging and no persistence of monopolistic positions between one innovation and another, which is a quite counterintuitive picture of what is going on in the real world.[2] A number of solutions to this paradox have been proposed, most of which are based on some technological advantage of the patentholder – for a survey, see Tirole (1988, Ch.10) - under such an advantage, the incentives of the current monopolist to invest in R&D rise and eventually counterbalance the Arrow effect. Despite these are reasonable explanations for the puzzle, they do not seem to tell the whole story, as we see monopolists investing in R&D even if they do not have a consistent technological advantage to the outsiders. This paper develops a new rationale for the persistence of a monopoly which is based on two ingredients: Stackelberg competition and free entry.

I study a patent race where the patentholder has the opportunity to make a strategic precommitment to a level of investment in R&D. This may happen through a specific investment in laboratories and related equipment for R&D, by hiring researchers or in a number of other ways. In the case of 'contractual costs' of R&D, that is, when a fixed initial investment determines the arrival rate of the innovation, the interpretation of a strategic precommitment for the incumbent monopolist is very standard. The leader can choose to invest before the other firms and, since the leader is by definition the firm who has discovered the latest technology, it is reasonable to assume that such a discovery was associated with a first mover advantage in the following patent race. More generally, our strategic assumption seems a natural one since the patentholder can be easily seen in a different perspective from all other entrants in the patent race. Moreover the first mover advantage could be a consequence of an even small technological advantage, so that our arguments should be seen as complementary to those based on technological advantages.

I first consider a single patent race with drastic innovations where each participant invests a flow of resources until the new technology is discovered. For a given number of firms, a patent race based on Stackelberg competition delivers an equilibrium in which the patentholder invests less than each other entrant. This happens because the investment choices are strategic complements as defined by Bulow *et al.* (1985). Hence, the leader uses the first mover advantage to induce a

[2] The empirical study by Blundell *et al.* (1999) witnesses a positive relationship between market power and innovation activity which is consistent with pre-emptive investment in R&D by the leaders. This result holds in a panel data with many sectors but especially in the pharmaceutical sector, which is a sector with a high R&D/sales ratio, strong patent protection and where firms typically recognise that they are in races to develop innovations.

reduction in the investment of the outsiders, accomplished by reducing its own investment in R&D below the Nash level. Overall investment is reduced and the lifespan of the current patent is lengthened. The Arrow effect is strengthened under Stackelberg competition and barriers to entry.

Under free entry, the results are completely changed and induce the main result of the paper. Indeed, the leader patentholder always invests in R&D and more so than any other firm, thus the Stackelberg assumption with free entry delivers a new rationale for the investment by incumbent monopolists in R&D and for the persistence of a monopoly.[3] The intuition is simple if one realises that the incumbent is now taking as given the probability of an innovation in the market, since any profitable opportunity for doing R&D left open by the leader will be seized by new entrants until the profits are zero. Hence the investment of the leader does not affect the expected lifespan of the current patent and the Arrow effect disappears. Now the leader can just use the first mover advantage to adopt the profit maximising investment for a given aggregate probability of innovation in the market. This is higher than the investment chosen by the entrants because the entrants take into account the effect of their investment on the aggregate probability of innovation, since they play Nash between themselves. This effect is positive and increases the effective discount on the net expected value of winning the patent race, reducing the investment of the entrants.[4] If we believe that Stackelberg competition is the right assumption in the study of patent races, we obtain very strong conclusions from this analysis. A market characterised by some persistence of monopoly is competitive, while one with continuous leapfrogging must be characterised by some barriers to entry! This is exactly the opposite conclusion to the one we obtain by assuming Nash competition, so we need to be very careful in deriving policy prescription from models of innovation if we are not sure of the market context in which they apply.

The paper is organised as follows. Section 1 describes the basic model, derives the equilibrium under Nash competition and compares it with the one under Stackelberg competition, obtaining our main results. Section 2 extends the basic model in many dimensions, such as nondrastic innovations, contractual costs of R&D, endogenous size of innovations and sequences of innovations and applies the last extension to describe the evolution of a sector subject to innovations in partial equilibrium and a Schumpeterian model of growth in general equilibrium. Section 3 concludes. All the technical details are left in the Appendix.

[3] The empirical results of Blundell *et al.* (1999) 'are in line with models where high market share firms have greater incentives to pre-emptively innovate'. Their conclusion is rather explicit: 'It is often asserted that the superior performance of large firms in innovating is because they have higher cash flows from which to finance investment in R&D. Our findings suggest that this is not the whole story - dominant firms innovate because they have a relatively greater incentive to do so. Firms with high market shares who innovate get a higher valuation *on the stock market* than those who do not.'

[4] The aggressive behaviour of the leader is an application of a more general result pointed out in Etro (2002*a*). In any symmetric model of Stackelberg competition with a fixed number of leaders and an endogenous number of entrants, each leader behaves aggressively. For instance, under quantity competition a leader produces more than each follower (and under weak conditions it completely deters entry), while under price competition with differentiated products a leader sets lower prices than any entrant (the opposite to that with a fixed number of firms since prices are strategic complements). The results holds also in presence of asymmetries between leaders and followers as in this paper.

## 1. The Model

Consider a market in which a monopolist with a patent on the leading edge technology is obtaining a flow of profits $\pi \in \mathcal{R}_{++}$ but a superior technology, if discovered, would give the right to a new patent whose value is $V \in \mathcal{R}_{++}$. I assume that the innovation is drastic, so that after the new technology is discovered, the current one is not used any more. The patent race for the next technology involves $n$ 'entrant' firms $i = 1,\ldots,n$ and the incumbent patentholder, $L$. Each firm can participate in the patent race by paying a fixed cost $F \in (0, V)$ and investing a constant flow of resources $z^i \in \mathcal{R}_+$, as to obtain an instantaneous probability of innovation:

$$h_i = h(z^i)$$

according to a standard Poisson process with $h'(z) > 0$, $h''(z) < 0$ for any $z \in \mathcal{R}_+$ where we assume $h(0) = 0$ and $\lim_{z\to 0} h'(z) > (V - F)^{-1}$. The aggregate arrival rate of innovation will be the sum of the individual arrival rates of the $n$ entrants plus the one of the incumbent:

$$p = \sum_{i=1}^{n} h(z^i) + h(z^L).$$

Using the properties of Poisson processes in a standard fashion, the objective function of entrant $i$ is:

$$\Pi^i = \frac{h(z^i)V - z^i}{\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right]} - F \tag{1}$$

where $r$ is the exogenous interest rate. The objective function of the incumbent monopolist is given by:

$$\Pi^L = \frac{h(z^L)V + \pi - z^L}{\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right]} - F. \tag{2}$$

### 1.1. *Nash Competition*

In this subsection we model competition in the Nash fashion, assuming that each firm chooses the investment in R&D, taking as given the one of the others and the interest rate. Each entrant chooses $z^i$ as to satisfy the first order condition:[5]

$$\left[h'(z^i)V - 1\right]\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right] = h'(z^i)\left[h(z^i)V - z^i\right], \tag{3}$$

[5] The second order condition is always satisfied thanks to the concavity of $h(\cdot)$. To verify that an interior equilibrium exists notice that we need that (3) holds for any $z^i$ in a small enough right neighbourhood of 0 and for any investment of the other firms, hence also when they do not invest. Hence, after some rearrangement of (3), we derive the condition:

$$\lim_{z\to 0} h'(z) > \lim_{z\to 0} \frac{r + h(z)}{Vr + z} = \frac{1}{V},$$

which always holds under our assumptions.

which provides a unique best response function for the flow of investment. Straightforward differentiation shows that this best response is increasing in terms of the expected value of innovation, the interest rate, the number of firms and the flow of investment of each other firm, implying strategic complementarity. Moreover, it can be shown that the uniqueness of the best response function for each firm implies equal investment between the entrants for a given number of firms. In other words, the equilibrium is symmetric between the entrants.

If the incumbent invests, its choice $z^L$ satisfies the first order condition:[6]

$$\left[h'(z^L)V - 1\right]\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right] = h'(z^L)\left[h(z^L)V + \pi - z^L\right], \tag{4}$$

which defines an analogous best response function decreasing in the flow of current profits $\pi$. This implies that, *ceteris paribus*, the incumbent invests less than each entrant and has lower expected profits from participating in the patent race (Reinganum, 1983). Finally, the investment of all firms is increasing in $r$, $V$ and $n$ while decreasing in $\pi$.

Assuming free entry and noticing that the expected profit functions of all firms are decreasing in the number of firms, we can conclude that the incumbent will stop researching if the number of firms is great enough – this is the well-known *Arrow* (*or replacement*) *effect*[7] – and the entrants will break even if the number of firms achieves a still higher bound. This bound is defined from the free entry condition:

$$h(z)V - z = F[r + nh(z)] \tag{5}$$

where I have used $z^L = 0$ and the symmetry of the equilibrium. Rearranging this equation, we can re-express the equilibrium flow of investment in the following implicit way:

$$h'(z)(V - F) = 1 \tag{6}$$

which is increasing in the difference between expected value of the innovation and fixed cost but independent from the interest rate. The equilibrium number of firms turns out increasing in $V$, but decreasing in $F$ and in the interest rate $r$.[8]

[6] For the incumbent, an interior solution always exists if:

$$\lim_{z\to 0} h'(z) > \lim_{z\to 0} \frac{r + \pi + h(z)}{V(r+\pi) + z - \pi} = \frac{r+\pi}{V(r+\pi) - \pi},$$

which may not hold under our assumptions for $\pi$ big enough. In that case, the incumbent does not participates to the patent race for any number of entrants.

[7] If the incumbent has some exogenous technological advantage in doing research, the Nash equilibrium may also entail more incentives to invest in R&D for the incumbent than the entrants. This is true even with free entry if the technological advantage is great enough. I thank Robert Barro for pointing this out.

[8] I ignore, as usual, the integer constraint on $n$ and consider it as a real number, hence the following results hold as good approximations for a great number of firms or, in other words, for small enough fixed costs. Otherwise, the equilibrium number of firms would be the largest integer below $n$. However, in this case, all firms make positive profits in equilibrium and, when fixed costs are high, it is possible that the incumbent invests, though still less than the entrants. Finally, notice that our assumptions deliver always an interior solution for the investment choice but, obviously, we need a small enough fixed cost to obtain at least one firm in equilibrium.

### 1.2. *Stackelberg Competition*

I will now drop the hypothesis of Nash behaviour and will assume that the the patentholder has the opportunity to make a strategic precommitment to a level of investment in R&D. This may happen through a specific investment in R&D laboratories, by hiring researchers or in a number of other ways. Our strategic assumption seems a natural one since the patentholder can be easily seen in a different perspective from all other entrants in the patent race.

The opportunity to make a strategic precommitment is exploited by the incumbent as to increase its expected profits but done so in dramatically different ways according to the competitive structure of the patent race. If the structure is characterised by a fixed number of firms, the incumbent leader will commit to a low level of investment because such a strategy will induce a reduction in the investment of the other firms and a longer expected lifespan of the current patent. This is a direct consequences of well-known results by Fudenberg and Tirole (1984) and Bulow *et al.* (1985) who have shown that in a Stackelberg duopoly the leader is more aggressive than the follower under strategic substitutability and accommodating under strategic complementarity.[9] In our patent race strategic complementarity holds hence, when the number of firms is fixed, the leader is accomodating and it may even not invest at all in R&D.

However, if entry in the patent race is free, the leader will commit to a high level of investment. Indeed, the investment of the leader perfectly crowds out that of the entrants, leaving constant the aggregate probability of innovation, as given by the free entry constraint. Hence the marginal cost of investment is lower for the leader than for the entrants, whose investment does affect the aggregate probability of innovation. Such a situation provides a leader investing and investing more than any other entrant. This is an application of a more general result by Etro (2002*a*), which shows that any Stackelberg game with free entry is characterised by a leader acting more aggressively than the follower regardless of strategic complementarity or substitutability. This holds under quantity competition or price competition, with asymmetries between the leader and the followers, with endogenous investment by the leader and even when there are many leaders.

More formally, I will consider a two stage patent race. In the first stage, the leader chooses whether to participate in the new patent race and, in the former case, its investment $z^L$. In the second stage all the entrants choose their own investment $z^i$ , knowing the investment of the leader and taking as given the investment of all other entrants. Obviously our equilibrium concept is subgame perfection with the entrants playing Nash in the second stage.

Let us consider the second stage. Each entrant chooses $z^i$ to maximise their expected profits. Its choice satisfies the first order condition:

$$\left[h'(z^i)V - 1\right]\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right] = h'(z^i)\left[h(z^i)V - z^i\right], \tag{7}$$

[9] The result holds for any number of followers playing Nash in the second stage (Dixit, 1987). The pathbreaking work on sub-game perfection by Dixit (1980) studied a related model with precommitment on capacity where the leader has an incentive to overexpand to bankrupt a follower.

which implies a reaction function for $z^i$ increasing in $z^L$, $\phi^i(z^L)$: the more aggressive the leader, the more aggressive the followers.[10] In the first stage, the choice of the leader $z^L$ satisfies the first order condition:

$$\left[h'(z^L)V - 1\right]\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right] = \left\{h'(z^L) + \frac{\partial\left[\sum_{j=1}^{n} h(z^j)\right]}{\partial z^L}\right\}\left[h(z^L)V + \pi - z^L\right]$$

and we assume that the second order condition is satisfied.

#### 1.2.1. *No free entry*

Let us take as given the number of entrants $n$ and assume that it is low enough that entry is actually profitable in equilibrium. The system (7)–(8) defines the equilibrium. Our preliminary result establishes some comparative statics for this equilibrium. Changes in $r$ and $\pi$ induce similar effects to the Nash case. However, in general, the comparative statics of the investment of each firm with respect to $n$ and $V$ is ambiguous. For instance, an increase in the number of entrants induces a direct positive effect on each firm's investment but it also has an ambiguous effect on $\partial\phi^i(z^L)/\partial z^L$ and hence an indirect ambiguous effect on the investment of the leader. If the latter induces a net reduction of the investment of the leader, there is a further negative effect on the investment of the entrants in the second stage, which may overturn the initial effect. Paradoxically, an increase in the value of the innovation also makes the entrants more aggressive, and this may induce a reduction of the investment of the leader, with ambiguous consequences.[11]

Proposition 1. *Stackelberg competition for a given number of firms implies an investment for each firm which is increasing in the interest rate, r, and decreasing in the flow of current profits, π, but ambiguously dependent on the value of the innovation, V, and the number of firms, n.*

Corollary 1. *Stackelberg competition for a given number of firms implies an aggregate investment in R&D which is increasing in r and decreasing in π, and an expected lifespan of the current patent which is decreasing in r and increasing in π, while the effects of changes in V and n are ambiguous.*

Finally, we can prove that Stackelberg competition induces less aggregate investment in R&D than Nash competition by each firm. A graphical explanation is shown in Figure 1, where I use the fact that in equilibrium all entrants

[10] Indeed, by totally differentiating (7), we have:

$$\frac{\partial\phi^i(z^L)}{\partial z^L} = \frac{-[h'(z^L)V - 1]h'(z^L)}{h''(z^i)\left\{V\left[r + \sum_{j\neq i} h(z^j) + h(z^L)\right] + z^i\right\}} > 0.$$

[11] After a version of this paper was finished I found interesting work by Reinganum (1985*b*) where similar results for the Stackelberg game with no free entry are obtained.

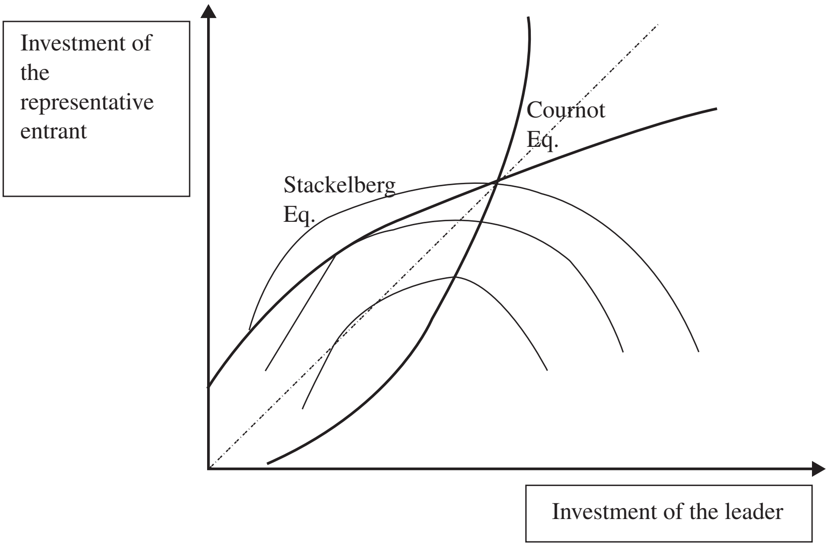


Fig. 1. *Nash versus Stackelberg Competition without Free Entry*

choose the same level of investment. I depict the reaction functions of the leader and of the representative entrant in the space $(z^L, z)$, with the isoprofit curves for the former. The Nash equilibrium is at the intersection of the two reaction functions, while the Stackelberg equilibrium is at the tangency of the lowest isoprofit locus of the leader with the reaction function of the representative entrant. Clearly, in both cases the leader invests less than the representative entrant but, in the Stackelberg case, the leader invests even less then in the Nash case and less than each other single firm. Moreover the representative entrant invests less in the Stackelberg equilibrium than in the Nash equilibrium. Since the number of entrants is given, it must be that the aggregate investment is reduced.

PROPOSITION 2. *Stackelberg competition for a given number of firms implies*

(*a*) *a lower investment than Nash competition for all the entrant firms and the leader patentholder, a smaller aggregate investment, a longer expected lifespan of the current patent, and*
(*b*) *lower investment for the leader patentholder than for each of the other firms.*

The intuition is that by investing less the incumbent reduces the incentives to invest for each entrant, since investment is characterised by strategic complementarity in this environment and this both decreases the probability of an innovation by a follower and increases the expected lifespan of the current patent. The incumbent uses its first mover advantage to reduce its own investment. Hence, Stackelberg leadership with a fixed number of firms due to some barriers to entry does not give a new rationale for incumbents' investment in R&D. Actually the opposite happens.

1.2.2. *Free entry*

Let us now consider the free entry case, in which the leader has to foresee the effects of its investment choice on the equilibrium number of entrants. Despite this complication, it turns out that it is quite easy to characterise the new equilibrium. In the second stage all the entrant firms choose the same flow of investment $z$. Using this symmetry, the zero profit condition becomes:

$$h(z)V - z = F[r + nh(z) + h(z^L)]. \tag{8}$$

Substituting (8) in the equilibrium first order condition of the entrants (7) we obtain an implicit expression for the entrant's investment:

$$h'(z)(V - F) = 1,$$

which provides (6) again and does not depend on the leader's decision. However, the equilibrium number of firms, given by:

$$n = \frac{V}{F} - \frac{z}{h(z)F} - \frac{r + h(z^L)}{h(z)} \tag{9}$$

does depend on the leader's choice.[12] Totally differentiating this condition – and using the fact that $z$ does not depend on $z_L$ – we can obtain the expected change of investment in R&D of each entrant for a change in the leader's investment:

$$\frac{\partial\left[\sum_{j=1}^{n} h(z^j)\right]}{\partial z_L} = \frac{\partial nh(z)}{\partial z_L} = \frac{\partial n}{\partial z_L} h(z) + nh'(z)\frac{\partial \phi(z^L)}{\partial z^L}$$
$$= -\, h'(z^L),$$

which has the opposite sign of the case without free entry. Despite an increase in the investment of the leader increasing the investment of each entrant, the effect on the equilibrium number of firms is negative and large enough to more than compensate the former. Substituting in (8) we obtain an implicit expression for the leader investment:

$$h'\left(z^L\right)V = 1, \tag{10}$$

which is greater than the investment of each entrant and thus of the investment of the patentholder under Nash competition. This also implies a lower number of entrants than in Nash equilibrium with free entry. It follows:

PROPOSITION 3. *Stackelberg competition with free entry implies*

(*a*) *the same investment as Nash competition for the entrant firms with a lower number of entrants and*

(*b*) *a higher investment for the leader patentholder than for each of the other firms.*

[12] Again, I ignore the integer constraint on the equilibrium number of firms. This is a good approximation for a great number of firms or, in other words, for small enough fixed costs. The analysis is more complex when this is not the case and the exact equilibrium number of firms may not simply be the largest integer below $n$. When the fixed cost is high enough and the profit flow is big enough, I could not even exclude special cases in which the leader invests less than the followers. I thank David de Meza for pointing this out.

Stackelberg competition with free entry induces the aggressive behaviour of the monopolist in the patent race, while under Nash competition the incumbent was not doing any research, the first mover advantage delivers a strong incentive to invest for the incumbent. The intuition is related to the perception the leader has of the entry process. It is understood that any profitable opportunity for doing R&D left open by the leader will be seized by new entrants until their profits are zero. The aggregate probability of innovation is determined by the free entry constraint independently of the investment of the leader and is thus taken as given by the same leader. So, the monopolist loses the strategic incentive to keep its investment low: the latter is not going to affect the expected lifespan of the current patent. Hence, the only purpose of investing in R&D for the leader is actually to win the patent race and the incentives to do it are now higher than those of any other entrant. An intuitive way to see this asymmetry again depends on the fact that the leader is taking as given the aggregate probability of innovation; so, the optimal investment of the leader maximises $h(z^L)V - z^L$ without taking into account the impact on the aggregate arrival rate of innovation. This impact, instead, is taken into account by each entrant and reduces their profits, explaining why entrants invest less than the leader.

Notice that the Arrow effect does not play any role in this equilibrium and the investment of the leader is independent from the current flow of profits. Under Stackelberg competition the Arrow effect disappears. To understand this, we need to reinterpret the Arrow effect. In a patent race, what this really says is that when all firms play Nash, the opportunity cost to invest is higher for the patentholder because higher investment increases aggregate investment and reduces the expected lifespan and expected profits of the current patent. But under Stackelberg competition the latter is constant, as we have seen, so the expected profits from the current patent are independent from the investment of the leader: hence, the Arrow effect disappears.

We finally derive some comparative statics in the following Proposition:

PROPOSITION 4. *Stackelberg competition with free entry implies:*

(*a*) *an investment for each entrant firm which is increasing in the value of the innovation $V$ and decreasing in the fixed cost $F$, while independent from interest rate $r$ and current profits $\pi$,*
(*b*) *an investment for the incumbent leader which is increasing in $V$ and independent from $r$, $\pi$ and $F$ and*
(*c*) *a number of firms which decreases in $r$, is independent from $\pi$ and is ambiguously affected by $V$ and $F$.*

COROLLARY 2. *Stackelberg competition with free entry implies an aggregate investment in R&D which is decreasing in $r$, independent from $\pi$ and ambiguously dependent from $V$ and $F$, and an expected lifespan of the current patent which is increasing in $F$ and decreasing in $V$, while it is independent from $r$ and $\pi$.*

Notice that as the value of innovation increases, the investment of the outsiders increases more than that of the leader and the two converge if $V$ is great enough or $F$ small enough.

#### 1.2.3. *Implications*

If we believe that Stackelberg competition is the right assumption in the study of patent races (which may be true in some specific contexts but not in all and is ultimately an empirical issue), we obtain some sharp conclusions from this analysis. Paradoxically, if we see a market for innovation dominated by the current monopolist and another one in which the current monopolist does not research (or does less research than the other entrants), the former is competitive, while the latter is not. In other words *a market characterised by high persistence of monopoly is competitive, while one with systematic leapfrogging must be characterised by some barriers to entry*! This is exactly the opposite conclusion than the one we obtain by assuming Nash competition.[13]

The model also implies that aggregate investment in R&D may be higher under Stackelberg competition, while the number of firms is typically higher under Nash competition. From a welfare point of view, since the externality associated with Nash behaviour, the business stealing effect and the consumer-surplus effect (due to the positive difference between social and private value of innovations) work in different directions, we cannot compare different equilibria, but it may well be the case that different policy prescriptions characterise them. Only a general equilibrium approach would allow a proper comparison. An example of such a general equilibrium approach is provided in Section 2.4.

More empirical work on the structure of the market for innovations seems desirable, not only to discriminate between alternative models of innovation but also to discriminate between the different policy prescriptions they deliver.

## 2. Extensions

In this Section I extend the basic model in different directions. First, I consider the case of contractual costs of innovation, in which all firms invest at the beginning of the patent race, and their initial investment determines their probability of innovation over time. Second, I drop the assumption of drastic innovations and assume some form of collusion between the new patentholder and the old one which allows both to obtain positive profits after the innovation. Third, I depart from the usual assumption of an exogenous size of innovations and study the case in which firms can increase the profitability of the future innovation with additional investment. Finally, I consider a sequence of innovations and apply the results to a couple of examples. The first shows the emergency of innovation cycles in a partial equilibrium context, the second develops a Schumpeterian growth model in general equilibrium where the market for innovations is characterised by realistic patent races. The purpose of this Section is to show that the main claim of the paper – persistence of monopoly under Stackelberg competition with free entry holds in a variety of applications and that

[13] One may wonder what would happen if the first mover is not the incumbent monopolist but one of the entrants. This case replicates our Stackelberg equilibrium with the new first mover taking the place of the incumbent and the latter not investing anymore in R&D. This shows how the identification between the first mover advantage and the patentholder is crucial to obtaining persistence of leadership.

the idea can be usefully exploited in more complex environments than the basic one previously studied.

## 2.1. *Contractual Costs of R&D*

Let us assume now that the probability of innovation is a function of the fixed cost initially paid by each firm. The instantaneous probability of innovation is now $h_i = h(z^i)$ where $h(0) = 0$, $h'(z) > 0$ and $h'(z) \gtrless 0$ for $z \lessgtr \hat{z}$: hence I allow for increasing returns to scale for low investment but I assume decreasing returns to investment greater than a cut off $\hat{z} > 0$. The objective function of firm $k$ is now:

$$\Pi^k = \frac{h(z^k)V + \pi^k}{\left[r + \sum_{j=1}^n h(z^j) + h(z^L)\right]} - z^k, \tag{11}$$

where $\pi^k = 0$ for any entrant $k = 1,\ldots,n$ and $\pi^L = \pi$ for the incumbent.

Under Cournot-Nash competition the first order conditions and the free entry condition imply a symmetric equilibrium between the entrants with investment implicitly given by:

$$h'(z^C)\left(1 - \frac{z^C}{V}\right) = \frac{h(z^C)}{z^C} \tag{12}$$

and investment of the leader implicitly given – if positive – by:

$$h'(z^{LC})\left\{1 - \frac{z^C}{V}\left[\frac{h(z^{LC})V + \pi}{Vh(z^C)}\right]\right\} = \frac{h(z^C)}{z^C} \tag{13}$$

while the equilibrium number of firms is:

$$n^C = \frac{V}{z^C} - \frac{r}{h(z^C)} - \frac{h(z^{LC})}{h(z^C)}. \tag{14}$$

Notice that the investment of the incumbent monopolist is decreasing in its flow of profits, which means that the incumbent invests less than any other firm, but, in this case, the assumption of initial increasing returns makes it possible for the incumbent to invest a positive amount in the Nash equilibrium with free entry.

With Stackelberg competition, the equilibrium first order condition and the free entry condition at the second stage are the same and they imply that $z^S = z^C$, so that the leader is now maximising:

$$\begin{aligned}\Pi^L &= \frac{h(z^L)V + \pi}{[r + nh(z^S) + h(z^L)]} - z^L \\ &= \frac{h(z^L)V + \pi}{h(z^S)V} z^S - z^L,\end{aligned}$$

from which the first order condition:

$$\frac{h'(z^{LS})}{h(z^S)} z^S = 1 \tag{15}$$

defines a local maximum when $h''(z^{LS}) < 0$, as we will assume. We also show that this investment is always a global maximum and higher than the investment of each one of the entrants. Henceforth, it follows:

PROPOSITION 5. *Stackelberg competition with free entry and contractual costs of R&D implies*

(*a*) *the same investment as Nash competition for the entrant firms with a lower number of entrants and*
(*b*) *a higher investment for the leader patentholder than for each of the other firms.*

Notice that in the Stackelberg equilibrium each entrant is investing under (local) increasing returns to scale, while the leader does it at a larger scale but under decreasing returns to scale. Also an increase in the value of innovation tends to promote investment from the entrants while decreasing that of the leader. Both the investments of the entrants and the leader, however, are independent from the interest rate and the current profit. The last result also confirms that with contractual costs of R&D the Arrow effect disappears.

### 2.2. *Nondrastic Innovations*

Now let us assume that the innovation is nondrastic, and if the incumbent loses the patent race, a duopoly between the winner and the incumbent sets in. The value of winning the patent race for the incumbent is denoted with $V^W$, while if an entrant wins, the previous incumbent obtains $V^L$ and the winner obtains $V^E$, which is obviously smaller than $V^W$. The standard assumption in this case is that, even if the duopoly is characterised by perfect collusion, the sum of the discounted profits obtained by the two duopolists cannot be greater than the discounted profits obtained by the incumbent who wins the patent race:

$$V^W \geq V^E + V^L > 0. \tag{16}$$

Notice that there is a special case of drastic innovations for $V^W = V^E \equiv V$ and $V^L = 0$. However, the direct incentives for the incumbent to invest are reduced, because, even when losing, the incumbent will make some profit forever. This is not true for the outsiders, who earn profits only in case they win the patent race. Indeed, the objective functions are now given by:

$$\Pi^i = \frac{h(z^i)V^E - z^i}{\left[r + \sum_{j=1}^n h(z^j) + h(z^L)\right]} - F \tag{17}$$

and:

$$\Pi^L = \frac{h(z^L)V^W + \pi - z^L + \sum_{j=1}^n h(z^j)V^L}{\left[r + \sum_{j=1}^n h(z^j) + h(z^L)\right]} - F. \tag{18}$$

Nevertheless, our main result is always valid:

Proposition 6. *Stackelberg competition with free entry and non drastic innovations implies*

(*a*) *the same investment as Nash competition for the entrant firms with a lower number of entrants and*
(*b*) *a higher investment for the leader patentholder than for each of the other firms.*

The result is analogous to the previous cases since the Arrow effect disappears. Now the investment of the leader is directly related to the net prospective value of innovating $V^W - V^L$, which is strictly higher than the one of the entrant $V^E$. This induces a stronger persistence of monopoly in the case of nondrastic innovations than in the case of drastic innovations.

### 2.3. *Endogenous Size of Innovations*

We now study the case in which the size of innovations is endogenous. Each firm can invest in two different ways: on one hand the firm can invest so as to increase the probability of innovation and, on the other, the firm can invest so as to obtain a greater profit from the innovation in the case the patent is obtained. The latter investment is also characterised by decreasing returns to scale. The higher is this investment, the more radical will be the innovation but the associated prize is assumed to increase in a less than proportional way.

For the sake of simplicity let us assume that the innovation is always drastic. Its value is now a function of a firm specific investment, $V(x^i)$ with $V(x) > 0$, $V'(x) > 0$ and $V''(x) < 0$ where $x \in \mathcal{R}_+$ is a flow of investment which is necessary to discover an innovation of quality $V(x^i)$. The objective function of firm $k$ becomes:

$$\Pi^k = \frac{h(z^k)V(x^k) + \pi^k - z^k - x^k}{\left[r + \sum_{j=1}^{n} h(z^j) + h(z^L)\right]} - F \tag{19}$$

where $\pi^k = 0$ for any entrant $k = 1,\ldots,n$ and $\pi^L = \pi$ for the incumbent. In the free entry equilibrium with Stackelberg competition the incumbent chooses to invest not only more, but also in higher quality improvements than the other firms under the conditions emphasised in the next Proposition:

Proposition 7. *Stackelberg competition with free entry and an endogenous amount of innovation implies*

(*a*) *the same investment and value of innovation as Nash competition for the entrant firms, positive investment for the leader patentholder, a lower number of entrants and*
(*b*) *a higher investment for innovations of greater value for the leader patentholder than for each of the other firms if the fixed cost is small enough.*

Hence, at least for small fixed costs, the incumbent invests in more 'radical' innovations. Indeed, each firm equalises the marginal productivity of the two investments, which end up being complementary. That is why our traditional incentive to invest a lot as the leader due to the absence of the Arrow effect also induces investment in innovations of greater value. The two forces strengthen each other. This is a surprising result since a widespread view claims that small firms would invest in more radical innovations.[14]

This Section tries to point out the complexity of the choices of firms racing for innovations. These and others should be focused in future research. For instance, an important choice faced by the leader concerns the allocation of investment between improving the profitability of the existing patent and doing the R&D to produce a new product. An example is the choice between higher chip speed or a new chip architecture.[15] Another important dilemma which many innovators face is the choice between patenting an innovation (which provides a sure flow of monopolistic profits as long as the patent lasts) and keeping it secret (which provides the same flow of profits until some other firms finds out and patent the same innovation or a better one). A famous example is the secret formula of Coca Cola, which since its introduction by a pharmacist from Atlanta in 1886, has never been patented and nowadays is still hidden in a safe deposit vault at the Trust Company of Georgia (according to the legend, not less than two picked people and no more than three ever know the ingredients at the same time and they never travel together!).[16] Future research may study the interdependence between these choices and the structure of the market for innovation.

### 2.4. *Sequences of Innovations*

I will now consider a dynamic model of multiple innovations with free entry. Consider a series of drastic innovations $k = 1,2,\ldots$ with associated flows of profits $\pi_k \in \mathcal{R}_+$. After any innovation, a patent race starts for the next one. Let us consider $n_k$ entrant firms $i = 1,\ldots,n_k$ plus the incumbent monopolist $L_k$ in the patent race for the innovation $k+1$. Each firm can participate in a single patent race by paying a fixed cost $F_k \in \mathcal{R}_+$ and investing a flow of resources $z_k^i \in \Re_{++}$ to obtain an instantaneous probability of innovation $h_{ik} = h(z_k^i)$. Let us denote with $V_k$ the value of innovation $k$ . The objective function of entrant $i$ is:

$$\Pi_k^i = \frac{h(z_k^i)\,V_{k+1} - z_k^i}{(r + p_k)} - F_k \tag{20}$$

where $p_k = \sum_{i=1}^{n_k} h(z_k^i) + h(z_k^L)$. The value function of being the monopolist with the technology $k$ is given by the Bellman equation:

[14] Nevertheless, empirical evidence by Blundell *et al.* (1999) is consistent with our result. They show that, after an innovation, the market value of firms increases more for firms with stronger market power, which is consistent with the argument that 'leading firms have a systematic tendency to produce innovations that are intrinsically of higher quality than smaller firms'.

[15] I thank a referee for this suggestion. In a more general model, one can imagine a trade-off for the leader between investments in the current and in the new technology. This would happen, for instance, if improving the current patent implies stricter requirements for the patentability of the next innovation (or, in other words, it reduces the rate of innovation of each firm for a given investment in R&D).

[16] See Etro (2002*b*) for further anecdotical evidence on the behaviour of innovators and some related theories. Another suggestive example which is consistent with radical innovation by leaders in patent races and even efforts to keep secret certain innovations can be found in sports based on advanced technologies as Formula 1 racing or the America's Cup.

$$V_k = \max_{z_k^L \geq 0} \left[ \frac{h(z_k^L) V_{k+1} + \pi_k - z_k^L}{r + h(z_k^L) + \sum_{i=1}^n h(z_k^i)} - F_k I(z_k^L > 0) \right], \tag{21}$$

where $I(z_k^L > 0)$ is an indicator function with value 1 if $z_k^L > 0$ and 0 otherwise.

The Cournot-Nash case is analogous to the static version. Let us index with $C$ its equilibrium variables. In each patent race, $z_k^{LC} = 0$, while the investment of each entrant is implicitly given by:

$$h'(z_k^C)(V_{k+1}^C - F_k) = 1 \tag{22}$$

and the equilibrium number of firms is:

$$n_k^C = \frac{V_{k+1}^C}{F_k} - \frac{z_k^C}{h(z_k^C) F_k} - \frac{r}{h(z_k^C)}. \tag{23}$$

Hence, the Bellman equation can be rewritten as:

$$\begin{aligned} V_k^C &= \frac{\pi_k}{[r + n_k^C h(z_k^C)]} \\ &= \frac{\pi_k F_k}{h(z_k^C) V_{k+1}^C - z_k^C}, \end{aligned}$$

where $z_k^C$ is the function implicitly defined by (22).

Let us now move to the Stackelberg case. In any patent race for innovation $k+1$, the value $V_{k+1}$ is taken as given by all firms, each entrant chooses $z_k^i$ at the second stage as to maximise its expected profits for a given investment of the leader, free entry determines the number of entrants $n_k$ and, in the first stage, the leader decides $z_k^L$ to maximise the value function (21). Hence the equilibrium in each patent race is analogous to the one considered in Section 2. The investments of entrants and leader satisfy:

$$h'(z_k^S)(V_{k+1}^S - F_k) = 1 \tag{24}$$

and

$$h'(z_k^{LS}) V_{k+1}^S = 1 \tag{25}$$

while the equilibrium number of firms is given by:

$$n_k^S = \frac{V_{k+1}^S}{F_k} - \frac{z_k^S}{h(z_k^S) F_k} - \frac{r + h(z_k^{LS})}{h(z_k^S)} \tag{26}$$

where we introduced the index $S$ to denote equilibrium variables of the Stackelberg case. Finally we can substitute (26) in the Bellman equation to obtain:

$$V_k^S = \left[ \frac{h(z_k^{LS}) V_{k+1}^S + \pi_k - z_k^{LS}}{h(z_k^S) V_{k+1}^S - z_k^S} - 1 \right] F_k, \tag{27}$$

where $z_k^S$ and $z_k^{LS}$ are the functions defined by (24) and (25).

A simple example I will use later on is obtained with the isoelastic arrival rate:

$$h(z_k) = (\phi_k z_k)^{\epsilon} \quad \text{with } \epsilon \in (0,1), \tag{28}$$

where $\epsilon$ is a measure of the degree of returns to scale in the market for innovation[17] and $\phi_k$ is parameter specific to the patent race. In this case, the equilibrium investments in the Nash and Stackelberg cases become:

$$z_k^C = \epsilon^{\frac{1}{1-\epsilon}} \phi_k^{\frac{\epsilon}{1-\epsilon}} \left(V_{k+1}^C - F_k\right)^{\frac{1}{1-\epsilon}}, \quad z_k^{LC} = 0, \tag{29}$$

$$z_t^S = \epsilon^{\frac{1}{1-\epsilon}} \phi_k^{\frac{\epsilon}{1-\epsilon}} \left(V_{k+1}^S - F_k\right)^{\frac{1}{1-\epsilon}}, \quad z_k^{LS} = \epsilon^{\frac{1}{1-\epsilon}} \phi_k^{\frac{\epsilon}{1-\epsilon}} V_{k+1}^{S\frac{1}{1-\epsilon}}. \tag{30}$$

This kind of model can be useful for different purposes. Here I will briefly discuss two of them: the patterns of innovation in a single sector, which I will show to be characterised by cycling investment, and aggregate endogenous growth driven by innovation of the monopolists as in Schumpeter's original idea of creative destruction (1942).

### 2.4.1. *Application 1: Innovation cycles*

I will now study the dynamic of investment in R&D in a single sector characterised by an infinite sequence of innovations.[18] To consider a stationary environment, I will simplify the analysis by assuming constant flows of profits ($\pi$) and fixed costs of entry in the patent race ($F$) for all the innovations – market size effects which increase profits from an innovation to the next one can be easily introduced.

In the Nash case, (24) provides the equilibrium path for $V_k^C$ and hence $z_k^C$ and defines a steady state for $V_{k+1}^C = V_k^C = V^C$ which is increasing in $\pi$ and $F$ (since a higher cost implies less aggregate R&D and hence a longer expected life of the patent). Inverting (24) to obtain $V_{k+1}^C = \varphi^c\left(V_k^C\right)$ we can verify that $\partial\varphi^c(V^C)/\partial V^C \in (-1{,}0)$ always holds, which implies a cycling convergence to the steady state.

In the Stackelberg case, (27) provides the equilibrium path for $V_t^S$ and hence all the other variables and defines a steady state for $V_{k+1}^S = V_k^S = V^S$. Inverting (27) to obtain $V_{k+1}^S = \varphi^s\left(V_k^S\right)$ we can verify when $\partial\varphi^s(V^S)/\partial V^S \in (-1{,}0)$ so that the steady state is achieved through an *innovation cycle*. This is indeed the typical case, even if, in general, I could not exclude other dynamics or biperiodal cycling due to a flip bifurcation.[19]

Under our functional form specification (28), with $\phi_k = 1$, the equations of motion for $V_k^C$ and $V_k^S$ become:

[17] Values for $\epsilon$ between 0.1 and 0.6 have been suggested in the empirical literature on innovation, for instance by Kortum (1993).

[18] Notice that this case is different from the one studied by Reinganum (1985*a*) not only because she studied a finite horizontal environment and Nash competition but also because she considered free entry at the initial stage rather than in every single patent race. Hence, she did not derive zero investment for the incumbent. Since the cutting edge technology is common knowledge for every firm, it seems more natural to assume that every time a new technology is made available, new firms can enter in the next patent race.

[19] Notice that without an initial condition for investment, the exact equilibrium is indeterminate. Obviously the problem would disappear if the horizon was finite: with $T$ periods, it is clear that $V_T^S = V_T^C = \pi/r$ and the exact equilibrium can be obtained recursively.

$$V_k^C = \frac{\pi F}{\epsilon^{\frac{\epsilon}{1-\epsilon}}\left(V_{k+1}^C - F\right)^{\frac{\epsilon}{1-\epsilon}} V_{k+1}^C - \epsilon^{\frac{1}{1-\epsilon}}\left(V_{k+1}^C - F\right)^{\frac{1}{1-\epsilon}}} \tag{31}$$

$$V_k^S = \left[\frac{\left(\epsilon^{\frac{\epsilon}{1-\epsilon}} - \epsilon^{\frac{1}{1-\epsilon}}\right)\left(V_{k+1}^S\right)^{\frac{1}{1-\epsilon}} + \pi}{\epsilon^{\frac{\epsilon}{1-\epsilon}}\left(V_{k+1}^S - F\right)^{\frac{\epsilon}{1-\epsilon}} V_{k+1}^S - \epsilon^{\frac{1}{1-\epsilon}}\left(V_{k+1}^S - F\right)^{\frac{1}{1-\epsilon}}} - 1\right] F. \tag{32}$$

For instance, assuming $\epsilon = 0.5$, $F = 0.1$, $r = 0.04$ and $\pi = 100$ implies a steady for the Stackelberg equilibrium with $V^S = 3.4$, $z^S = 2.7$, $z^{LS} = 2.9$, $n = 16$ and cycling convergence to it.

If, for some reason, expectations focus on a value of the incumbent firm which is different from the steady state value,[20] *innovation cycles* emerge. They actually have a straightforward intuition. If the value of being the next monopolist is expected to be great, there is a lot of investment and entry in the current patent race, increasing the current effective discount rate and reducing the value of being the current monopolist. A low value of the current monopolistic position is consistent with a low investment in the last patent race and so on. As shown in Section 1, when the value of the next innovation increases, the investment of the leader increases more than the one of the outsiders, suggesting that periods of high investment in R&D are also associated with high probability of replacement of the incumbent monopolist. In other words, the degree of persistence of monopolistic positions is following a cycling pattern. It would be interesting to test these implications in particular sectors.

### 2.4.2. *Application 2: Schumpeterian growth*

The idea that growth is driven by the innovation of the monopolists due to Schumpeter (1942) has been recently formalised by Aghion and Howitt (1992, 1998), Grossman and Helpman (1991), Barro and Sala-i-Martin (1995) and others.[21] In these models, the engine of growth is innovation and the engine of innovation is the possibility to replace the current monopolist and obtain a flow of profits until someone else will replace you. In the real world, however, the value of a monopolistic position is associated not only with a flow of profits but also with a preferential position in the race for the next technology. Thus, the engine of innovation is something different if monopolistic positions are persistent rather than not.[22]

[20] This may reasonably happen if there was uncertainty on the future flows of profits. Self-fulfilling prophecies driven by espectations would arise.

[21] Romer (1987, 1990) introduced the idea of innovations driven by monopolistic profits as the engine of growth. For other related models of Schumpeterian growth see Acemoglu (2000), Howitt (2000) and Zeira (2003).

[22] Cozzi (2002) has noticed that, in the standard Schumpeterian model with linear R&D technology and Nash competition between researchers, the patentholders are actually indifferent between investing or not in R&D, but if they do it the other features of the equilibrium are not affected. However this kind of indeterminacy only holds under linear technology. As I make clear below, the Arrow effect is always present with decreasing returns to scale in the innovation sector and Nash Competition. Stackelberg competition is instead crucial to obtain always persistence of monopolistic positions.

Here I study a model of endogenous growth based on the previous discussion. The sector analysed in the previous subsection is one of $N$ sectors producing intermediate goods $j = 1,\ldots,N$. The only final good - which can be used for consumption or production of intermediates - is produced according to the CRS function:

$$Y = AL^{1-\alpha}\sum_{j=1}^{N}(q^{\kappa_j}X_j)^{\alpha}, \tag{33}$$

where $L$ is fixed labour supply and I normalise $A = L = 1$ for simplicity, $X_j$ is the intermediate good $j$ of quality $k_j$, $q > 1$ and $\alpha \in (1/2,1)$. The market for final goods is perfectly competitive while each intermediate good is produced by a single firm - with a patent on it - and sold at a monopolitistic price. Profit maximisation by the monopoly provider of intermediates of vintage $\kappa_j$ implies the monopolistic unit price $1/\alpha$ and, for simplicity, we will assume that only the highest vintage intermediates are sold on the market. The aggregate quantity produced of intermediate good $j$ can be determined as $X(\kappa_j) = \alpha^{2/(1-\alpha)}q^{\kappa_j\alpha/(1-\alpha)}$. Substituting, we obtain the output of final goods $Y = \alpha^{2\alpha/(1-\alpha)}Q$, where we have introduced the aggregate quality index, $Q \equiv \sum_{j=1}^{N} q^{\frac{\kappa_j\alpha}{1-\alpha}}$. The flow of profit for each intermediate good producer is:

$$\pi_{\kappa_j} = \left(\frac{1}{\alpha}-1\right)X(\kappa_j) = \left(\frac{1-\alpha}{\alpha}\right)\alpha^{2/(1-\alpha)}q^{\kappa_j\alpha/(1-\alpha)}. \tag{34}$$

We assume (28) again, so that investments by the firms is given by (29) in the Nash case and (30) in the Stackelberg case, while the equilibrium number of entrants can be obtained residually from the free entry conditions. To guarantee that the relative amount of resources used in R&D does not follow a trend, we need to assume that later innovations are more difficult to discover and that the fixed cost of entry increases between subsequent patent races. In particular I will assume that:

$$\phi_{\kappa_j} = \alpha^{\frac{-2}{(1-\alpha)}}q^{\frac{-(\kappa_j+1)\alpha}{(1-\alpha)}}.$$

Moreover, the fixed cost can be thought of as an expenditure in the construction of prototypes and samples of the next generation intermediates, so it is proportional to their future cost of production. Hence, I will assume that the fixed cost is a constant fraction of the expected cost of production with the new technology:

$$F_{\kappa_j} = \frac{\eta X(\kappa_j+1)}{\left(r + p_{\kappa_j+1}\right)}$$

where $\eta \in (0,(1-\alpha)/\alpha)$. This assumption captures the idea that the larger the scale of expected production of a firm, the larger the fixed costs necessary for the discovery, the development of the associated technology and the infrastructure needed to adopt this technology (think of new assembly lines, new training of workers,...). Our assumptions are consistent with a constant and common arrival rate of innovations in each sector, so that $p_{\kappa_j} = p_{\kappa_j+1} = p$ for any $k_j$ and any $j$.

The model is closed in general equilibrium by assuming a representative agent with utility:

$$U = \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta} \mathrm{e}^{-\rho t} \mathrm{d}t \tag{35}$$

with $\theta$ elasticity of substitution and $\rho$ rate of time preference. Standard arguments – developed in Barro and Sala-i-Martin (1995) – imply that the growth rate of consumption is given by the Euler condition

$$g = \frac{r-\rho}{\theta}$$

where $r$ is the interest rate at which agents and innovating firms borrow and lend, and that the growth rate of output must be the same and given by $g = p[q^{\alpha/(1-\alpha)} - 1]$. We now have all the elements to solve for the general equilibrium values of growth rate, interest rate, arrival rate of innovation, number of firms investing in R&D per sector and the investment of each one. Further details are in the Appendix where I also solve the social planner problem.

In the case of Cournot-Nash competition each entrant invests:

$$z_{\kappa_j}^C = \frac{\epsilon\eta\left(\frac{1-\alpha}{\alpha} - \eta\right)}{\left[1 - \epsilon\left(\frac{1-\alpha}{\alpha} - \eta\right)\right]} X(\kappa_j + 1)$$

which is increasing in the elasticity of revuenue to investment $\epsilon$ and in the effective mark up $(1-\alpha)/\alpha - \eta$. However, one can prove that a social planner would choose $X^*(\kappa_j + 1) = X(\kappa_j + 1)\alpha^{-\frac{1}{1-\alpha}} > X(\kappa_j + 1)$ and an investment in R&D:

$$z_{\kappa_j}^* = \frac{\epsilon\eta}{(1-\epsilon)} X^*(\kappa_j + 1),$$

which is always greater than the equilibrium one. This implies that subsidies to R&D are always optimal in this model – while ambiguity is the typical result in general models of this literature, as in Barro and Sala-i-Martin (1995). However the number of firms may well be more than optimal, implying the necessity of entry fees in the patent race. Indeed, the equilibrium growth rate turns out to be:

$$g^C = \frac{\epsilon^\epsilon\left(\frac{1-\alpha}{\alpha} - \eta\right)^\epsilon \left[\frac{1 - \epsilon\left(\frac{1-\alpha}{\alpha} - \eta\right)}{\eta}\right]^{1-\epsilon} - \rho}{\theta + [q^{\alpha/(1-\alpha)} - 1]^{-1}},$$

while the optimal one is:

$$g^* = \frac{1}{\theta}\left[\epsilon^\epsilon\left(\frac{1-\epsilon}{\eta}\right)^{1-\epsilon} \alpha^{\frac{-\epsilon}{1-\alpha}}\left(1 - q^{-\frac{\alpha}{1-\alpha}}\right)\left(\frac{1-\alpha}{\alpha}\right) - \rho\right],$$

which is greater than the equilibrium if $\theta$ is small enough.

The Stackelberg case is slightly more complicated and provides a system of four equilibrium equations in four unknowns $r^S$, $p^S$, $n^S$ and:

$$g^S = g^S(q, \epsilon, \eta, \alpha, \rho, \theta)$$

without a closed form solution but easy to solve numerically. Despite general comparisons with the optimal solution are not possible, in Etro (2001) I show that also in this case a combination of entry fees and subsidies to R&D can induce the optimal allocation of investment in R&D.

I will summarise my findings by means of a simulation. Since the model is still quite stylised the numbers should be taken *cum grano salis* but I will try to calibrate the parameters in a realistic fashion. In particular, let us assume $\rho = 0.02$, $q = 1.01$, $\alpha = 2/3$, $\epsilon = 0.5$ and, with only the purpose of fitting reasonable growth rates, $\eta = 0.02$. In the realistic case in which $\theta = 2$ the equilibrium growth rate under Nash competition is 5.7% – corresponding to an interest rate of 9.4%, though it would be optimal to grow at 2.6%. The equilibrium number of firms is 36 for each sector, though it would be optimal to have just 3 of them. Nevertheless, each firm in equilibrium invests 9.3% of the optimal per firm flow of investment. Clearly it would be optimal to restrict entry while subsidising investment in R&D. Under Stackelberg competition there are 11 firms investing 28% of the optimal per firm investment, while the incumbent invests 30% of that: the resulting growth rate is 3.3% – corresponding to an interest rate of 4.6%.[23]

We can also calculate a *persistence index PI*, that is the probability that an incumbent will discover the next technology and remain the leader, $PI = h(z^{LS})/p^S$. In our example $PI = 8.4\%$. This index is low because the value of being leader is largely dominated by the simple value of monopolistic profits from the current technology, while the option value of the following discovery (given by the participation in the next patent race) is quite small. A more realistic model should take into consideration technological advantages of innovation for the patentholder, and an important step in this direction has been done by Aghion *et al.* (2001). Our model could be extended in many different directions. Since it is characterised by a more complex and hopefully realistic description of the market for innovations, it would be interesting to study technological diffusion in opening economies with different technological frontiers, issues of international patenting and trade of intermediate goods.

## 3. Conclusion

In this paper I have developed a new rationale for the persistence of monopolies by assuming that an incumbent monopolist has a first mover advantage in the patent

[23] See Etro (2001) for a wider discussion and simulation in a more general model. Notice that Barro and Sala-i-Martin (1995, Ch.7) also obtain a lower growth rate when the incumbent does research alone. Obviously, they just assume persistence of leadership and do not derive it from the structure and technology in the market for innovation. Robert Barro (in personal comunication) and Denicolo' (2001) have independently obtained research only by the incumbent assuming a first mover advantage but maintaining the assumption of constant returns to scale ($\epsilon \to 1$ and $\eta \to 0$ in my notation). It seems that a realistic model should predict that incumbents do some of the research but not all of it. This is possible only if there are decreasing returns to scale.

race for the next innovation and there is free entry for the outsiders. I have used this result for different applications. An important consequence I have shown is that some of the implications of endogenous growth models with innovation are changed if we take into account decreasing returns to scale and endogenous persistence of monopolistic positions. Even if these results are suggestive, much further theoretical and empirical research is needed to understand the delicate implications of patent race equilibria in single industries and in the global economy.

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